

Study on the Scale Criterion of Radial Scaling on Land Ejection Test

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Abstract: In this thesis, through the application of the dimensional analysis method, we get the scaling criterion of the movement parameters of the cylinder in the land ejection scaling test, and by the application of the equation analysis method, we get the scaling criterion of the gas generator injection into the cylinder. On this basis, a radial scaling model suitable for the land ejection test is established, a scaling scheme is proposed, and the motion parameters of the scaling model are calculated. Compared with the full-size model, the accuracy of the reduced scale model is verified. At the same time, the main factors affecting the accuracy of radial scale tests are analyzed.

1. Introduction

According to the demand of some research work, it is necessary to break through the combined charge gas flow control technology and carry out the scale ejection test and full-scale prototype design. Therefore, While demonstrating the overall scheme, it is necessary to carry out the research on the scaling criterion and facilitate the planning of the test scheme to guide the design and production of the follow-up test products.

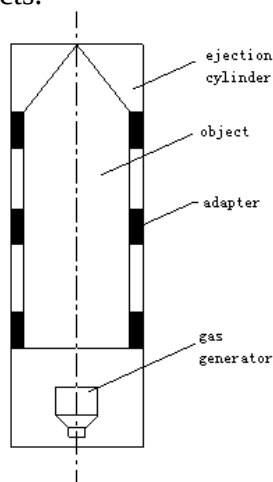


Figure 1 Ejection schematic.

In that physical process, one or more gas generators are pulsed ignited or operate, and working medium gas enters the ejection cylinder to overcome resistance such as the weight of an object, friction force and atmospheric pressure. The object moves in the cylinder and finally exits the cylinder at a certain speed. After exiting the cylinder, the object ignites, as shown in Figure 1. The physical process can be divided into two stages: a) the working medium gas enters the bottom of the ejection cylinder; b) the working medium gas pushes the object in the cylinder until it moves out of the cylinder. If we want to achieve a good corresponding relationship between the scale test system and the full-scale system motion parameters, we need to make the main physical quantities and the full-scale model in these two stages meet a similar relationship according to the test purpose^[1].

The main purpose of the scale test is to verify whether the movement parameters of the full-size model meet the requirements. Therefore, the scaling test shall mainly meet the reduced scale model and the full-size model, which have similar movement parameters out of the barrel.

The scaling model is an incompressible object, whose similarity in the cylinder must satisfy the motion equation. On the other hand, the working medium gas at the bottom of the object is compressible fluid. It should satisfy not only the equation of motion but also the equation of energy and the equation of states.

2. Calculation of movement parameters of the full-size model out of cylinder

In the case of known parameters such as the movement parameters of the full-size model and the weight of the object, the multi-chamber combined flow scheme is adopted to control the ignition of different gas generators, to control the gas flow, make the gas flow match with the object movement during ejection, and ensure the trajectory indexes such as the acceleration in the barrel and the speed out of the barrel of the object meet the requirements. Assuming that the mass of ejected object in the full-scale model is 28t, the diameter is 1.1m, the stroke in the cylinder is 20m, and the ignition time of two pulses is 0s and 0.32s. The calculation results of the main motion parameters of the full-size model are shown in Table 1.

Table 1 Main calculation results of main motion parameters of full-size mode.

Propellant	Composite propellant		
	Upper deviation	Median	Lower deviation
Outlet velocity (m/s)	28.89	26.67	24.28
Maximum acceleration (g)	2.377	2.177	2.124
Peak pressure in the cylinder(MPa)	0.945	0.937	0.933

3. Establishment of a similar model of gas-powered cold ejection

3.1 Similarity model of movement in gas-powered cold ejection cylinder

According to the principle of cold catapult with gas work, the establishment method of a similar model is studied. The key to judging the similarity between the two phenomena is to determine the scaling criterion. There are two main methods for determining the scaling criteria: the equation analysis method based on the similarity theory and the dimensional analysis method based on the π theorem. In this section, the dimensional analysis method based on π theorem is used to establish a similar model of the cylinder motion of a gas-powered cold ejection system^[2].

The movement in the barrel of the object is determined by the following parameters: m_0 -mass of the object, l -characteristic size, ρ -density of the ambient medium, g -gravitational acceleration, p_0 -

characteristic pressure, μ -gas kinematic viscosity, T_0 -initial temperature, J -rotational inertia of the object, k -adiabatic index of working gas, V_c -velocity of motion of the object, t -current time, ξ -energy utilization^[3].

From the 12 values that make up the list, four primary values have independent scales: l , ρ , p_0 , T_0 . According to the similarity theorem, the number of dimensionless parameters is equal to 8. The four variables l , ρ , p_0 , T_0 and the other eight variables are formed into a dimensionless equation in turn. After solving the equations, we can get the scaling criteria in the ejection cylinder of the gas-powered land cold ejection system as follows:

$$\left. \begin{aligned} &a) \frac{g\rho l}{p_0}; b) \frac{\mu}{l} \sqrt{\frac{\rho}{p_0}}; c) \frac{m_0}{\rho l^3}; d) \frac{J}{\rho l^5}; \\ &e) k; f) \frac{p_0}{\rho v^2}; g) \xi; h) \frac{t}{l} \sqrt{\frac{p_0}{\rho}} \end{aligned} \right\} \quad (1)$$

The letter m denotes the scale ratio of the physical quantity of the full-scale model and the scale model. For example, if $m_l = 3$, it indicates that the geometric scale of full-scale mold storage and scale model is 3. By using the scaling criterion condition (1), the scale with relation factors is compared with the principal scale m_{p_0} , m_l , m_{T_0} , m_ρ . Connected:

$$\left. \begin{aligned} &a) m_{p_0} = m_g m_\rho m_l; b) m_\mu = m_{p_0}^{1/2} m_\rho^{-1/2} m_l; \\ &c) m_{m_0} = m_\rho m_l^3; d) m_J = m_\rho m_l^5 \\ &e) m_k = 1; f) m_\xi = 1; \\ &g) m_v = m_{p_0}^{1/2} m_\rho^{-1/2} \\ &h) m_t = m_{p_0}^{-1/2} m_\rho^{1/2} m_l \end{aligned} \right\} \quad (2)$$

In addition, since the medium density is the same, the gravitational acceleration is the same, and the initial temperature is assumed to be the same, the following limitations are added to the numerical scale with independent factors:

$$m_\rho = 1, m_{T_0} = 1, m_g = 1 \quad (3)$$

Value m_l is determined by the size of the scale test device. By bringing (3) into (2), other physical value scales can be obtained:

$$m_{p_0} = m_l, m_\mu = m_l^{3/2}, m_{m_0} = m_l^3, m_J = m_l^5, m_k = 1, m_v = m_l^{1/2}, m_\xi = 1, m_t = m_l^{1/2}$$

Strictly fulfilling the above conditions is very complicated. The complete similarity is not possible when $m_l \neq 1$ ^[4]. And only approximate completion conditions can be formulated. The effect of the time of ejection and the effect of criterion b) on ejection parameters can be ignored.

3.2 Generator mass flow similarity model

Through dimensional analysis, the similitude relation of the cylinder exit motion of the scaling model is obtained. In addition, the similitude relation of the working medium gas injected into the cylinder by the gas generator should be analyzed. In the physical process of the gas generator injecting

working gas into the cylinder, the parameters of the propellant gas are the same as those of the full-scale model gas. In this process, the most important are the gas generator's operating time t_c and the mass flow $\dot{G}$ [5].

In order to ensure that the pressure p_l at the bottom of the ejection cylinder is similar, it shall be satisfied that the reduced scale model is the same as the $\frac{\dot{G}t_c}{\rho_l V_l}$ of the full-size model. Herein, t_c is the gas generator's operating time, ρ_l is the density of the working medium gas at the bottom of the cylinder, and V_l is the volume of the bottom of the cylinder.

$$\frac{\dot{G}_H t_{cH}}{\rho_{tH} V_{tH}} = \frac{\dot{G}_M t_{cM}}{\rho_{tM} V_{tM}} \quad (4)$$

The full-size model is marked with H, and the scale model is marked with M, yielding:

$$m_{\dot{M}} = \frac{\dot{G}_H}{\dot{G}_M} = \frac{t_{cM} \rho_{tH} V_{tH}}{t_{cH} \rho_{tM} V_{tM}} = m_l^{3.5} \quad (5)$$

In addition, the mass flow rate of the working medium gas injected into the cylinder by the gas generator is:

$$\dot{G} = \frac{P_c \sqrt{k_c}}{\sqrt{R_c T_c}} A_c^* \left(\frac{2}{k_c + 1} \right)^{\frac{k_c + 1}{2(k_c - 1)}} \quad (6)$$

Here, p_c is the internal pressure of the gas generator, and A_c is the throat diameter of the gas generator. The burning rate will change under different p_c , and there is no certain similar relationship. Therefore, in order to ensure that the main parameters of gas generator working time t_c and mass flow $\dot{G}$ are similar in the design of the gas generator scale model, $P_{cH} = P_{cM}$ is taken. There are $m_{A_c} = m_l^{3.5}$.

3.3 Comparison of similarity between objects in gas-powered cold projectiles

Table 2 Scale relationship table.

Project	Linear scale m_l	Pressure scale m_{p_0}	Speed scale m_{v_c}
Scaled scale	As determined by the test setup	$m_{p_0} = m_l$	$m_{v_c} = m_l^{1/2}$
Project	Acceleration scale m_v	Time scale m_t	Mass scale m_{m_0}
Scaled scale	$m_v = 1$	$m_t = m_l^{1/2}$	$m_{m_0} = m_l^3$
Project	Force scale m_F	Gas generator mass scale flow $m_{\dot{G}}$	Gas generator throat area scale m_A
Scaled scale	$m_F = m_l^3$	$m_{\dot{G}} = m_l^{3.5}$	$m_A = m_l^{3.5}$

In the above section, we analyzed the similarity relationships of the main parameters and listed the

main similarity relationships as shown in Table 2.

Table 2 shows a similar relationship during the test. This scale model is called the full-scale model in this report, which meets a similar relationship in terms of geometry, pressure, speed, etc. In the whole scaling process, the atmospheric pressure at the ejection barrel mouth also needs to be scaled, but the outdoor conditions of the shooting range can not meet similar conditions of atmospheric pressure, so we need to find another way to solve this problem.

4. Establishment and evolution of radial scale model

Some non-critical parameters should be abandoned, and further analysis should be carried out around the target of scaling when the scaling test is carried out under atmospheric pressure without affecting the ballistic performance scaling. Since the atmospheric pressure cannot be changed, it is necessary to realize that the pressure scaling coefficient between the full-size and scaling model is 1, that is, under the premise of $m_{p_0} = 1$. $m_{p_0} = 1$, $m_l = 1$ is considered on the basis of the scaling principle in Table 2; other scaling factors are 1. This obviously does not achieve the purpose of the scaling test.

Table 3 Relation table of the radial scaling model.

Project	Ejection direction geometric scale m_L	Pressure scale m_{p_0}	Speed scale m_{v_c}
Model evolution process	-	$m_{p_0} = m_L$	$m_{v_c} = m_L^{1/2}$
Radial scaling model	$m_L = 1$	$m_{p_0} = 1$	$m_{v_c} = 1$
Project	Acceleration scale m_v	Time scale m_t	Mass scale m_{m_0}
Model evolution process	$m_v = 1$	$m_t = m_L^{1/2}$	$m_{m_0} = m_L m_R^2$
Radial scaling model	$m_v = 1$	$m_t = 1$	$m_{m_0} = m_R^2$
Project	Force scale m_F	Gas generator mass flow scale m_G	Gas generator throat area scale m_A
Model evolution process	$m_F = m_L m_R^2$	$m_G = m_L^{3/2} m_R^2$	$m_A = m_L^{3/2} m_R^2$
Radial scaling model	$m_F = m_R^2$	$m_G = m_R^2$	$m_A = m_R^2$

We assume that under the radial scaling model, m_L is the geometric scale in the ejection axis direction, and m_R is the geometric scale in the diameter direction of the ejected cylindrical object. So, we take the linear scale m_l in the previous section and split it into the above two geometric dimensions. The ejection process of the object is a one-dimensional motion, and the equation of motion of the ejection process is one-dimensional along the ejection axis. The scaling scale of parameters such as pressure, velocity, and time involved in the equation is mainly related to m_L . Mass, force, gas generator mass flow and throat diameter area are all related m_L and m_R , and the scaling relation expression is split according to the expression in Table 2, and is shown in the second column of Table 3 after splitting.

If $m_{p_0} = 1$ is satisfied, $m_L = 1$ is required. After the evolution of m_l , the geometry scale of the

ejection axis $m_L = 1$ can be realized. m_R values can be taken according to the actual situation, and scale test devices can be carried out accordingly. The evolved radial scaling model is shown in column 3 of Table 3. Else, the assumption condition of the evolution process of the radial scale model is consistent with that of the full-scale model before evolution, which is $m_p = 1, m_{t_0} = 1, m_g = 1, m_\xi = 1, Re = 1$.

From Table 3, it can be seen that the radial scale model has the same axial scale as the full-size model, and only the radial scale is reduced. When the radial scale model and the full-size model use the same charge, all the movement parameters measured by the radial scale model will be equal to the movement parameters of the full-size model. That is, the ratio coefficient of the two is 1:1. It can be seen that the motion parameters measured by such a model intuitively reflect the performance of the prototype. Because of the same working time and internal pressure of the gas generator, the grain used in the model test can be the same as that used in the full-size test, which dramatically simplifies the charge manufacturing process.

Because $m_L = 1$, then the length of the ejection cylinder and the ejected object of the scaled model are the same as those of the full-size model. Under the condition of the same relatively long full-size models, the cost of manufacturing scaled models with the same length is relatively large. In order to reduce the test difficulty and expenditure, the length of the ejection cylinder and the ejected object in the radial scaling model can be reduced on the premise of meeting the main key performance indexes of verification.

The scale test can be based on the actual conditions of the exiting test device, mainly considering that the stroke can cover all the ignition requirements of the gas generator and simulate partial trajectory under the condition of maximum overload^[6].

5. Analysis of factors affecting the accuracy of radial scale test

The main factors affecting the accuracy of the scale test are: a) there is a deviation between the energy utilization ratio after scale and the full size^[7]; b) the test may be slightly tilted to ensure safety; c) resistance change arises from slenderness ratio change due to radial shrinkage ratio and axial on-shrinkage ratio.

In this model, the energy utilization coefficient of the full-size model is the same as that of the scaled model, but the proper energy utilization coefficient shall be selected according to the actual scaled model test. This is because the scaled model in the experiment mainly considers a similar relationship after the energy of gunpowder is converted into kinetic energy. According to the previous experimental experience, when m_L is between 3 and 5, the energy utilization coefficient of the scaled model will be about 0.02-0.04 smaller.

In the scale test, since that object does not fly, after ejection from the barrel, to ensure that the object does not hit the launcher when falling back, it is necessary to carry out small angle oblique ejection. In the case of oblique launch, the resistance of the object will change (mainly gravity and friction resistance of the adapter), and the movement parameter of the scaled model out of the cylinder will produce non-essential deviation from the full-size model under the condition that the bottom pressure of the cylinder remains unchanged. In that design of the main parameter of the adapter, the design calculation should be carried out with the parameters of inclination along the direction of the cylinder, and the weight of the scaled model should be calculated and corrected along the direction of the launching cylinder. It aims to make sure that the force state along the ejection direction of the object is consistent with the full-size model. Taking the second section model $m_R = 4$ and oblique angle 10° for example, the velocity error is less than 0.2% when compared with the vertical oblique ejection after calculation and correction.

The slenderness ratio of the object is changed by setting different scale coefficients in the diameter direction and the stroke direction of the object. The slenderness ratio has an effect on the fluid resistance term, but the effect is small in air and at low velocity. In the case of the scale test and the simulation of partial stroke, the length of the scale model object to be ejected will be shortened by utilizing the existing conditions. In this case, the slenderness ratio of the full-size model and the reduced model will be different. Taking the section 2 model $m_r = 4$, the cylinder stroke of the scaled model is 1/5 of the full size, the slenderness ratio of the full size is 12, the slenderness ratio of the scaled model is 27, the resistance coefficients are about 0.83 and 0.92, respectively, and the maximum resistance difference is less than 30N, which is negligible compared with gravity and friction.

6. Theoretical verification of scale parameters of test protocol

We scale down the full-scale model mentioned in section 2. Two scaling schemes are provided, as shown in Table 4. The in-cylinder stroke simulates partial stroke under the condition of covering the maximum in-cylinder pressure, maximum acceleration, and other key parameters. In addition, the scaled gas generator scheme is the same as the full-size solution, using the same propellants and types as the full-size solution.

Table 4 Comparison of two scaling test schemes.

	Full-scale model	Scaling model scheme 1 $m_r = 4$	Scaling model scheme 2 $m_r = 2$
In-cylinder stroke L (m)	20	10 (1/2 stroke)	8 (1/2.5 stroke)
Mass m (t)	28	1.75	7
Body diameter R (m)	1.7	0.375	0.75
Body Speed V (m/s)	V	V	V
Body Acceleration $\dot{V}$ (m/s^2)	$\dot{V}$	$\dot{V}$	$\dot{V}$
Gas generator mass $\dot{G}$ (kg/s)	$\dot{G}$	$\dot{G}$	$\dot{G}$
Pulse ignition sequence	0,0.32	0,0.32	0,0.32

In consideration of economic cost, site and other factors, scheme 1 is selected to carry out relevant tests. Table 5 shows the comparison between the ballistic index calculation of Scheme 1 and the full-size model.

It can be seen from Table 5 that all indexes of the ejection power system in the scale test are basically consistent with the full-size scheme.

Table 5 Comparison of the reduced ballistic index and median value of the full-size model.

	Full-size	Scale model	Scale number
When the scaling model is out of cylinder speed (m/s)	15.84 (same distance in cylinder movement)	15.85	About 1
Maximum acceleration (g)	2.277	2.216	About 1
Peak pressure in the cylinder (MPa)	0.939	0.914	About 1

7. Conclusion

Model tests based on similarity theory have always been an important means to solve new or complicated problems. In this paper, the scale criterion of the land ejection scale test is obtained using a dimensional analysis method. By means of equation analysis, the shrinkage test criterion of the gas generator injection cylinder is obtained. On this basis, the full-scale model and radial scale model are established. Through analysis and comparison, the radial scale model is chosen as the scale model of the land test, and the movement parameters of the test scheme are calculated, which verifies the correctness of the scale model. In this paper, the multi-chamber flow combination scheme is taken as an example to analyze the power system, and there is no essential difference between it and single-chamber single-flow and single-chamber double-flow in the scaling relation. The scale test criterion can be applied to the scale test research of missiles, rockets and other land launches, which can obtain relatively accurate full-size model dynamic parameters at low cost. It seeks to assess the feasibility of new technologies in the launch stage.

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