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One Question, One Lesson: Hit the Essence, Build a System

-- A Teaching Example of Solving Problems with Abstract Functions

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Abstract: "One problem, one lesson" is a new problem-solving teaching paradigm in the era of mathematical literacy, which has unique value in focusing on mathematical contexts and constructing students' cognitive structures. Through the analysis of lesson cases on abstract function problem-solving instruction, the “one question, one lesson” teaching model is found to play a significant role in uncovering the essence of mathematics and supporting the construction of students’ conceptual knowledge structures. The lesson cases take abstract functions as the core, introduce concepts by reviewing basic elementary functions, and use typical questions and various teaching methods to guide students to master the methods of abstract function problem-solving. The teaching process covers multiple links such as context introduction, example explanation, and summary to help students understand the connection between abstract functions and basic elementary functions, build a knowledge system, and enhance logical reasoning, abstract thinking and other abilities.

1. Teaching objectives

1.1 Knowledge and skills objectives

Students have a thorough understanding of the concept of abstract functions and are proficient in the abstract forms and properties of basic elementary functions; Be able to apply the properties of abstract functions flexibly to solve problems such as finding the function value, judging the monotonicity and parity of the function, and determining the zero point of the function.

1.2 Process and method objectives

Effectively develop students' logical reasoning ability, abstract thinking ability and knowledge transfer ability through in-depth analysis of typical examples and interactive exploration between teachers and students; Teachers guide students to master the thinking method from the specific to the general, from the concrete to the abstract, and significantly enhance students' comprehensive ability to analyze and solve problems.

1.3 Emotional attitudes and values objectives

Fully stimulate students' strong interest and enthusiasm for learning mathematics, and focus on cultivating students' spirit of daring to explore and innovate; Let students deeply experience the joy of success in the process of actively participating in classroom interaction and successfully solving problems, and truly enhance their confidence and sense of achievement in learning mathematics.

2. Key points and difficulties in teaching

2.1 Teaching focus

Master the abstract forms of elementary functions and their core properties; students are able to skillfully and flexibly apply the properties of abstract functions to solving various practical problems efficiently.

2.2 Teaching difficulties

Deeply understand the essence of abstract functions and clearly sort out the intrinsic connections between abstract functions and basic elementary functions; students are able to apply the properties of abstract functions flexibly in complex problem situations to support rigorous reasoning and precise calculation.

3. Teaching methods

Lecture method, discussion method, question guidance method, interactive inquiry method.

4. Teaching process (teacher-student interaction)

4.1 Situational introduction

Teacher: Students, we have learned a lot of basic elementary functions, such as linear functions, quadratic functions, exponential functions and logarithmic functions, etc. Now，I will give you some descriptions of functions that do not have specific expressions but have special operational relationships. For example, for a function, satisfy. $f(x)$ $f(x+y)=f(x)+f(y)$ Think: Is this somewhat similar to the property of any of the basic elementary functions we've learned?

Student: (Think, discuss) It feels a bit like a linear function.

Teacher: Very good, everyone's association is correct. This is actually a form of an abstract function. So what is an abstract function? An abstract function, in simple terms, is a function defined by properties or operational relationships that are not explicitly given specific expressions. Today, let's delve into the problem-solving methods of abstract functions.

Design intent

By reviewing the basic elementary functions that students are familiar with, The teacher introduces the concept of abstract functions, starting from students' existing knowledge and experience, reduce the strangeness of abstract functions, stimulate students' curiosity and thirst for knowledge, and naturally introduce the topic of this lesson.

4.2 Explanation of examples

Example 1: Given the domain of the function $f(x)$ is R, and $f(x+y)+f(x-y)=f(x)f(y)$,then $\sum_{k=1}^{13} f(k)=(\quad)$

A. -3 B. -2 C. 0 D. 1

Teacher: Students, when we get this question, let's first observe the relationship of this abstract function $f(x+y)+f(x-y)=f(x)*f(y)$. What methods can we use to start solving it?

Students: (think) assign values.

Teacher: Excellent! Then what value shall we assign first? You can give it a try.

Student: Ling. $x=1$, $y=0$.

Teacher: Alright, let's do the calculation together. Substitute $x=1$ and $y=0$ into the equation $f(x+y)+f(x-y)=f(x)*f(y)$,thus we have $f(1)+f(1)=f(1)*f(0)$,Given that $f(1)=1$,Who can tell me,what the value of $f(0)$ is?

Student: $f(0)=2$.

Teacher: Exactly right! Then let's try another assignment, If we set x equal to 0,what will we get?

Student: $f(y)+f(-y)=f(0)*f(y)=2f(y)$.

Teacher: Yes, after moving the terms, we can obtain $f(-y)=f(y)$. what property does the function $f(x)$ have?

Student: It's an even function.

Teacher: Exactly! Now let's get $y=1$. We will obtain $f(x+1)+f(x-1)=f(x)f(1)=f(x)$ ① Then we replace x with $x+1$. What can we obtain from this? Please try to write it down by yourselves.

Student: (calculate) $f(x+2)+f(x)=f(x+1)$ ... ②

Teacher: Very good! Now let's add ① and ② together and see what we can find.

Student: (Think, calculate), that is. $f(x+2)+f(x-1)=0$, which means $f(x+3)=-f(x)$

Teacher: Amazing! So ,what else can we conclude from this?

Student: $f(x+6)=-f(x+3)=f(x)$, therefore the period of the function f(x) is 6.

Teacher: Exactly correct! Next we can find the value of the function within a period. Given that $f(1)=1$ and $f(0)=2$, using the equation $f(x+1)+f(x-1)=f(x)$, let $x=1$, we get $f(2)+f(0)=f(1)$. Therefore, what is the value of $f(2)$?

Student: $f(2)=f(1)-f(0)=1-2=-1$.

Teacher: Yes! Let's set $x=2$, Then, $f(3)+f(1)=f(2)$. So, what is $f(3)$?

Student: $f(3)=f(2)-f(1)=-1-1=-2$.

Teacher: Excellent! Using this method, continue to calculate $f(4)$, $f(5)$, $f(6)$ (Guides the students to perform the calculations).

Student: (After calculating) $f(4)=-1$, $f(5)=1$ $f(6)=2$.

Teacher: In one cycle, $f(1)+f(2)+f(3)+f(4)+f(5)+f(6)=1-1-2-1+1+2=0$. So, what is the quotient and remainder when 13 is divided by 6?

Student: Quotient 2 remainder 1.

Teacher: What is the value of $\sum_{k=1}^{13} f(k)$?

Student: $2\times 0+f(1)=1$,The answer is D.

Teacher: Very good! Everyone has a very good understanding of this question. This problem mainly uses the assignment method to find the function value of the special point, to determine the parity and periodicity of the function, and then to find the sum of the sequence.

Design Intent

Through close interaction between teachers and students, guides students to analyze problems step by step, allowing students to deeply understand problem-solving ideas and methods in the process of thinking and answering questions.The teachers develops students' logical reasoning and computational abilities while enabling them to recognize the critical role of assumptions in solving abstract function problems and to appreciate the process of systematically deriving conclusions from given conditions.

Example 2: Given that the function $f(x)$ satisfies $f(x+2)=f(x)$ for all x in R and when x is in the interval [0, 2], $f(x)=\begin{cases}2^x, 0\le x\le 1\\ 2-x, 1<x\le 2\end{cases}$, if the function $g(x)=f(x)-\log_a(x+2)(a>0$ and $a\ne 1)$ exists for any and at that time, if the function has 4 zeros in the interval (-2, 6], then the range of values for a is _______.

Teacher: Students, for this problem, we know that $f(x+2)=f(x)$. What property does this tell us about the function $f(x)$?

Student: The period is 2.

Teacher: Right! Then let's first draw the graph of $f(x)$ over the interval $[0,2]$. When $0\le x\le 1$, $f(x)=2^x$. What is the monotonicity of this function?

Student: Monotonically increasing.

Teacher: When $1<x\le 2$, $f(x)=2-x$. What is the monotonicity here?

Student: Monotonically decreasing.

Teacher: OK, let's draw the graph of $f(x)$ on the interval $(-2,6]$ based on its periodicity. Then, the function $g(x)=f(x)-\log_a(x+2)$ has 4 zeros on the interval $(-2,6]$. What does this mean?

Student: The graphs of $y=f(x)$ and $y=\log_a(x+2)$ have 4 intersection points in the range (-2, 6].

Teacher: Exactly correct! Now let's discuss it in different cases. When $a>1$, what is the monotonicity of $y=\log_a(x+2)$?

Student: Monotonically increasing.

Teacher: In order for the graphs of two functions to have four intersection points, what conditions do we need to satisfy? You can try to list the inequalities.

Students: (Think, discuss). $\begin{cases}\log_a(6+2)<2\\ \log_a(4+2)\ge 1\end{cases}$.

Teacher: Excellent! Now let's solve the first inequality: $\log_a(6+2)<2$, which simplifies to $\log_a 8<\log_a a^2$, since $a>1$ and the logarithmic function is monotonically increasing. So what can we conclude from this?

Student: Since $a^2>8$, we can solve for a as $a>2\sqrt{2}$.

Teacher: Now let's solve the second inequality: $\log_a(4+2)\ge 1$, which is $\log_a 6\ge \log_a a$ What can we obtain from this?

Student: $a\le 6$.

Teacher: at that time, the range of values was. What was the monotonicity of at that time?

Teacher: So when $a>1$, the range of values for a is $2\sqrt{2}<a\le 6$. When $0<a<1$, What is the monotonicity of $y=\log_a(x+2)$?

Student: Monotonically decreasing.

Teacher: At this point, how many intersections can there be between the two function graphs? Student: There are 2 intersections. But this doesn't meet the requirements of the question, so we'll discard it.

Teacher: In conclusion, the range of value for a is $(2\sqrt{2},6]$.

Design intent

Through teacher-student interaction, the teacher guides students to start with the properties of the function, analyze the characteristics of the function graph, and then transform the function zero point problem into the function graph intersection point problem. Teacher cultivates students' conceptual understanding of integrating algebraic and geometric reasoning as well as classification-based analysis, enhance students' ability to solve comprehensive problems, and deepen their understanding of and proficiency in applying function properties

4.3 Summarizing and concluding

Teacher: Everyone, we just solved two typical examples together. Now let's summarize the common types and solving methods of abstract functions. Please recall, for an abstract function in the form of $f(x+y)=f(x)+f(y)$, which basic elementary function is similar to it?

Student: Linear function.

Teacher: Exactly! This often reminds us of the linear function $f(x)=kx$. Using the substitution method, we can also prove properties such as $f(nx)=nf(x)$ (n is an integer). What about the form f $f(xy)=f(x)+f(y)$?

Student: Similar to logarithmic functions.

Teacher: Excellent! It is often related to the properties of logarithmic functions. And $f(x+y)=f(x)f(y)$,what function is this similar to?

Student: Exponential function.

Teacher: Exactly! Similar to the properties of exponential functions. $f(x)=a^x(a>0,a\neq 1)$ So what are the general steps we take to solve the problem of abstract functions? First, observe the form of the abstract function, and then what?

Student: Find the function values of some special points by assignment.

Teacher: Yes, for example, $f(0), f(1)$ etc. And then?

Student: Judge the properties of a function based on the given conditions, such as parity, periodicity, monotonicity, etc.

Teacher: Excellent! And the last?

Student: Use the properties of the function to solve problems such as finding the function value, solving inequalities, determining the number of zeros of the function, etc.

Teacher: Everyone has summarized very comprehensively. I hope that in the future when you encounter problems with abstract functions, you will be able to think and solve them by following these methods and steps.

Design intention

Through the joint summarization of teachers and students, students can sort out the relevant knowledge and problem-solving methods of abstract functions, and form a systematic knowledge system. Teacher cultivates students' ability to summarize and engage in abstract thinking, enable them to elevate specific problem-solving experiences into general methods and strategies, and enhance their learning efficacy and problem-solving efficiency.

4.4 Extension and expansion

Teacher: Now I'll give you some extended questions to see if you can solve them using what you've learned. Given the function $f(x)$, satisfies $f(x+1)=\frac{1+f(x)}{1-f(x)}$, $f(1)=2$, find the value of $f(2025)$,Let's think about where we can start.

Students: (thinking, discussing) We can first calculate the value of $f(2)$.

Teacher: Alright, then how do we calculate $f(2)$?

Student: Substitute $x=1$ into $f(x+1)=\frac{1+f(x)}{1-f(x)}$, yields $f(2)=\frac{1+2}{1-2}=-3$.

Teacher: Excellent! Now, let's find out the value of $f(3)$, shall we?

Student: Substitute x = 2, then $f(3)=\frac{1+(-3)}{1-(-3)}=-\frac{1}{2}$.

Teacher: What about $f(4)$?

Student: $f(4)=\frac{1-\frac{1}{2}}{1+\frac{1}{2}}=\frac{1}{3}$.

Teacher: What about $f(5)$?

Student: $f(5)=\frac{1+\frac{1}{3}}{1-\frac{1}{3}}=2$.

Teacher: Have you all discovered any patterns?

Student: The period of the function $f(x)$ is 4.

Teacher: Yes! So what is the remainder when 2025 is divided by 4?

Student: Quotient 506 remainder 1.

Teacher: So, what is the value of $f(2025)$?

Student: $f(2025)=f(1)=2$.

Teacher: Exactly correct! Everyone did a great job. Now let's look at another question. Here is an abstract function $f(x)$,Which is an odd function, and $f(x+2)=-f(x)$. Let's try to construct a specific function that satisfies these properties.

Students: (Think, discuss, try to construct the function)

Teacher: (Patrolling and guiding students to think) We can start with functions we've learned before, such as the sine function.

Student: (After thinking and trying). $f(x)=\sin\left(\frac{\pi}{2}x\right)$

Teacher: Very good! Everyone's thinking is very flexible. I hope that through these extended questions, you will have a better grasp of the knowledge and methods of abstract functions.

Design Intent

Further consolidate students' understanding and application of knowledge of abstract functions through extension questions, and cultivate students' ability to transfer knowledge and innovative thinking. In the process of solving problems, let students deepen their application of the properties of functions, improve their ability to analyze and solve problems, and at the same time stimulate their interest in learning and spirit of exploration.

4.5 Class summary

Teacher: Dear students, today we have delved deeply into the problem-solving methods for abstract functions. Starting from the abstract form of the functions, by analyzing real questions from the college entrance examination, we have summarized the common types of abstract functions. Could any student please summarize what we have learned today?

Student: We learned about the concept of abstract functions, how to find the value of special point functions by assignment, how to determine the parity, periodicity, monotonicity and other properties of functions, and how to use these properties to solve problems such as finding function values and function zeros.

Teacher: It's very comprehensive! We also learned about the connection between abstract functions and basic elementary functions. For example, $f(x+y)=f(x)+f(y)$ is similar to linear functions and so on[1]. I hope that in your future studies, when you encounter problems with abstract functions, don't be afraid and use the methods you learned today to analyze and solve them. After class, you should review in time and do more related exercises to consolidate today's knowledge.

Design Intent

By having students summarize and teachers supplement, help students sort out the key points of this lesson, strengthen students' recognition and understanding of abstract functions, and enable students to form a complete knowledge system. Teachers emphasize the application of knowledge and review after class to cultivate students' good study habits.

5. Refinement and consolidation of the abstract form of basic elementary functions

Teacher: students, we have already gained some problem-solving experience with abstract functions through examples. Now, let's further refine the abstract forms of basic elementary functions so that we can better understand and remember them systematically. Let's start with the linear function. The abstract form is $f(x+y)=f(x)+f(y)+b$ (b is a constant), and when $b=0$, $f(x+y)=f(x)+f(y)$. As we previously mentioned, this is related to the properties of the linear function $f(x)=kx$ (k is a constant). Could any student explain how to prove $f(nx)=nf(x)$ (n is an integer) using this abstract form?

Student: We can use mathematical induction. First, prove that when $n=1$, $f(x)=1\times f(x)$, which is clearly true. Assume that for $n=k$ (k is an integer), $f(kx)=kf(x)$ holds true. Then, for $n=k+1$, we have $f[(k+1)x]=f(kx+x)=f(kx)+f(x)=kf(x)+f(x)=(k+1)f(x)$. Therefore, $f(nx)=nf(x)$ holds true.

Teacher: Excellent, your answer is very precise! Now, for the abstract form of the quadratic function, $f(x+y)+f(x-y)=2f(x)+2f(y)$, if we set $y=0$, what will we get?

Student: So. $f(x)+f(x)=2f(x)+2f(0)$,Therefore $f(0)=0$.

Teacher: Right. Then we can study its symmetry and other properties through appropriate assignment and transformation. This is related to some properties of the quadratic function $f(x)=ax^2+bx+c(a\neq 0)$. Now let's look at the abstract form of the exponential function $f(x+y)=f(x)f(y), f(0)=1(f(x)\neq 0)$. This form is consistent with the property of the exponential function $f(x)=a^x(a>0,a\neq 1)$, $a^{x+y}=a^x\cdot a^y$. Based on this abstract form, how can we derive $f(nx)=[f(x)]^n$ (n is an integer)? Please think about it.

Student: When $n=1$, $f(x)=[f(x)]^1$ holds true. Assuming that when $n=k$, $f(kx)=[f(x)]^k$ holds true, then $n=k+1$, $f[(k+1)x]=f(kx+x)=f(kx)f(x)=[f(x)]^k f(x)=[f(x)]^{k+1}$, then, so it was. $f(nx)=[f(x)]^n$

Teacher: Excellent, your reasoning is very clear! Now look at the abstract form of the logarithmic function: $f(xy)=f(x)+f(y), f(1)=0$. This corresponds to the property of the logarithmic function $f(x)=\log_a x(a>0,a\neq 1)$, where $\log_a(xy)=\log_a x+\log_a y$. We can use this property to study its monotonicity, domain, and other characteristics. Finally, for the power function's abstract form $f(xy)=f(x)f(y)$, when $x=y=1$, we have $f(1)=f(1)f(1)$, which implies $f(1)=0$ or $f(1)=1$,this is related to the properties of the power function $f(x)=x^{\alpha}$. You should remember these abstract forms and their connections with basic elementary functions, as they are very helpful in solving problems involving abstract functions.

Design Intent

Through in-depth analysis and derivation of the abstract form of basic elementary functions through teacher-student interaction, students will gain a clearer understanding of the intrinsic connection between abstract functions and basic elementary functions, and deepen their understanding of the concept and properties of functions. Teacher emphasizes the application of knowledge and review after class to cultivate students' good study habits.

Teacher cultivates students' logical reasoning ability and capacity for knowledge induction, help them build a coherent function knowledge system, and enable them to quickly identify the corresponding basic elementary function model when encountering abstract function problems, thereby facilitating the identification of an appropriate solution approach.

6. Exercises in class

(1) Given that the function $f(x)$ satisfies the equation $f(x+y)=f(x)f(y)$, and $f(1)=2$, then $\frac{f(2)}{f(1)}+\frac{f(4)}{f(3)}+\frac{f(6)}{f(5)}+\cdots+\frac{f(2024)}{f(2023)}$ equals( )

A. 2022 B. 2023 C. 2024 D. 2025

(2) It is known that the function $f(x)$ is an odd function defined on the set of real numbers, and $f(x+4)=f(x)$. When x belongs to the interval $(0,2)$, $f(x)=2x^2$, then $f(7)$ equals ( ).

A. -2. B. 2 C. -98 D. 98

Design intention

Through in-class exercises, students' knowledge and methods learned in this class will be consolidated in a timely manner, and their understanding and application ability of the properties of abstract functions will be tested. In the process of reviewing exercises, the teacher further strengthens students' problem-solving thinking and strategies, help them identify their own misconceptions and gaps, and address and refine these areas promptly.

7. Assign homework

(1) Given the function $f(x)$ that satisfies $f(xy)=f(x)+f(y)$, and $f(2)=1, f(3)=a$, find the value of $f(36)$.

(2) Let the function $f(x)$ be an even function defined on the set of real numbers, and satisfy $f(x+3)=f(x-1)$. When x xbelongs to the interval $[-2,0]$, $f(x)=\left(\frac{1}{2}\right)^x$. Calculate the value of $f(2025)$.

(3) Thinking: For an abstract function $f(x)$, if the relationship between $f(x+1)$ and $f(x)$ is

known, how can we determine the function? What is the periodicity of $f(x)$? Could you provide an example to illustrate it?

Design intent

By assigning homework, give students the opportunity to further consolidate and deepen their understanding and application of abstract functions after class. The first two questions help students consolidate the problem-solving methods and skills they have learned, while the third questions guide students to think deeply about the method of judging the periodicity of functions, develop students' independent inquiry ability and innovative thinking, and at the same time lay the groundwork for discussion and learning in the next class.

8. Discussion

In this teaching session, the "one question, one lesson" model was adopted, based on the abstract form of basic elementary functions in the textbook, and the teaching was carried out in combination with typical examples, achieving good teaching results. Through frequent and in-depth interactions between teachers and students, students actively participated in classroom discussions and problem-solving, gained a deeper understanding of the concept and properties of abstract functions, and were able to solve some abstract function problems using assignment methods, function properties, etc., and their logical reasoning ability and abstract thinking ability were effectively exercised.

However, there are some shortcomings in the teaching process. In the example explanation session, although most students can follow the train of thought, there are still some students with a weaker foundation who have difficulty understanding some complex reasoning processes. More targeted tutoring and practice should be provided in the subsequent teaching to ensure that every student can master the basic problem-solving methods[2-3]. In the extension section, some students' thinking is not broad enough and they cannot quickly apply the knowledge they have learned to new problem situations. This suggests that in future teaching, more attention should be paid to cultivating students' knowledge transfer ability and innovative thinking. For example, more open-ended questions can be set to guide students to think about problems from different perspectives.

In addition, the time management can be further optimized. For example, in the classroom practice session, some group discussions can be appropriately added to allow students to exchange ideas and methods of solving problems with each other. This will not only increase students' participation but also promote mutual learning among students. At the same time, more practical examples can be introduced to help students better understand the connection between the abstract form and the concrete function for the extraction part of basic elementary functions.

In response to these issues, in future teaching, more attention should be paid to individual differences among students, stratified teaching should be adopted to meet the learning needs of different students[4].Teachers strengthen the development of students’ thinking abilities and encourage them to question boldly and explore actively; they continually refine their teaching methods and strategies, enhance the effectiveness and quality of classroom instruction, and enable students to master abstract function concepts more effectively and develop related competencies.

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