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Multibeam Line Finding Problem Based on Geometric Analysis and Genetic Algorithm

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Abstract: In response to the problem of multi beam survey lines, based on the working principle of multi beam survey lines, this article first analyzes the coverage width and overlap rate of adjacent survey lines by calculating geometric knowledge, and establishes a model for coverage width and overlap rate of adjacent survey lines at a slope of. The results show that in a fixed seabed with a depth of west and shallow east, the distance between the survey line and the center point is inversely proportional to the depth of seawater, coverage width, and overlap rate of adjacent survey lines, and the change is stable. Secondly, genetic algorithm is used to design and plan the optimal path for the entire sea area to be tested.

1. Introduction

With the rapid development of technology, technologies such as seabed topography and channel measurement have also developed efficiently. As a high-tech measurement system, multi beam has good practical measurement results. Water depth measurement has gradually transformed from the earliest manual to intelligent, and has been developed into single beam technology. In recent years, this technology has developed from the single beam point line measurement mode to the current full coverage measurement mode based on multi beam measurement. Compared to previous artificial and single beam measurements, it greatly improves the accuracy and efficiency of measurement. It combines the advantages of previous water depth measurement technologies, including high-precision, high-resolution High efficiency, wide coverage, automated mapping, etc. The multi beam bathymetry system consists of multiple fixed angle forward transducers that emit sound waves underwater, and then obtain a beam perpendicular to the navigation direction through the receiver, thereby detecting underwater terrain data[1]. At present, this technology is widely used in various aspects such as benchmark and plane control, water level control, survey line layout control, etc., providing a basis for transportation industry, maritime surveying and mapping, etc[2].

2. Problem Analysis

The development of multi beam bathymetry technology has driven progress in seabed exploration, water level control, and more. Zhang Dihua[3] extracted and organized the seabed contour lines based on the point cloud data of the multi beam bathymetric system; Lu Dong[4] studied the key technologies of shallow water multi beam sonar and proposed relevant theories based on the development status and actual needs of shallow water multi beam sonar equipment at home and abroad at that time; Scholars such as Zhao Bin[5] used multi beam bathymetry technology to study marine sedimentology, demonstrating the important role of high-precision multi beam data in this field; Li Dagang introduced the principles of traditional multi beam bathymetry systems and implemented wireless communication technology based on this system; Grz ą Dziel A, W ąż[6] In 2018, M elaborated on the development and invention of multi beam bathymetry technology, revolutionizing hydrological surveys and proving to be an effective means of hydrological and ocean surveys; Guan Xiaohan[7] discussed the advantages of multi beam bathymetry technology and new ideas for ocean surveying in his book "Advantages of multi beam bathymetry technology and new ideas for ocean surveying", providing new ideas for ocean surveying; Yan Hongzhe[8] proposed during his research on the key technologies of multi beam bathymetry systems that whether multi beam bathymetry can accurately measure the seabed depends on his core technology, the signal processing system; Multi beam bathymetry technology plays an important role in ocean and waterway surveys, so it is necessary to establish more effective models to provide a theoretical basis for the improvement of technology and promote its further development.

3. Modeling and solving

3.1 Analysis and Solution of Question 1

3.1.1 Model theoretical preparation

The multi beam measurement system measures water depth through linear sound waves. During this process, sound waves propagate at a uniform speed in the water. A sound wave transmitter is used to emit a certain width of covering sound waves underwater, and the reflection principle is used to receive the reflected sound waves. The time required for this process, combined with the propagation speed of sound waves in seawater, is used to measure the depth of seawater. When the multi beam sounding system is working, multiple survey lines are parallel to each other.

The slope is defined as α, where multiple measuring lines are parallel to each other on the same plane, and its front view is shown in Figure 1. As the measuring line moves from left to right, while the opening angle of the transducer remains unchanged, the depth of seawater decreases. $J-I$ is the overlapping part of adjacent bands, and the $Q-T$ coverage width decreases gradually.

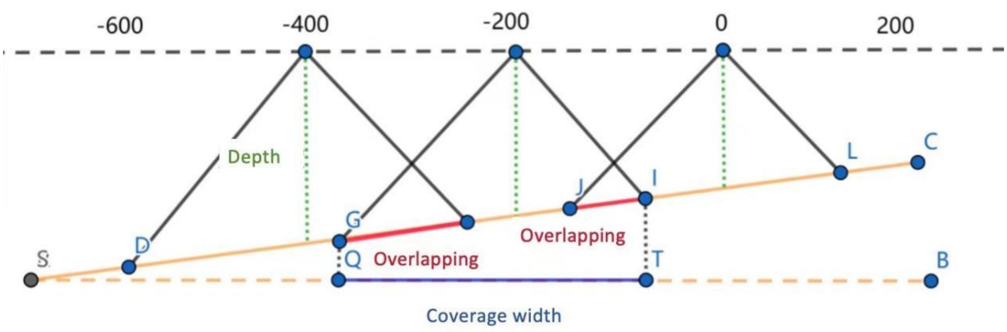


Figure 1: Front view of multi beam bathymetry system operation

3.1.2 Model Establishment

(1)Calculating seawater depth

As shown in Figure 2, given the depth D' of seawater at the center point of the sea area, due to the presence of slope α, the depth of seawater changes as the measuring lines move parallel, denoted as D'. The distance between adjacent measuring lines is d, and the distance from the center point is d. If there is $\angle MFJ = \alpha$, the model for the depth D' of seawater at non center points of the sea area is:

$$D' = D - d' \cdot \tan\alpha \tag{1}$$

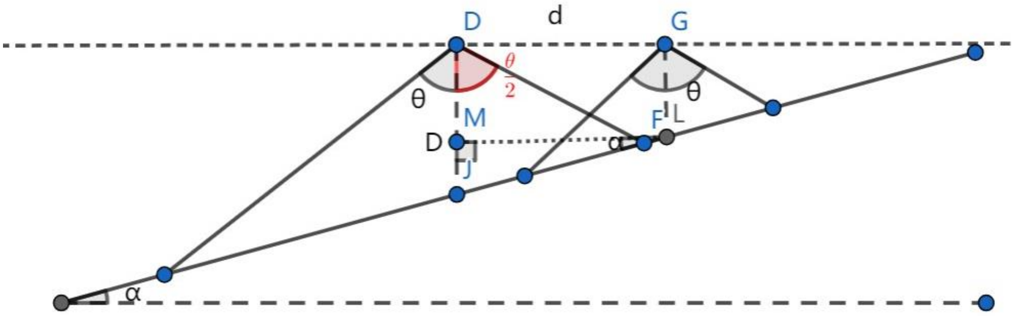


Figure 2: Chart of changes in seawater depth

(2)Coverage width and overlap rate

When the slope is α, multiple beams emit hundreds of beams in a plane perpendicular to the track to measure a fully covered water depth strip with a certain width centered on the ship's survey line. The coverage width is defined as: the plane formed by the emitted multiple beams intersects with the seabed slope to form two intersecting lines. The projection distance between the intersecting lines on the seabed plane is the coverage width, as shown in Figure 3, where $H_3 - W_1$ is a coverage width.

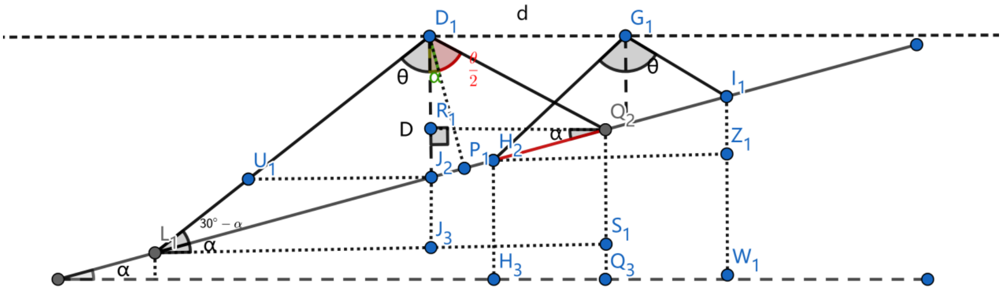


Figure 3: Schematic diagram of coverage width

Based on the above definition of coverage width, for the coverage surface with the first ship survey line as the axis in the above figure 3, its projections on the seabed plane are two rays with L_1 、 Q_2 as endpoints, respectively. Therefore, the coverage surface width is the projection L_1S_1 of L_1Q_2, and the relationship equation is:

$$L_1S_1 = (L_1J_2 + J_2Q_2) \cdot \cos\alpha \tag{2}$$

According to formula (1), it is known that:

$$L_1J_2 = \frac{D\cos\alpha}{\cos(\frac{\pi}{2} - \frac{\theta}{2} - \alpha)} \tag{3}$$

Similarly, it can be concluded that

$$J_2Q_2 = \frac{D\cos\alpha}{\cos(\frac{\theta}{2}+\alpha)} \tag{4}$$

The coverage width model obtained is:

$$W = (\frac{\cos\frac{\theta}{4}}{\cos(\frac{\theta}{2}-\alpha)} + \frac{\cos\frac{\theta}{4}}{\cos(\frac{\theta}{2}+\alpha)}) \cdot D^{'} \cdot \cos\alpha \tag{5}$$

If the survey lines are parallel to each other and the seabed terrain is flat, the overlap rate between adjacent strips is:

$$\eta = 1 - \frac{d}{W} = \frac{W-d}{W} \tag{6}$$

In this problem, due to the presence of slopes, the coverage width is different, that is, W takes $w_1, w_2, ..., w_n$ and the values are not equal. According to the meaning of formula (6), the overlap rate between adjacent bands is the ratio of the overlap part to the coverage width. Based on the geometric relationship, the overlap rate model is:

$$\eta = \frac{1}{2} \cdot (\frac{w_i - d}{w_{i+1}}) \tag{7}$$

Among them $i = 1,2,..., n$, n is the number of measuring lines.

3.1.3 Model solving

The opening angle θ of the transducer is 120°, the slope is 1.5°, and the depth D of the seawater at the center point of the sea area is 70m. From the given distance value from the center point, it can be seen that the distance d between adjacent measuring lines is 200m. The above values are brought into the model (1), (5), and (7) to obtain the following formula:

$$\begin{cases} D^{'} = 70 - d^{'} \cdot \tan 1.5^{\circ} \\ W = (\frac{\cos 30^{\circ}}{\cos(60^{\circ}-\alpha)} + \frac{\cos 30^{\circ}}{\cos(60^{\circ}+\alpha)}) \cdot D^{'} \cdot \cos\alpha \\ \eta = \frac{1}{2} \cdot (\frac{w_i - 200}{w_{i+1}}) \end{cases} \tag{8}$$

The final results obtained through Python software are shown in Table 1, with three decimal places reserved for the obtained results.

Table 1: Calculation Results of Question 1

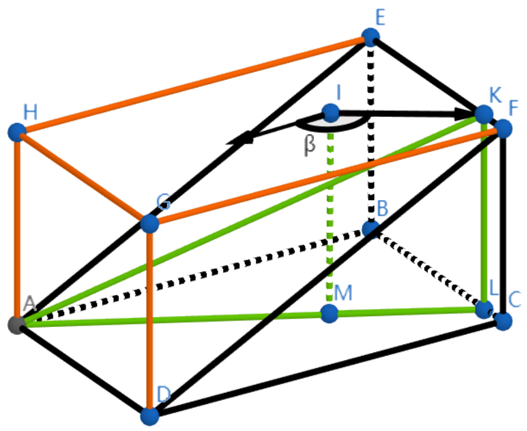
distance/m	-800	-600	-400	-200	0	200	400	600	800
depth of seawater/m	90.949	85.712	80.474	75.237	70.000	64.763	59.526	54.288	49.051
coverage width/m	315.705	297.526	279.346	261.166	242.987	224.807	206.628	188.448	170.269
Overlap rate with the previous line/%	---	19.445	17.456	15.191	12.586	9.561	6.003	1.759	-3.392

When the distance between the measuring line and the center point is 800m, the overlap rate is -3.3922%, that is, n<0. At this point, the measuring line does not overlap with its adjacent measuring line, resulting in missing measurements, which is a blind spot for detection and leads to a decrease in measurement quality. When the distance between the measuring line and the center point is -600m, -400m, -200mm, or 0m, the overlap rate is between 10% and 20%, which can effectively ensure the convenience of measurement and the practicality of the data. However, when $\eta < 10\%$, the overlap rate is small, which cannot ensure the accuracy of the data; At $\eta > 20\%$, excessive overlap results in reduced measurement efficiency and complex data, which increases workload.

3.2 Analysis and Solution of Question 2

3.2.1 Model Establishment and Solution

In question 2, considering the situation where the exploration ship is traveling in different survey line directions, i.e. when the angle β between the survey line direction and the normal direction of the seabed slope projected on the horizontal plane changes, the intersection line of the plane parallel to the survey line direction and the seabed slope forms a diagonal line with an angle α' with the horizontal plane, and the slope is recorded as α'. The line segment IK is perpendicular to the horizontal plane, and obtain the section AKL with the slope surface. The slope between this section and the horizontal plane is α', as shown in Figure 4.



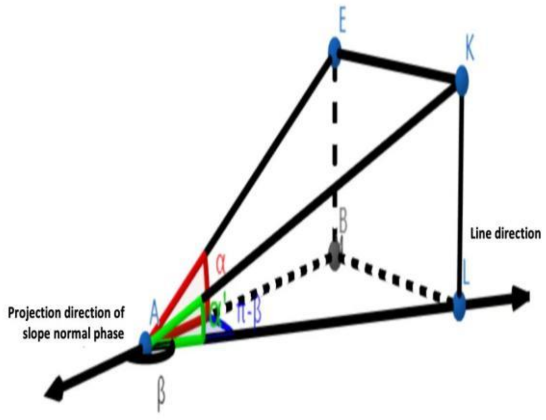


Figure 4: Schematic diagram of the model when the direction of the measuring line changes.

Figure 5: Model after Line Direction Change

3.2.2 Slope model after changing the direction of the survey line

The geometric model obtained after changing the direction of the measuring line is shown in Figure 5. Assuming the slope length AE is x and the slope height h=EB=KL, based on the trigonometric function relationship and the slope $\alpha = 1.5°$ given in the question, the following three equations can be established:

$$\begin{cases} KL = EB = x \cdot \sin\alpha \\ AL = \dfrac{AB}{\cos(\pi - \beta)} = \dfrac{x \cdot \cos\alpha}{\cos(\pi - \beta)} \\ \tan\alpha' = \dfrac{KL}{AL} \end{cases} \tag{9}$$

Compile and calculate the available data

$$\tan\alpha' = \cos(\pi - \beta) \cdot \tan\alpha \tag{10}$$

From this, it can be inferred that

$$\alpha' = \arctan(\cos(\pi - \beta) \cdot \tan\alpha) \tag{11}$$

It can be seen that there is a certain functional relationship between α' and β changes, which can be recorded as $\alpha' = f(\beta)$. When the detection ship is traveling in different survey line directions, that is, when β changes, the slope α' changes.

3.2.3 Sea water depth model

Based on the known information provided in the question, it can be seen that the depth of seawater at the center point of the sea area is D=120 m. As shown in Figure 6, from left to right along the measuring line direction, the distance between the measuring vessel and the center point of the sea area gradually increases, and the depth of seawater gradually decreases. The distance at 0 is the center point of the sea area.

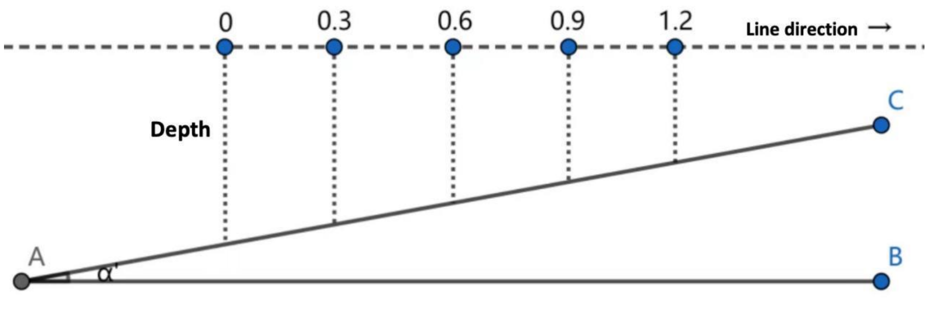


Figure 6: Plan view of the plane parallel to the measuring line direction

The distance between the measuring vessel and the center point of the sea area is expressed as b. Based on 1 nautical mile=1852 meters, the distance b is first converted into units. Secondly, based on the seawater depth model in question 1, the seawater depth models of the measuring vessel in different survey line directions can be obtained as follows:

$$Dep = D - b \cdot \tan\alpha' \tag{12}$$

Substituting formula (10) into can obtain

$$Dep = D - b \cdot \cos(\pi - \beta) \cdot \tan\alpha \tag{13}$$

$$Dep = D - b \cdot \cos(\pi - \beta) \cdot \tan\alpha = D + b \cdot \cos\beta \cdot \tan\alpha \tag{14}$$

$$Dep = D - b \cdot \cos(\pi - \frac{\pi}{2}) \cdot \tan\alpha \tag{15}$$

3.2.4 Coverage width model

Based on the known information given in question 1 and the coverage width model established in question 1, where the opening angle $\theta = 120°$ of the multi beam transducer is 120, the depth and slope of seawater can be obtained using formulas (11) and (12). The following coverage width model can be established:

$$Wid = \left(\frac{\cos\frac{\theta}{4}}{\cos(\frac{\theta}{2} - \alpha')} + \frac{\cos\frac{\theta}{4}}{\cos(\frac{\theta}{2} + \alpha')}\right) \cdot Dep \cdot \cos\alpha' \tag{16}$$

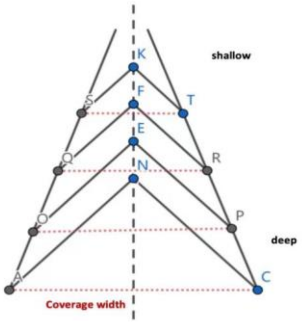


Figure 7: Coverage Width at 0 °and 180 °

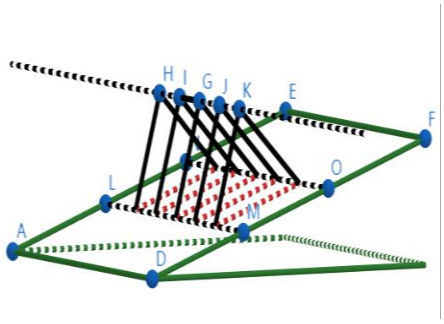


Figure 8: Coverage Width at 90 °and 270 °

Among them, the coverage width of $\beta = 0°$ and $\beta = 180°$ is shown in Figure 7. When $\beta = 0°$, the direction of the measurement ship is from the shallowest to the deepest point of the sea, and the distance from the center point of the sea gradually increases. The depth of the sea gradually increases, and the coverage width gradually increases; On the contrary, when $\beta = 180°$, the direction of the measuring vessel is from the deepest to the shallowest part of the sea, and the distance from the center point of the sea gradually increases. The depth of the sea gradually decreases, and its coverage width gradually decreases. In this case, it is equivalent to calculating the coverage width on a horizontal plane. According to Figure 8, the calculation formula is:

$$Wid = 2 \cdot Dep \cdot \tan\frac{\theta}{2} \tag{17}$$

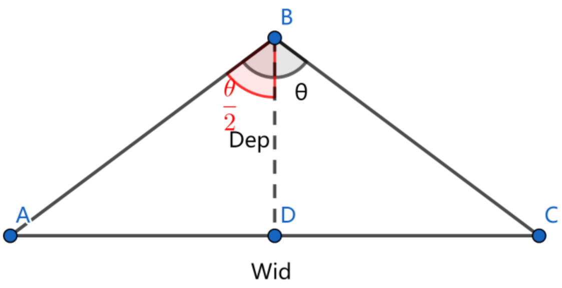


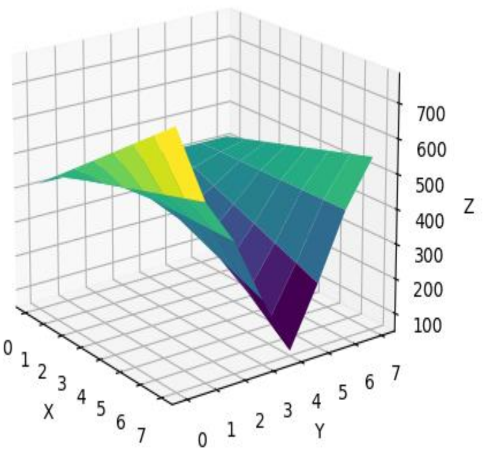
Figure 9: Geometric Calculation of Coverage Width on a Horizontal Plane

The case of $\beta = 90°$ and $\beta = 270°$ is problem 1, and its coverage width is shown in Figure 9. In both cases, the coverage width is the same in the direction of the measurement ship, both of which are the distance between the parallel lines LM and NO, and is calculated using formula (5).

3.2.5 Model Solution and Results

Table 2: Calculation Results of Question 2

Coverage width/m		Measure the distance between the ship and the center point of the sea area							
		0	0.3	0.6	0.9	1.2	1.5	1.8	2.1
Angle between measuring line directions/°	0	416.542	466.847	517.151	567.455	617.760	668.064	718.369	768.673
	45	415.808	434.325	452.843	471.360	489.878	508.396	526.913	545.431
	90	416.000	416.000	416.000	416.000	416.000	416.000	416.000	416.000
	135	415.928	389.438	362.947	336.456	309.966	283.475	256.984	230.494
	180	416.549	366.046	315.544	265.041	214.538	164.035	113.533	63.030
	225	415.928	389.438	362.947	336.456	309.966	283.475	256.984	230.494
	270	416.000	416.000	416.000	416.000	416.000	416.000	416.000	416.000
	315	415.808	434.325	452.843	471.360	489.878	508.397	526.913	545.431



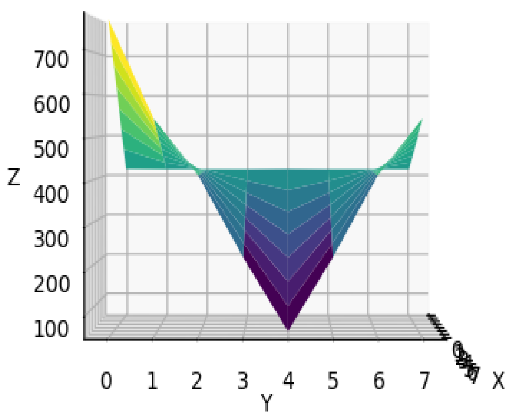


Figure 10: Schematic diagram of three-dimensional results Figure 11: Relationship between Line Direction and Coverage Width

As shown in Figure 10, at each distance between the measuring vessel and the center point of the sea area, the coverage width and the angle degree in the direction of the survey line show a symmetrical distribution with an axis of 180 °, with a distribution trend of 45 °to 315 °. The coverage width first decreases and then increases, and the coverage width is the smallest at the angle degree in the direction of the survey line at 180 °. And the farther the measuring vessel is from the center point of the sea area, the greater the influence of the angle between the measuring line direction and the coverage width, as shown in Table 2 and Figure 11.

3.3 Analysis and Solution of Problem Three

3.3.1 Model Establishment and Solution

According to the wiring principles of relevant literature and the measurement characteristics of multiple beams, this article only considers the situation where the exploration ship is traveling in the direction of the survey line parallel to the isobath, that is, the angle between the survey line direction and the normal direction of the seabed slope projection on the horizontal plane is $\beta = 90°$ and $\beta = 270°$. Therefore, calculating the shortest measurement path is the number of times to calculate the direction path of the measurement line parallel to the isobath. With the center point of the sea area as the initial point, the length of the measurement path is divided into two parts: from the center point to the shallow water area and from the center point to the deep water area. Multiple genetic algorithms are used to calculate the distance between the measurement lines, and finally an optimal path sequence is found, making the measurement length the shortest and fully covering the entire sea area to be measured.

3.3.2 Basic principles of genetic algorithms

The essence of genetic algorithm is to simulate the process of biological evolution, conduct continuous random search in space, continuously generate new solutions during the search process, and retain better solutions. When using genetic algorithms to solve optimization problems, the first step is to encode each solution in the solution space to determine the initial population. Then, based on the size of the fitness function value, chromosomes are selected, crossed, mutated, and combined to select a certain proportion of individuals from all current chromosomes as the next generation population, continuously generating new solutions, and then entering the cycle again.

3.3.3 Population initialization

The generation of the initial population should have a certain degree of diversity to fully explore the search space.

3.3.4 Determination of fitness function

The fitness function, also known as the objective function, describes the corresponding relationship between individuals and their fitness, and is also the key to solving problems in genetic algorithms. In the planning of survey lines in the sea area to be surveyed, the fitness function can be determined by evaluating indicators such as the length of survey lines, coverage, and overlap between adjacent strips.

Taking the point at the depth of D_{i-1} as the center point of the sea area, and a certain number of randomly generated survey lines with spacing d_i as the initial population, based on the coverage width model $W_{i+1} = (w_1 + w_2) \cdot \cos\alpha$ and overlap rate model $\eta = \frac{W_i - d}{2W_{i+1}}$ established in question 1, it can be obtained that

$$w_1 + w_2 = \frac{W_i - d_{i+1}}{2\eta\cos\alpha} \tag{18}$$

When moving from the center point to the shallow water zone, the following four fitness functions can be established:

$$\begin{cases} eq_1 = \dfrac{D_i - D_{i+1}}{\tan\alpha} - d_i \\ eq_2 = \dfrac{W_i - d_i}{\eta\cos\alpha} - (w_1 + w_2) \\ eq_3 = \dfrac{w_1\cos(\frac{\theta}{2} - \alpha)}{\cos\frac{\theta}{4}} - D_i \\ eq_4 = \dfrac{w_1\cos(\frac{\theta}{2} + \alpha)}{\cos\frac{\theta}{4}} - D_i \end{cases} \tag{19}$$

When moving from the center point towards the deep water area, eq_1 and eq_2 should be changed to:

$$\begin{cases} eq_1 = \dfrac{D_{i+1} - D_i}{\tan\alpha} - d_i \\ eq_2 = \dfrac{d_i + 2\cdot\eta\cdot W_{i+1}}{\cos\alpha} - (w_1 + w_2) \end{cases} \tag{20}$$

The objective function is to minimize the absolute sum of the four fitness functions mentioned above, which is

$$\min\ fit = |eq_1| + |eq_2| + |eq_3| + |eq_4| \tag{21}$$

3.3.5 Cycle to obtain optimal solution

According to the requirements of the question, set the overlap rate to 15% for model solving. After iteration, the optimal solution for the distance between seawater depth D_{i-1} and D_i is obtained as d_i. Therefore, multiple genetic algorithms were performed to obtain the optimal solution d_{i+1} for the distance between seawater depth D_i and D_{i+1}, and the optimal solution d_{i+2} for the distance between seawater depth D_{i+1} and D_{i+2}. By analogy, assuming that when n d values are obtained in the direction from the center point to the shallow water zone, full coverage of half of the sea area in this direction can be achieved, the termination condition can be set as:

$$\sum_{i=1}^{n} d_i \geq h \tag{22}$$

Among them, h is half of the east-west width of the sea area. Based on the given information, it can be seen that in a rectangular sea area with a length of 2 nautical miles from north to south and a width of 4 nautical miles from east to west, h=2 nautical miles=3704 meters. The direction from the center point to the deep water area is the same.

4. Model Solution and Results

After iterative solution, the number of d_i in both directions is added to obtain 35 parallel measuring line paths. Therefore, the shortest measuring path L=35 * 2=70 nautical miles=129640 meters. The schematic diagram of its shortest path is shown in Figure 12:

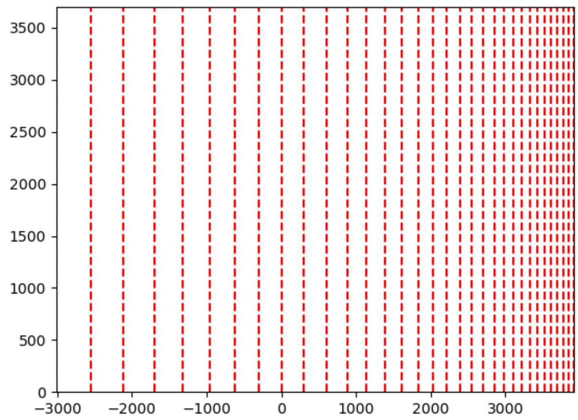


Figure 12: Schematic diagram of the shortest path

According to the schematic diagram of the path in Figure 12 and the specific data in Table 3, the horizontal axis represents the east-west width of the sea area of 4 nautical miles, the path passing through the center point of the sea area at point 0, and the vertical axis represents the north-south length of the sea area of 2 nautical miles. Looking from right to left in the horizontal axis direction, it means that from the shallowest part of the seawater to the deepest part of the seawater, the spacing between adjacent bands gradually increases, the coverage width gradually increases, and the density of the paths increases from dense to sparse, totaling 35 paths. Therefore, the shortest path measured is L=35 * 2=70 nautical miles=129640 meters.

Table 3: Shortest Path Related Indicator Data (Partial)

	depth/m	strip spacing/m	coverage width/m	Distance from center point/m
1	22.115	—	82.139	3978.731
2	23.663	63.246	87.888	3915.484
3	25.319	67.673	94.039	3847.812
4	27.091	72.409	100.621	3775.402
5	28.987	77.477	107.664	3697.925
...	...	...	...	...
31	164.534	384.373	536.198	-1700.680
32	175.255	409.419	571.137	-2110.100
33	186.674	436.097	608.352	-2546.196
34	198.838	464.513	647.992	-3010.709
35	211.794	494.780	690.216	-3505.490

References

[1] Yu Qiyi. Submarine Topographic Survey Based on Multibeam Sounding Technology [J]. Surveying and Spatial

Geographic Information, 2022, 45 (09): 263.
[2] Zhang Chuqi. Application of multi beam bathymetry system in the bathymetry of Beigang, Hainan Port [J]. Journal of Ezhou University, 2022, 29 (05): 104.
[3] Zhang Dihua. Research on seabed contour extraction and organization technology based on multi beam bathymetric data [D]. Southwest Jiaotong University, 2014:1.
[4] Ludong. Research on Key Technologies of Shallow Water Multi beam Sounding Sonar [D]. Harbin Engineering University, 2015:1-3.
[5] Zhao Bin, Liu Shengxuan, Guan Yongxian. Exploration of the Application of Multibeam Sounding Technology in Marine Sedimentology Research: Taking the Ledong Sea Area in the Beibu Gulf of the South China Sea as an Example [J]. Geological Research of the South China Sea, 2015 (00): 54-62.
[6] Grzą Dziel A, Wąż M. The Invention and Development of Multibeam Echosound Technology [J] Polish Hyperbaric Research, 2018, 62 (1).
[7] Guan Xiaohan. Advantages of multi beam bathymetry technology and new ideas for ocean surveying [J]. China Water Transport, 2019 (05): 53-54.
[8] Yan Hongzhe. Research on Key Technologies of Multi beam Sounding System [D]. Zhejiang Ocean University, 2019.