

# *High-Dimensional Multi-Objective Optimization Strategy Based on Decision Space Oriented Search*

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**Abstract:** Traditional multi-objective evolutionary algorithms (MOEAs) have good performance for low-dimensional continuous multi-objective optimization problems, but with the increase of the target dimension of the optimization problem, the optimization difficulty will also increase sharply. The main reasons are: the algorithm itself. Insufficient, the selection pressure becomes smaller when the dimension increases, and the convergence and distribution conflicts are difficult to balance. This paper proposes a directional search strategy in decision space by using the characteristics of continuous multi-objective optimization problem to optimize the high-dimensional multi-objective optimization. The strategy can be combined with the MOEA based on dominance relationship. DS firstly samples and analyzes the problem, and analyzes the problem characteristics to obtain the convergence subspace control vector and the distributed subspace control vector. The algorithm search process is divided into the convergence search phase. And the distributed search phase, which corresponds to the convergence subspace and the distribution subspace respectively, and uses the sampling analysis pair to make a macroscopic influence on the region of the generation of the individual generation in the different stages of the search. Convergence and distribution are considered in stages to avoid convergence. Sexuality and distribution are difficult to balance, and at the same time, search for funds in a certain stage. The relative concentration of the source increases the search ability of the algorithm to some extent. In the experimental part, the NSGA-II and SPEA2 algorithms combined with the DS strategy are compared with the original NSGA-II and SPEA2 algorithms, and DS-NSGA-II is taken as an example. Compared with other high-dimensional algorithms MOEAD-PBI, NSGA-III, Hype, MSOPS and LMEA, the experimental results show that the performance of DS strategy is significantly higher than that of NSGA-II and SPEA2 algorithms. DS-NSGAI has strong competitiveness compared with the existing classic high-dimensional multi-target algorithm.

## 1. Introduction

There are many multi-objective optimization problems in the real world. There are conflicting relationships between different targets in these multi-objective optimization problems. There is only one optimal solution for single-objective optimization problems, and the most for multi-objective

optimization problems. The optimal solution is a set of equilibrium solutions. The multi-objective algorithm is a kind of iterative optimization algorithm based on swarm intelligence. Because it can find a group in a single run, it is widely used to solve multi-objective optimization problems. The multi-investigator's note. The goal of MOEAs is to find a set of equilibrium solutions that are as close as possible to the optimal equilibrium solution set. This set of decision vector solutions is called Pareto Set (PS), corresponding to PS. The target vector set in the target space is called the Pareto Front (PF) [1].

Traditional dominance-based algorithms, such as NSGA-II and SPEA2, have a good effect in solving low-dimensional multi-objective optimization problems, but as the target dimension increases, the problem is difficult to optimize when the number of targets is greater than 3. The main reasons are: 1) The search ability of the algorithm itself is insufficient, 2) The selection pressure is insufficient, the optimization conflict between different targets is intensified after the target dimension is increased, and the convergence distribution is difficult to balance. The optimal PF of the optimization problem is more complicated and the target dimension is large, it is difficult to properly decompose the problem. The index-based algorithm for the high-dimensional multi-objective optimization increases exponentially with the target dimension increasing the time complexity of the algorithm. At the same time, due to the characteristics of the indicators, the algorithm will prefer some special points in the PF [2].

This paper mainly studies the algorithm based on dominance relationship. Based on the above two points, we do further work on how to generate solutions. We use the characteristics of multi-objective optimization problems to overcome the multi-objective optimization algorithm encountered in high-dimensional problems. The difficulty proposes a directed search strategy based on decision space (referred to as DS), which affects the convergence of the algorithm by affecting the generation region of the offspring. The basic idea is to divide the whole optimization process of the algorithm into three stages: At the first stage, the problem is sampled and analyzed, and the convergence control vector and the distribution control vector of the problem decision space are judged. In the subsequent search process, the convergence subspace and the distribution subspace are determined based on the sampling analysis result; the second stage algorithm is along Convergence subspace search, so that a few solutions reach the real PF nearby; the third stage algorithm searches along the distributed subspace, so that the population covers the real PF as close as possible. Based on the problem sampling analysis results, the search in the convergence subspace is first partially concentrated computing resources, but also overcome the problem of insufficient selection pressure in high-dimensional problems, when less the number of individuals approaching the real PF and then through the search in the distributed subspace makes the population evenly cover the entire PF.

## 2. Motivation and Related Definitions Are Given

### 2.1 Characteristics of Continuous Multi-Objective Optimization Problems

According to the Kuhn Tucker condition (KKT), it can be concluded that for a continuous multi-objective optimization problem, PS is a piecewise continuous  $m-1$  dimensional fluid, and  $m$  is the dimension of the target space. This property is called multi-target. Optimize the characteristics of the problem.

Theorem 1: Assume that the objective function  $f(x)$  when (the dimension of  $\Omega$  is  $n$ )

Since PS is a fluid of  $m-1$  dimension, in order to simplify understanding, we regard the distance of an individual from a point on the real PS in the decision space as the computing resource needed to search for that point, then we can conclude that it is true. Parents near PS are more likely to generate offspring individuals located elsewhere in the PS.

## 2.2 Computing Resource Allocation

The common practice of the existing multi-objective optimization algorithm is to try to ensure the convergence and distribution of the entire population in the target space in an iterative process. The convergence mechanism and the distribution retention mechanism of the algorithm play a role at the same time, so the whole population is in the continuous iterative optimization process, the whole is approaching the whole real PF. The degree of convergence and uniformity of the individual in the solution set obtained by MOEAs in each iteration is not the same, but for the sake of easy understanding, it is understood to be roughly the same for the time being is allocated to the allocation [3].

## 2.3 Convergence and Distribution

The goal pursued by the multi-objective optimization algorithm is to obtain the final solution set as close as possible to the real PF while covering the real PF as uniformly and broadly as possible. The former is convergence and the latter is distributed. In the multi-objective optimization process, especially. It is a high-dimensional multi-objective optimization problem, and most of the generated individuals are not dominated by each other, and the degree of convergence is different. The meanings of some nouns are given below:

**Convergence:** If we layer the target space, the real PF is the innermost layer, and each layer is parallel to the real PF structure. The position of the layer where the individual is located represents the degree of convergence. The closer to the real PF layer, the higher the convergence. .

**Homogeneous individuals:** Individuals with the same degree of convergence (located on the same floor) at different locations are called homogenous individuals.

**Isomorphic individuals:** Individuals with the same relative position but different degrees of convergence in each layer are called co-distributed individuals.

## 3. Ds: Directed Search Strategy Based on Decision Space

### 3.1 Basic Idea

According to the characteristics of the continuous multi-objective optimization problem, that is, the real PS of the multi-objective optimization problem is an fluid of  $m-1$  dimension, it can be concluded that if there is a solution reaching the vicinity of the real PF, the solution can be faster. Find other good solutions, and then through the distributed retention strategy, the entire population will eventually cover the entire PF surface. Although the overall idea is simple, it has the following advantages: firstly, the characteristics of the optimization problem and the characteristics of the gene recombination operator to generate the children around the parent individual determine the feasibility of the first convergence and post-distribution search strategy. The DS strategy is fully utilized. The characteristics of continuous multi-objective problems are used to enhance the search efficiency of the algorithm. Secondly, the convergence and distribution are searched at different stages, which resolves the difficulty of convergence and distribution in high-dimensional multi-objective optimization problems, making the strategy high-dimensional. There will be better performance in the optimization problem; the final search and distribution separately search can make the computing resources more concentrated in specific sub-parts at specific moments, and enhance the search ability of the algorithm.

## 3.2 Algorithm Flow

The problem sampling analysis is to some extent an analysis of the problem characteristics. The sampling analysis of the problem yields the directional trend feature of the convergent subspace and the distributed subspace in the solution space, ie the subspace control vector. It should be noted that the convergence is in convergence. The search phase is to make a few solutions quickly search to any position on the real PF, so at the beginning of the population, a solution is randomly generated, and the convergence subspace control vector is combined with the one to determine the convergence subspace. The initial population is generated in space, and then iteratively searched for convergence in the convergence subspace. The initialization of the distributed search segment is similar to the convergence, and the difference is centered on the optimal one of the optimal solutions obtained during the convergence search phase. Combine the distributed subspace control vector to determine the distributed subspace.

The algorithm combined with the DS strategy does not change the convergence and distribution retention mechanism of the original algorithm. The algorithm still uses the environment selection and matching selection mechanism of the original algorithm. The DS strategy is mainly through the search area of the control algorithm (ie, the generation of the individual. The region is generated to affect the convergence and distribution of the algorithm. In the convergence search phase, the algorithm mainly searches in the convergence subspace. The individuals in the population are mostly the same distribution individuals. At this time, the algorithm focuses on the convergence of the population. The convergence mechanism of the original algorithm works, centralizing the computational resources to quickly optimize the minority solutions to or near the real PF; in the distributed search phase, the algorithm mainly searches in the distributed subspace, and the individuals in the population are mostly the same convergence individuals. The algorithm focuses on the distribution of the population, at this time mainly due to the distribution mechanism of the original algorithm, so that the population can cover the entire PF well.

## 4. Experiment Analysis

### 4.1 Experimental Parameter Setting

In order to compare all the optimization algorithms fairly, this paper takes the recommended parameter values as the parameter values of the algorithm comparison. The algorithm involved in this paper uses the method of simulating binary crossover and polynomial variation to generate the progeny population. As recommended in the paper, cross-distribution The number of indicators ( $N_c$ ) is set to 20, the number of distribution indicators ( $N_m$ ) of the mutation is set to 20, and the probability of intersection ( $P_c$ ) is set to 1.0, and the probability of change ( $P_m$ ) is set to  $1/D$ , where  $D$  is the decision The number of variables. In order to avoid the generated weights being distributed on the boundary of the Pareto frontier, the NSGA-III recommended two-layer reference point generation strategy is used to generate uniformly distributed MOEA/D weight vectors and NSGA-III reference points. The sampling analysis part takes a  $J$  value of 8.  $J$ . The larger the value, the more accurate the sampling analysis result. For most problems,  $J$  8 can more accurately determine the control vector of the subspace. The other algorithms in this paper use the same population size for the same problem. For the 4-10 dimensional test problem, set the population size to 120. For the 15 and 20-dimensional test questions, set the population size to 220. Hype, MSOPS, and LMEA all use the parameter settings recommended by the original text. All test questions of the method needle were performed in 30 independent repeated experiments. This paper takes the total number of evaluations of individuals as the end condition of the algorithm.

The value range of  $r$  is divided into 10 equal parts (parameter variation gradient is 0.1) for experiment ( $r$  is 0, ie no convergence search is obviously not conducive to the optimization of the problem, so the experiment is based on the value of  $r$  from 0.1). And the DS-NSGA-II algorithm changes the IGD value under the different values of the parameter  $r$ , and repeats the experiment 30 times independently. As shown in Fig. 3, the DTLZ3, the degradation problem DTLZ5, the discontinuous PF problem DTLZ7 are given. The experimental results on the 8-dimensional target. The value of  $r$  reflects the computational resources allocated during the convergence search phase.

## 4.2 Experimental Parameter Setting

1) DS-NSGA-II VS NSGA-II & DS-SPEA2 VS SPEA2DS strategy can be combined with multi-objective evolutionary algorithm based on dominance relation. This paper combines the classical algorithm NSGA-II and SPEA2 based on dominance relationship with DS strategy respectively. The algorithms DS-NSGA-II and DS-SPEA2 are obtained and tested by DTLZ series test problems. The effectiveness of DS is verified by comparing the performance of the algorithm before and after the introduction of DS strategy. NSGA-II and SPEA2 algorithm in low-dimensional multi-objective optimization problem. The performance is better, and DS is mainly a strategy for high-dimensional multi-objective optimization problems, so in this section, the DTLZ series test questions are compared in the four dimensions of 4, 5, 6, and 8. Experiment. IGD is used as the evaluation index, and the experimental results are given in Table 2. The smaller the average IGD is, the better the performance is.

2) Compare with other algorithms. This section is mainly to further verify the validity of the DS strategy, and to prove the feasibility of considering the convergence and distribution separately. The representative algorithm in the MOEAS is a comparative experiment. Because it is large, we use DS- in this section. NSGA-II is further compared with untyped algorithms. Hype, MSOPS, and LMEA.MOEAD are decomposition-based algorithms, NSGAI governs relationships, and LMEA is a large-scale multi-objective algorithm for decision variable classification. For comparison experiments, the DTLZ problem uses the IGD algorithm to perform better. The larger the HV index value, the better the performance of the algorithm. The HV indicator test results of the WFG series test problem.

## 5. Conclusions

1. According to the characteristics of continuous multi-objective optimization problem, it provides a new perspective for multi-objective optimization: phased optimization of the convergence and distribution of the population. This can greatly weaken the convergence of high-dimensional multi-objective problems. Conflict between distribution and distribution.

2. The comparison between individuals in the population is: the convergence between the same distribution individuals, or the distribution between the same individuals. This can increase the individual selection pressure when optimizing the environment selection in high-dimensional multi-objective problems. .

3. According to the sampling analysis of the optimization problem, there is a macroscopic influence on the generation of the offspring in the process of algorithm optimization, which makes the purpose of the search stronger. The problem of insufficient search ability for high-dimensional multi-objective optimization problems is purposeful. Sexual search concentrates on computing resources, avoiding the wasted computing resources caused by searching in unnecessary areas.

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