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Teaching Design for the Concept of Trigonometric Functions from the Perspective of Mathematical Modeling Core Competency

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Abstract: This article is based on the core competencies of mathematical modeling and SOLO classification theory to design a teaching plan for the high school lesson on 'The Concept of Trigonometric Functions'. Aimed at the cognitive development process of students from intuitive understanding to abstract comprehension, it uses the motion of a Ferris wheel as a real-world context to guide students in establishing geometric models in both Cartesian and polar coordinates, exploring the generalization of trigonometric function definitions from acute angles to any angles. Through dynamic software demonstrations, guided question chains, and tiered exercises, it helps students understand the essence of trigonometric functions as mathematical models that characterize periodic phenomena, achieving a cognitive leap from structural understanding to relational understanding, thus promoting the unity of knowledge construction and competency development.

1. Introduction

Mathematical modeling is one of the six core mathematical competencies. It requires students to establish connections between the real world and the mathematical world, using mathematical language to describe and solve practical problems. As an important component of mathematical core literacy, mathematical modeling literacy can help students comprehensively apply the knowledge they have learned to solve problems, and promote the development of comprehensive qualities such as logical thinking, innovation, and practical abilities [1]. Cultivating this competency is not achieved overnight; it requires teachers to meticulously design teaching segments, allowing students to personally experience and actively construct knowledge through the process of "doing mathematics." The Structure of the Observed Learning Outcome (SOLO) taxonomy, proposed by Biggs and Collis, is a model for evaluating students' cognitive development levels. It classifies students' understanding of a knowledge point into five hierarchical levels: pre-structural, unistructural, multistructural, relational, and extended abstract. This theory can not only be used for learning assessment but also inversely guide the design of teaching objectives and the organization of teaching activities, ensuring targeted instruction that promotes the deepening of students' thinking levels.

The "concept of trigonometric functions" holds a unique position within the high school function system. Its definition breaks away from the familiar "variable-correspondence" model students are accustomed to, shifting towards a construction method based on geometric ratios. This shift constitutes a significant cognitive leap for students and also provides an opportunity to integrate mathematical modeling thinking. Traditional teaching often directly presents the unit circle definition, supplemented by extensive computational practice, leading to students being able to perform mechanical operations but struggling to understand the origin and necessity of the concepts, let alone grasp their substantive meaning and powerful functionality as mathematical models. Therefore, integrating the SOLO taxonomy with mathematical modeling thinking to design the teaching of the "concept of trigonometric functions" aims to enable students to achieve a spiral rise in both knowledge understanding and thinking levels simultaneously during the process of constructing mathematical models.

2. Textbook Analysis

This teaching design is selected from the Standard High School Mathematics Textbook-Compulsory Book 1 (People's Education Press, 2019 Edition A). The "concept of trigonometric functions" is located in the first section of Chapter 6, "Trigonometric Functions," serving as the starting point for students' systematic learning of trigonometric functions in high school [2]. Its positioning aligns highly with the progressive demand for thinking levels in the SOLO taxonomy. The textbook content starts by reviewing acute angle trigonometric functions from junior high school, gradually expanding to the definition of trigonometric functions for any angle in the rectangular coordinate system, and then abstracting the coordinate representation on the unit circle. This organizational logic clearly reflects the hierarchy of thinking. It corresponds precisely to the cognitive development path of the SOLO theory: from unistructural (grasping single knowledge points), to multistructural (connecting multiple elements), and then to relational (understanding internal connections between knowledge points). This provides students with a step-by-step thinking ladder, facilitating the cognitive leap from concrete operation to abstract understanding.

3. Teaching Objectives

Based on students' cognitive levels, the requirements of the new curriculum standards, and the SOLO taxonomy [3], the teaching objectives for this lesson are formulated as follows:

(1) From Pre-structural to Unistructural: Students can identify the commonality of periodic phenomena from life experiences, preliminarily establish the mathematical concept of "periodicity," and form a single cognitive point.

(2) From Unistructural to Multistructural: Students can use the rectangular coordinate system to describe the position of a point, understand polar coordinate representation, and establish connections between different representation methods.

(3) From Multistructural to Relational: Students can derive the relationship between acute angle trigonometric functions and coordinates, generalize this relationship to any angle, and understand the consistency of the definition.

(4) From Relational to Extended Abstract: Students can systematically establish the conceptual system of trigonometric functions, understand the functional relationship where ratios change with the angle, and grasp the advantages of the unit circle definition.

4. Teaching Methods

This lesson's teaching design integrates various teaching methods to promote the progressive development of students' thinking levels.

Visual Demonstration Method: Using GeoGebra dynamic mathematics software to demonstrate the motion process of the terminal side of any angle, visually displaying the correspondence between point P's coordinates and the change in angle α, helping students establish an intuitive connection between the angle and coordinate values.

Inquiry-Based Teaching Method: Guiding students to autonomously explore the formation process of trigonometric function concepts through a progressively structured chain of questions, such as "What is the connection between rectangular and polar coordinates?" and "Can the acute angle relationship be generalized to any angle?"

Practice Consolidation Method: Designing tiered practice problems, such as finding trigonometric function values given a point or determining function values considering the quadrant, to help students solidify their conceptual understanding.

Heuristic Questioning Method: Prompting students to think about the essence of functions and the advantages of the unit circle through key questions like "What is the relationship between the ratios and the changing angle?" and "How can we simplify the definition form?"

Collaborative Discussion Method: Organizing group discussions during the concept formation process to explore the internal connections between different coordinate representations.

These teaching methods work together to form a complete cognitive process from concrete to abstract and from specific to general, aligning with the progressive thinking levels emphasized by the SOLO taxonomy and aiding students in gradually constructing their conceptual system of trigonometric functions.

5. Teaching Process

5.1 Situational Creation and Problem Posing

[Teacher-Student Activities Design]

Activity 1: Teacher asks: "In the real world, many motions and changes have a cyclical nature, like the rotation of a Ferris wheel or the movement of clock hands. What common characteristic do these phenomena share [4]?" Students think and answer: "They all repeat over and over." Teacher further guides: "Excellent. We call this regular pattern of change, like the changing seasons or the phases of the moon, 'periodicity.' To better study this periodic phenomenon in mathematics, what method can we use to describe this pattern of change?" Students discuss amongst themselves.

Figure 1: Ferris wheel

Activity 2: Teacher asks: "As shown in Figure 1, the passenger cabin rotates continuously in cycles. What mathematical model should we use to describe this unique motion of the cabin? Which model can accurately reflect the cabin's true trajectory and pattern in the air? Discuss and answer."

[Design Intent] Introducing the lesson with the real-world context of "Ferris wheel motion" aims to stimulate student interest and guide their transition from the SOLO pre-structural level (mere life impression) to the unistructural level-initially focusing on the core question: "How to describe positional change?" By translating practical problems into mathematical descriptions, students not only enhance their problem analysis and solving abilities but also deeply appreciate the necessity and practical significance of introducing trigonometric functions as new mathematical tools during the modeling process, laying the foundation for subsequent development to higher thinking levels.

5.2 Model Establishment and Function Discovery

[Teacher-Student Activities Design]

Activity 1: Teacher guides: "To describe the dynamic situation of the passenger cabin more accurately, we can abstract the Ferris wheel into a circle (Figure 2), and the cabin into a point P on the circle. The problem becomes: How to describe the motion of point P on the circle?" Student anticipated response: "By establishing a rectangular coordinate system, using the coordinates (x,y) of point P to describe its position."

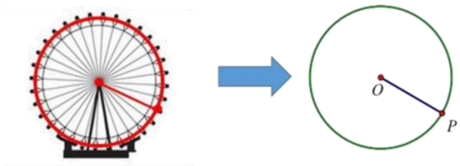

Figure 2: The ferris wheel is abstracted into a circle

Activity 2: Teacher follows up: "How to establish a rectangular coordinate system that fits the problem conditions?" Student anticipated response: "Use the center of the Ferris wheel as the origin (Figure 3). Then the ordered pair (x,y) can accurately describe point P's position." Students, under teacher guidance, establish the coordinate system.

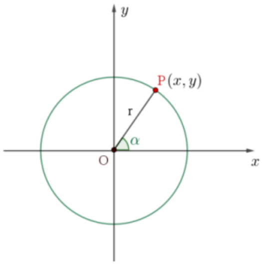

Figure 3: Cartesian coordinate system

[Design Intent] This activity guides students to abstract the practical problem into a mathematical graph and proactively select the coordinate system as a tool for quantitative description. This process embodies the core idea of mathematical modeling. From the perspective of SOLO taxonomy, it promotes the progression of students' thinking from the unistructural level (focusing solely on the Ferris wheel itself) to the multistructural level (connecting multiple elements such as the circle, coordinate system, and point coordinates), thereby laying the foundation for subsequently establishing functional relationships.

Activity 3: Teacher asks: "Besides rectangular coordinates (x,y), can other ordered pairs represent point P's position?" Teacher guides students to think about polar coordinates. Students discuss and answer: "In Figure 3, using the angle α between the positive x-axis and ray OP, the ordered pair (r,α) can describe point P's position." Teacher probes further: "What is the internal connection between the coordinates $P(x,y)$ and $P(r,\alpha)$ obtained from the rectangular and polar coordinate systems?" Further guides thinking: "When point P moves on the circle, angle α can be arbitrary. We should first study under what circumstances α, x, y, r have definite relationships, and then generalize. We first assume that α is an acute angle in our study."

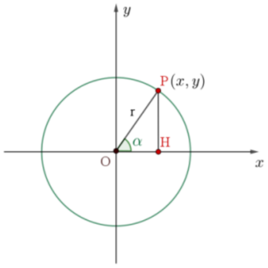

Figure 4: The Cartesian coordinate system describes position

Activity 4: Teacher guides students to think about the relationships between variables x, y, r α when α is acute. Students through coordinate system setup, derive preset relations: $r=\sqrt{x^2+y^2}$, $\sin\alpha=y$, $\cos\alpha=x$, $\tan\alpha=\frac{y}{x}$. Teacher asks how these are derived. Using Figure 4, and the Pythagorean theorem ($OP^2=OH^2+PH^2 \rightarrow r^2=x^2+y^2$), combined with the concept of acute angle trigonometric functions, the teacher guides the derivation. Teacher summarizes: "For acute α, by constructing a right triangle, we can translate the trigonometric ratios into ratios involving the coordinates of P."

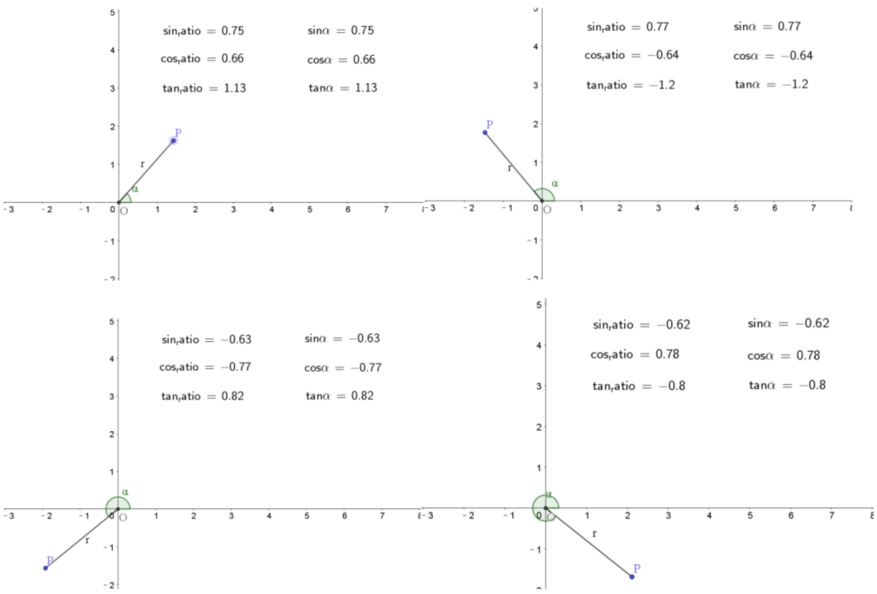

Figure 5: Trigonometric function values of any angle

Activity 5: Teacher questions students: "When angle α is any angle, what is the relationship between α and x, y, r? Do the previous formulas still hold?" Students think: Generalizing α to any angle, the relations still hold: $\sin\alpha=\frac{y}{r}$, $\cos\alpha=\frac{x}{r}$, $\tan\alpha=\frac{y}{x}$. Teacher uses GeoGebra software for demonstration (Figure 5) to let students intuitively reach the conclusion.

[Design Intent] This guides students from the SOLO unistructural level (knowing only rectangular coordinates (x, y)) towards the multistructural level by introducing polar coordinates (r,α) and exploring their connection. It helps students establish links between different representation methods. Further, by analyzing the generalization from acute to any angle, it facilitates the transition towards the relational level, helping students understand the consistency of the trigonometric function definitions, thus completing the mathematical concept construction process from concrete to abstract and specific to general.

5.3 Function Construction and Concept Formation

[Teacher-Student Activities Design]

Activity: Teacher poses the problem: "$\frac{y}{r}$, $\frac{x}{r}$, $\frac{y}{x}$ change as α changes. What kind of correspondence is this?" Students recall prior knowledge of the function concept and answer. Teacher then presents the concept of trigonometric functions: "For any angle α, the ratio $\frac{y}{r}$ is called the sine of α, denoted $\sin\alpha=\frac{y}{r}$; the ratio $\frac{x}{r}$ is called the cosine of α, denoted $\cos\alpha=\frac{x}{r}$; the ratio $\frac{y}{x}$ $(x\neq0)$ is called the tangent of α, denoted $\tan\alpha=\frac{y}{x}$ $(\alpha\neq\frac{\pi}{2}+k\pi(k\epsilon Z))$." Teacher follows up: "Can we take a specific point P on the terminal side to simplify $\sin\alpha=\frac{y}{r}$, $\cos\alpha=\frac{x}{r}$, $\tan\alpha=\frac{y}{x}$?" Teacher guides students to think and operate: Take OP = r = 1, take the intersection point P(x,y) of the unit circle and the terminal side(Figure 6).

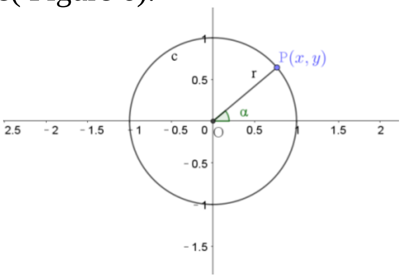

Figure 6: Simplify the values of trigonometric functions

1) The ratio y is called the sine function of α, that is, $\sin\alpha=y$.

2) The ratio x is called the cosine function of α, that is, $\cos\alpha=x$.

3) The ratio $\frac{y}{x}$ $(x\neq0)$ is called the tangent function of α, that is, $\tan\alpha=\frac{y}{x}$ $(\alpha\neq\frac{\pi}{2}+k\pi(k\epsilon Z))$.

[Design Intent] This guides students from the SOLO multistructural level (understanding the correspondence between ratios and the angle) towards the relational level. By analyzing the functional relationship where ratios change with the angle, it helps students systematize the concept of trigonometric functions. Furthermore, through the specialization of the unit circle, students understand the simplifiability and essential unity of the definitions, establishing the functional correspondence between angles and coordinate values, thus completing the abstract construction and internal connection of the mathematical concepts.

5.4 Practice and Consolidation

[Teacher-Student Activities Design] The teacher designs exercises based on student levels and lesson progress.

Exercise 1: The terminal side of angle α passes through point P(-3,4). Students are asked to find the values of sinα, cosα, tanα.

Solution:

$$r=\sqrt{x^2+y^2}=\sqrt{-3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5.$$

$$\sin\alpha=\frac{y}{r}=\frac{4}{5},\ \cos\alpha=\frac{x}{r}=\frac{-3}{5}=-\frac{3}{5},\ \tan\alpha=\frac{y}{x}=\frac{4}{-3}=-\frac{4}{3}.$$

[Design Intent] This exercise aims to help students develop from the unistructural to the multistructural level. Students need to apply the formulas $\sin\alpha=\frac{y}{r}$, $\cos\alpha=\frac{x}{r}$, $\tan\alpha=\frac{y}{x}$, where $r=\sqrt{x^2+y^2}$. Through calculation, students understand the definition of trigonometric functions for any angle and consolidate the relationship between coordinates and trigonometric values.

Exercise 2: If the terminal side of angle α is in the second quadrant and $\sin\alpha=\frac{1}{2}$, find $\cos\alpha$ and $\tan\alpha$.

Solution:

From $\sin\alpha=\frac{y}{r}=\frac{1}{2}$, we can assume y=1, r=2 for a point on the terminal side.

By the Pythagorean Theorem, $x=\pm\sqrt{r^2-x^2}=\pm\sqrt{4-1}=\pm\sqrt{3}$.

Since α is in the second quadrant, $x<0$, $y>0$, so $x=-\sqrt{3}$.

$\cos\alpha=\frac{x}{r}=\frac{-\sqrt{3}}{2}=-\frac{\sqrt{3}}{2}$, $\tan\alpha=\frac{y}{x}=\frac{1}{-\sqrt{3}}=-\frac{\sqrt{3}}{3}$.

[Design Intent] This exercise aims to guide students from the multistructural to the relational level. Students need to determine possible values based on $\sin\alpha=\frac{1}{2}$ and then combine this with the properties of the second quadrant (cosine negative, tangent negative) to arrive at the correct result.

5.5 Review and Summary

[Teacher-Student Activities Design]

The teacher guides students to review the lesson content through questioning and facilitates a summary.

[Design Intent] Students can cultivate the good habit of summarizing during the learning process through self-reflection. This also helps some students organize their knowledge and review or supplement any missed parts.

5.6 Homework Assignment

[Teacher-Student Activities Design]

The teacher assigns tiered homework based on student levels and classroom performance feedback.

[Design Intent] Assigning homework according to students' actual needs allows them to achieve development more suited to their individual levels in different mathematical tasks, respecting individual differences.

6. Teaching Reflection

This lesson successfully stimulated student interest through the Ferris wheel scenario. Through step-by-step guidance, students understood the definition of trigonometric functions from concrete to abstract. During the practice session, students could apply their knowledge to solve problems. However, some students in the classroom understood the generalization to any angle slightly slower. Subsequent teaching should pay more attention to these students, providing more examples to help them digest the concepts.

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Teaching Cases of Secondary School Mathematics Curriculum and Textbook Research Based on the New Curriculum Reform. (2025)

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