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Mathematical ideas and teaching implications of trigonometric functions in high school mathematics

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Abstract: Mathematical thinking methods are a crucial component of core mathematical competencies, permeating the entire process of high school mathematics teaching. Trigonometry serves as an important link between elementary and advanced mathematics, and is also a high-frequency examination point in the college entrance examination. This article focuses on the educational value of mathematical thinking and the position of trigonometry within the discipline. It delves into core thinking methods such as combining numbers and shapes, reduction and transformation, classification and discussion, functions, and equations. Through specific examples like the Ferris wheel model and ancient poetry scenarios, it illustrates the integration of these ideas into teaching. Additionally, it proposes teaching strategies like problem-driven learning and situational creation, aiming to provide some reference for classroom teaching of trigonometry.

1. Introduction

1.1 The Importance of Mathematical Thinking in Mathematics Education

Mathematical thinking and methods refer to the understanding of the essence and patterns underlying mathematical facts, concepts, principles, and techniques. These are mathematical perspectives abstracted, generalized, and refined from specific mathematical content and the process of acquiring mathematical knowledge. They are not explicitly visible but rather embedded within mathematical knowledge, requiring teachers to uncover them through concrete mathematical topics. Placing the teaching of mathematical thinking and methods at a prominent position in mathematics instruction and emphasizing the foundational nature of mathematics courses helps ensure students master fundamental mathematical ideas, basic knowledge, essential skills, and competencies. This contributes significantly to fostering a comprehensive understanding of the value of mathematics among students[1].

1.2 The Role of Trigonometric Functions in High School Mathematics

Trigonometric functions, as an important component of the high school mathematics knowledge system, have always been a mandatory part of the college entrance examination. From the perspective of knowledge structure, it serves as a bridge connecting elementary mathematics and higher mathematics. In high school, based on the trigonometric functions learned in junior high school, further in-depth research is conducted on the definitions, graphs, and properties of functions such as sine, cosine, and tangent. This part of content lays the foundation for subsequent learning in modules such as vectors, complex numbers, and analytic geometry. The derivation and operation of trigonometric formulas are necessary for solving many problems[2]. At the geometric application level, trigonometric functions can effectively solve practical problems such as the relationship between sides and angles of triangles, as well as distance and height measurements. In physics, it is also an important mathematical tool for analyzing periodic phenomena such as simple harmonic motion and alternating current. At the same time, comprehensive questions that integrate trigonometric functions with algebra and geometry appear frequently in the college entrance examination, mainly testing students' ability to comprehensively apply knowledge, logical reasoning, and solve problems through computation. In addition, the learning process of trigonometric functions itself helps cultivate students' abstract thinking and the idea of combining numbers with shapes, which has a positive effect on improving mathematical modeling and mathematical operation skills. This is of great significance for enhancing students' comprehensive mathematical literacy.

2. Exploring the mathematical ideas embedded in trigonometric functions.

The knowledge of trigonometric functions contains various mathematical ideas and methods. In high school mathematics teaching on trigonometry, common mathematical thinking approaches include the integration of numbers and shapes, reduction to simpler forms, case analysis, and the relationship between functions and equations[3,4].

2.1 The Integration of Numbers and Shapes: Using Graphs to Connect Trigonometric Expressions and Their Visual Representations

In the teaching process of trigonometric functions, the idea of combining numbers and shapes plays a crucial role as a link. Taking the sine function $y = \sin x$ as an example, its graph exhibits a wavy pattern. Students can intuitively identify abstract properties such as periodicity and monotonic intervals by observing the fluctuation pattern of the graph. This process actually visualizes complex numerical characteristics, making the derivation of properties and problem-solving ideas clearer and providing practical assistance to students in overcoming obstacles in abstract thinking.

2.2 Reduction to Simpler Forms: Simplifying Complexity to Solve Trigonometric Problems

The idea of reduction is central throughout the study of trigonometry. When faced with complicated problems involving multiple angles, non-standard forms, or intricate operations—such as simplifying and evaluating expressions like $\sin 2\alpha + \cos(\alpha + 6\pi)$ students can apply identities such as double-angle and sum formulas to convert them into familiar single-angle or simpler forms. This transforms unfamiliar challenges into solvable problems based on previously learned knowledge, effectively reducing complexity to simplicity.

3. Case Design of Mathematical Thinking Methods in Trigonometric Function Teaching in High School

3.1 Application Case of the Integration of Numbers and Shapes in Trigonometric Function Teaching

Topic: Generation of Trigonometric Function Graphs.

As shown in Figure 1, the radius of the Ferris wheel is Am$(A > 0)$, The Ferris wheel rotates uniformly in a counterclockwise direction, with an angular velocity of ω rad$(\omega > 0)$, If you were on a Ferris wheel Calculate the time starting from point P_0 in the figure. Please determine the vertical coordinate y of point P at time x min in the coordinate system shown in the figure 1.

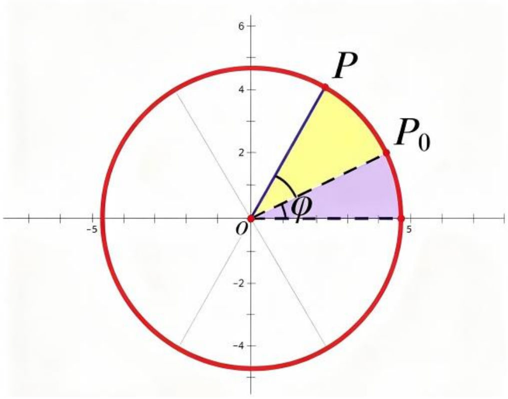

Figure 1: Schematic diagram of the Ferris wheel rotation

[Design Intent] Observing the world through the lens of mathematics, we perceive that the function $y = A\sin(\omega x + \varphi)$is a common mathematical model for depicting periodic phenomena in nature, endowed with a rich natural background. It is natural, clear, and understandable to comprehend the graphical properties of the function $y = A\sin(\omega x + \varphi)$ through its practical significance. First, place point P_0n the positive half-axis of the x-axis, and use the definition of the sine function to obtain $y = A\sin(\omega x)$; then, place point P_0at the position shown in the figure, and obtain the ordinate y of point P at time x min as $y = A\sin(\omega x + \varphi)$.

Key question 1:Based on our past experience, what is the usual way to explore or understand the properties of a function? The answer is: graph.

It is not difficult to understand that when the parameters A, φ, ω take different real values, different analytical expressions for the function will be obtained, and corresponding changes will occur in the function graph. Among this family of functions, is there a function that you are familiar with? The answer is: $y = \sin x$。

Key question 2: How to derive the graph of $y = \sin x$from the graph of $y = A\sin(\omega x + \varphi)(A > 0, \omega > 0)$?

Teacher-student activity: Guide students to develop research plans, with the teacher writing the plans on the blackboard. Summary: Based on previous comparison and discussion, the students determine the research plan for this class, that is, relatively fix two of them, with only one variable, and first explore the effects of A, φ, ωon the graphs of functions $y = \sin(x + \varphi)$ and $y = A\sin x(A > 0), y = \sin\omega x(\omega > 0)$, and then synthesize.

[Design Intent]. When facing problems, the teacher guides students to actively design research paths, with the focus on inspiring students to think about problem-solving strategies. For scenarios involving multiple variables, students learn to use the control variable method to transform complex problems into simple ones, and perceive the general idea of conducting research from the shallow to the deep, and from the simple to the complex.

Key question 3: How to get the graph of $y = \sin(x + 1)$ from the graph of $y = \sin x$?

Conclusion: The graph of $y = \sin(x + 1)$ can be viewed as the graph obtained by shifting all points on the graph of $y = \sin x$ to the left by one unit along the horizontal axis, while keeping the vertical coordinates unchanged, as illustrated in Figure 2.

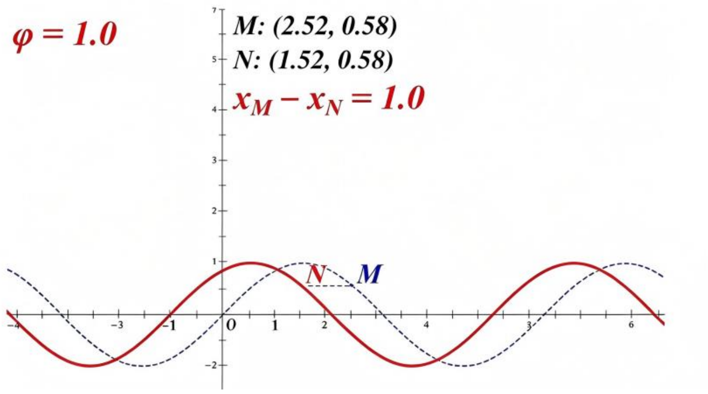

Figure 2: Image transformation of the graph from y=sinx to y=sin(x+1)

Key question 4: How to obtain the graph of $y = A\sin x(A > 0)$from the graph of $y = \sin x$？

Conclusion: The graph of $y = A\sin x(A > 0)$ can be viewed as the graph obtained by multiplying all points on the graph of $y = \sin x$ by A in the vertical direction, while keeping the horizontal coordinates unchanged, as illustrated in Figure 3.

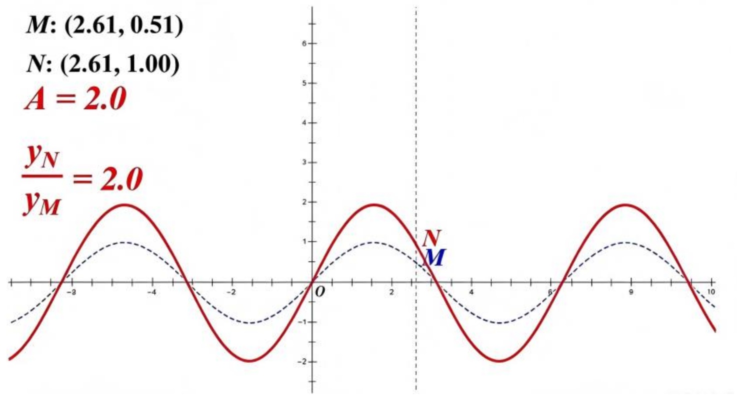

Figure 3: Image transformation of the graph from y=sinx to y=Asinx(A>0)

Key question 5: How to obtain the graph of $y = \sin\omega x(\omega > 0)$ from the graph of $y = \sin x$?

Teacher-student activity: Have students independently explore the method by analogy with previous methods, and then exchange ideas

Conclusion: The graph of $y = \sin\omega x(\omega > 0)$ can be regarded as the graph obtained by multiplying all points on the graph of $y = \sin x$ by $\frac{1}{\omega}$ in the horizontal direction, while keeping the vertical coordinate unchanged (Figure 4).

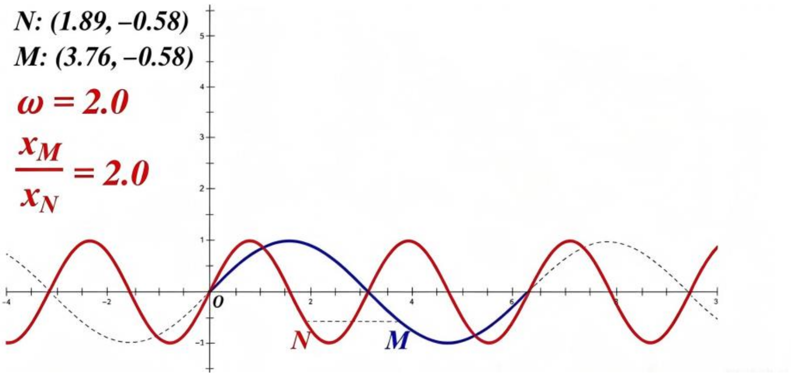

Figure 4: Image transformation of the graph from $y = \sin x$ to $y = \sin \omega x(\omega > 0)$

Summary: φ causes a translation transformation of the image, while A, ω cause a scaling transformation of the image. The essence of image transformation lies in the change of position of each point on the image, and the change of point position corresponds to the change of point coordinates. Therefore, to study the transformation rules of a function graph, it is only necessary to study the change rules of the coordinates of each point on the graph.

3.2 Application case of the idea of transformation in trigonometric function teaching

Classmates, this is a poem learned in the fourth grade of primary school, "The Song of the Frontier" by Lu Lun, a poet of the Tang Dynasty. In the dark of the moon, wild geese fly high; the chieftain flees in the night. I want to send light cavalry to chase, but heavy snow covers the bows and swords. Among them, the flying formation of geese is very orderly. What is the reason for this? Teacher plays a short video on geese migration on the computer: The angle between each side of the "V" formation formed by geese migrating south and the direction of advance is about 55°, and the bond angle of carbon-carbon bonds in diamond crystals is also about 55°. Is this a coincidence or a "tacit understanding" of nature?

Key question 1: What is the angle of the "V" formation? What kind of mathematical relationship is contained in it? Can we use the trigonometric formula for the sum and difference of two angles to derive the double-angle trigonometric formula? How to derive it?

[Design intention] Guide students to explore and derive the double-angle formula, develop their core competency of logical reasoning, and cultivate their ability to explore new knowledge. What other simple transformations can be made to the formula $\sin 2\alpha + \cos 2\alpha = 1$ for $\cos 2\alpha = \cos 2\alpha - \sin 2\alpha$ .Do you have other methods to derive $T_{2\alpha}$, What are the special requirements for the angles in $T_{2\alpha}$? Teacher writes the derived formulas on the blackboard as shown in Figure 5. Students observe the structure and characteristics of this set of formulas to facilitate their memorization and application.

$$S_{2\alpha}: \sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$C_{2\alpha}: \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$$
$$T_{2\alpha}: \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

Figure 5: Formula board

Consolidation exercise: Example 1: Given that angle α is in the second quadrant and $\cos\alpha = -\frac{3}{5}$., find the values of $\sin 2\alpha$, $\cos 2\alpha$ and $\tan 2\alpha$.

Variation exercise: Find the values of:

$$\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} \quad (1)$$

$$\sin\frac{\pi}{8}\cos\frac{\pi}{8} \quad (2)$$

$$1\text{-}2\cos^2 30^{\circ} \quad (3)$$

$$\frac{tan75°}{1\text{-}tan^2 75°} \quad (4)$$

4. Teaching Enlightenment Based on the Mathematical Ideas Involved in Trigonometric Functions

4.1 Problem-driven Teaching

In trigonometric function teaching, teachers should be adept at designing key questions that progress layer by layer, guiding students to actively explore, rather than directly imparting conclusions. The problem-driven teaching approach not only fosters students' logical reasoning and mathematical abstraction skills, but also assists them in comprehending the general approach to exploring problems from simple to complex, thereby seamlessly integrating mathematical thinking methods.

4.2 Create specific contexts to stimulate students' interest

Trigonometric functions have a rich natural and cultural background. Teachers should make full use of these real-life situations to enable students to appreciate the application value of mathematics. Integrating situational teaching into daily teaching can not only arouse students' enthusiasm for learning but also help enhance their mathematical modeling skills.

4.3 Strengthen the teaching of combining numbers with shapes

The concept of combining numbers and shapes can be described as a "key link" in the teaching of trigonometric functions, as it connects the originally abstract trigonometric function formulas with intuitive graphs. In practical teaching, teachers should consciously emphasize the strategy of "using graphs as a medium to bridge the gap between trigonometric function formulas and graphs". Specifically, it means that teachers should not separate formula derivation from graph analysis. Instead, they should enable students to visualize the corresponding graph when they see a trigonometric function formula, and conversely, to write down the corresponding analytical expression when they see a graph. Only by truly integrating numbers and shapes can students' understanding of the properties of trigonometric functions move beyond mere mechanical memorization. Instead, they can read numbers from shapes, think of shapes from numbers, and gradually develop a habit of thinking in terms of translating numbers and shapes into each other.

4.4 Utilizing geometric intuition to solve algebraic problems

In the teaching of trigonometric functions, the idea of transformation is the core strategy to solve complex problems. Teachers guide students to make good use of the mathematical idea of geometric intuition, transforming unfamiliar algebraic problems into known geometric models or familiar formulas. Using geometric intuition to solve algebraic problems can, on the one hand, reduce the cognitive difficulty in the learning process of trigonometric functions, and on the other hand, help cultivate students' core competencies such as logical reasoning, mathematical operations, and mathematical modeling. In general, geometric intuition and the idea of transformation are of great significance for improving students' comprehensive mathematical literacy.

5. Conclusion

Trigonometric functions are a core content in high school mathematics. For the teaching of this part, it is not enough to mechanically teach formula memorization and mechanical operations. More importantly, it is crucial to explore the mathematical ideas behind it. In the teaching process of trigonometric functions, if we can consciously integrate methods such as combining numbers and shapes, reduction, and classification discussion into the classroom, it will be a very crucial path to enhance students' core mathematical competencies. For example, using a Ferris wheel model to introduce the concept visually will enable students to naturally understand those abstract function properties, and at the same time, they will more easily perceive the connection between mathematics and the real world. Furthermore, by driving the learning process through questions, designing a series of key questions that progress step by step can guide students to explore how knowledge is generated, helping them gradually shift from "learning" to "learning how to learn". In real teaching practice, teachers emphasize the teaching of mathematical thinking methods and also use diverse teaching strategies to enable students to grasp basic knowledge while also personally perceiving the essence and laws of mathematics. The educational value of mathematics education is realized through daily teaching.

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