# Research on swimming force model of athletes based on Bernoulli's law 

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#### Abstract

There are many kinds of postures in swimming, but different postures have a significant effect on the speed at which swimmers move forward. This paper takes the swimming environment as the ideal state, without considering the change of the swimmer's leg strength during the swimming process and the influence of other forces not considered in addition to this article, we can accurately solve the best posture when swimming, so that the swimmer swims as fast as possible. Firstly, by combining Bernoulli's law and fluid continuity theorists, this paper establishes the lift model. Then, we solve the relationship between the angle of the athlete's body axis and horizontal surface and the swimming speed change. Finally, the optimal swing amplitude of the athlete's body axis and horizontal surface is derived, which helps the coaches improve the athlete's swimming posture.


## 1. Introduction

Swimming is a skill that people float upward under the buoyancy of water, and make the body move regularly in the water by means of the buoyancy. It is also a sport that can enhance the physical fitness of the human body, there are many kinds of swimming posture, and common swimming postures are: breaststroke, climbing, backstroke, and butterfly. In the actual movement process, different swimming posture movements determine different swimming speed and different stroke thrust. Swimming is a sport in the water environment. The density of water is greater than that of air. Therefore, when people move in it, they need to overcome greater resistance. At the same time, water also causes greater pressure on the human chest, increasing the resistance around the body. In addition, the water temperature of swimming pool is usually $26-28^{\circ} \mathrm{C}$, and its heat conduction is fast. Therefore, compared with the land project, swimming needs to consume more energy. In competitive swimming, speed is the key factor to measure the outcome. So how to adjust the swimming posture to improve the speed of swimming? Then we need to analyze and calculate the force of human body when swimming. By looking at some relevant data on the forces of swimming speed, we can find that the factors that affect swimming speed are resistance, thrust, gravity, buoyant force.

In order to facilitate the analysis of the problem, we make the following assumptions: Suppose the strength of the swimmer's legs does not change during the watering process; Suppose the swimmer's swimming environment meets the conditions of Bernoulli's law; Suppose the swimmer meets the flat throwing movement at the moment he jumps into the pool; Suppose that only one cycle of swimming action is considered; It is assumed that the speed of swimming during swimming is only affected by
the forces taken into account and not by other forces；
Combined with the principle of fluid continuity，and the Bernoulli principle，we established the lift model and overall force model of swimmers to adjust the swimmer＇s swimming posture，thereby increasing the speed of swimming．And solves the relationship between the angle of the athlete＇s body axis and horizontal surface and the swimming speed change image，the law of time and speed change and the time，angle and velocity by using Matplotlib and Noumpy to analyze the data and draw the functional relationship image obtained by the model．In turn，the optimal swing amplitude of the athlete＇s body axis and horizontal surface is derived，which helps the coaches improve the athlete＇s swimming posture．

## 2．Establishment and solution of the model

## 2．1 The construction of the lift model

## 2．1．1 The buoyant force of a person while swimming

By Archimedes＇buoyant principle：

$$
\begin{align*}
& F_{B}=\boldsymbol{F}_{\text {下 }}-\boldsymbol{F}_{\text {上 }} \\
& F_{B}=\rho g \boldsymbol{h}_{\text {下 }} S-\rho g \boldsymbol{h}_{\perp} S  \tag{1}\\
& F_{B}=\rho g \Delta h S
\end{align*}
$$

## 2．1．2 The the therm of continuity of the fluid

The continuity theority of a fluid means that when a fluid flows continuously and steadily through a pipe of varying weights，the mass of the fluid flowing into any face is equal at the same time because the fluid in any part of the pipe cannot be interrupted or squeezed．

The continuity theorology describes the relationship between the flow rate of the fluid in the flow and the pipeline facet．In the flow of fluid，not only the flow rate and pipe facet are interrelated，but also the flow rate and pressure are also interrelated．

Take a micro tube in the flow field（as shown in Figure 1）with a long edge $\boldsymbol{d}_{x}, \boldsymbol{d}_{y}, \boldsymbol{d}_{z}$ ，Center point O flow rate is $\left(\boldsymbol{U}_{x}, \boldsymbol{U}_{y}, \boldsymbol{U}_{z}\right)$

Take the x －axis direction as an example：
Left surface flow rate：

$$
\begin{equation*}
u_{M}=u_{x}-\frac{1}{2} \frac{\partial u_{x}}{\partial x} d x \tag{2}
\end{equation*}
$$

Right surface flow rate：

$$
\begin{equation*}
u_{N}=u_{x}+\frac{1}{2} \frac{\partial u_{x}}{\partial x} d x \tag{3}
\end{equation*}
$$

The quality flow in and out of the direction is poor per unit of time：

$$
\begin{align*}
M_{\text {右 }}-M_{\text {左 }} & =\left[\rho u_{X}+\frac{1}{2} \frac{\partial\left(\rho u_{X}\right)}{\partial x} d x\right] d y d z-\left[\rho u_{X}-\frac{1}{2} \frac{\partial\left(\rho u_{X}\right)}{\partial x} d x\right] d y d z  \tag{4}\\
& =\frac{\partial\left(\rho u_{X}\right)}{\partial x} d x d y d z
\end{align*}
$$

In the x－axis direction：

$$
\begin{equation*}
\frac{\partial\left(\rho u_{x}\right)}{\partial x} d x d y d z \tag{5}
\end{equation*}
$$

The sum of the differences in fluid mass between outflow and flow into heteomes per unit of time should be equal to the mass reduced by density changes in hetees, i.e.:

$$
\begin{equation*}
\left[\frac{\partial\left(\rho u_{x}\right)}{\partial x}+\frac{\partial\left(\rho u_{y}\right)}{\partial y}+\frac{\partial\left(\rho u_{z}\right)}{\partial z}\right] d x d y d z=-\frac{\partial \rho}{\partial t} d x d y d z \tag{6}
\end{equation*}
$$

When it is a constant flow:

$$
\begin{equation*}
\therefore \quad \frac{\partial\left(\rho u_{x}\right)}{\partial x}+\frac{\partial\left(\rho u_{y}\right)}{\partial y}+\frac{\partial\left(\rho u_{z}\right)}{\partial z}=0 \tag{7}
\end{equation*}
$$

### 2.1.3 The specific application of Bernoulli's law

The equation for Bernoulli's law is:

$$
\begin{equation*}
p+\rho g h+\frac{\rho V^{2}}{2}=C \tag{8}
\end{equation*}
$$

It is not difficult to see by Bernoulli's law that the greater the speed of water passing through an athlete, the greater the resistance. and the smaller the speed of water passing through an athlete, the less resistance. According to the fluid continuity theority, it can be seen that the volume of water flowing from the left side of the athlete in a unit of time is the same as that of the right side. And because the athlete's body will swing left and right when swimming, the volume of water flowing on the left and right sides at the same time is not the same, then the force produced on the athlete is also different, so that the water produces a force to push the athlete forward.

It can be dedesed by (8):

$$
\begin{equation*}
p_{2}-p_{1}=\frac{\rho\left(\nu_{1}^{2}-v_{2}^{2}\right)}{2} \tag{9}
\end{equation*}
$$

That is:

$$
\begin{equation*}
F_{T}=\Delta P S \tag{10}
\end{equation*}
$$

### 2.2 The construction of a force model for athletes swimming

The athlete's center of gravity in the water doesn't usually match the floating heart. But a good athlete can adjust the angle of his body, so that the body's center of gravity moves forward moderately, so that the body can be balanced into a line type.


Figure 1: Analysis of the force of athletes swimming

Through the analysis of the force of the athletes during swimming (as shown in Figure 1), and the orthodective decomposition of $F_{T}$ and $F_{D}$, the sports model of swimming is established.

$$
\begin{gather*}
F_{T 1} \cos \theta+F_{T 2} \cos \theta-F_{D}=m a_{x}  \tag{11}\\
F_{T 1} \sin \theta+F_{F}-F_{T 2} \sin \theta-G=m a_{y} \tag{12}
\end{gather*}
$$

### 2.2.1 Horizontal modeling

Bring (11) to (12) get:

$$
\begin{equation*}
\frac{\rho\left(V_{1}^{2}-V_{2}^{2}\right) S}{2} \sin \theta+\frac{C \rho S^{\prime} V^{2}}{2} \cos \theta=m a_{x} \tag{13}
\end{equation*}
$$

Set here: $V_{1}=v \alpha, V_{2}=v \beta$, and take the average speed of an athlete's movement for a very short period of time as the instantaneous speed,That is: $a=\frac{d v}{d t}$ thus becomes the following style:

$$
\begin{equation*}
\frac{\rho\left(v^{2} \alpha^{2}-v^{2} \beta^{2}\right) S}{2} \sin \theta+\frac{C \rho S^{\prime} v^{2}}{2} \cos \theta=m \frac{d v}{d t} \tag{14}
\end{equation*}
$$

The transfer item finishing formula is:

$$
\begin{align*}
& v^{2}\left[\frac{\rho\left(\alpha^{2}-\beta^{2}\right) S \sin \theta+C \rho S^{\prime} \cos \theta}{2}\right]=m \frac{d v}{d t}  \tag{15}\\
& {\left[\frac{\rho\left(\alpha^{2}-\beta^{2}\right) S \sin \theta+C \rho S^{\prime} \cos \theta}{2 m}\right] d t=\frac{d v}{v^{2}}} \tag{16}
\end{align*}
$$

Finally, the time t of the athlete's movement in the horizontal direction is derived, and the angle $\theta$ relationship equation is:

$$
\begin{equation*}
f(\theta, t)=\frac{2 v_{0} m}{\left(\rho S C_{1} \sin \theta+\rho S C \sin \theta \cos \theta\right) t} \tag{17}
\end{equation*}
$$

### 2.2.2 Vertical modeling

Bringing (1), (11) and (12) into (14):

$$
\begin{equation*}
m g+\frac{\rho S v^{2} C \sin \theta}{2}-\rho g \nabla h-\frac{\rho v^{2}\left(\alpha^{2}-\beta^{2}\right) S}{2}=m a_{y} \tag{18}
\end{equation*}
$$

Take the average speed of an athlete's movement in a very short period of time as the instantaneous speed, that is:

$$
\begin{gather*}
m g+\frac{\rho S v^{2} C \sin \theta}{2}-\rho g \nabla h-\frac{\rho v^{2}\left(\alpha^{2}-\beta^{2}\right) S}{2}=m \frac{d v}{d t}  \tag{19}\\
v^{2} C_{2}+C_{3}=\frac{d v}{d t} \tag{20}
\end{gather*}
$$

The result is the time $t$ for the athlete to move in the vertical direction,The angle $\theta$ relationship equation is:

$$
\begin{equation*}
f(\theta, t)=\frac{\sqrt{\frac{C_{2}}{C_{3}}} v_{0} \sin \theta+\tan C_{3} t}{\sqrt{\frac{C_{2}}{C_{3}}}-\frac{C_{2}}{C_{3}} v_{0} \sin \theta+\tan C_{3} t} \tag{21}
\end{equation*}
$$

$S^{\prime}$ for the athlete's water area, because the athlete and thehorizontal surface have a certain angle $\theta$

$$
\begin{equation*}
S^{\prime}=S \sin \theta \tag{22}
\end{equation*}
$$

### 2.3 The solution to the model

According to many formulas and algorithms applied earlier, a mathematical model is established to solve the angle between the athlete's body axis and the horizontal surface, and the results are as follows:


Figure 2: Result
For swimmers, swimming in the water so that the body axis and horizontal angle of 45 degrees can improve the speed of athletes swimming.

In addition, the way you row water also has an effect on the speed of your swim. Therefore, in order to improve the speed of swimming, athletes should bend their elbows when rowing water, through different depths, routes (such as "S" type), to get more thrust. Experiments have shown that when the palm and hand forward direction is 40 degrees, the upward thrust is the largest, in climbing the water, athletes can adjust the angle of the palm at any time, in order to obtain maximum thrust. Athletes can length longer the water distance and water-facing area in the area directly below the body, can use the curved arm "Z" or "S" type rowing route, and make the palm of the palm slightly,
increase the watering effect. Speed up the stroke frequency, the curved arm can reduce the upper limb rotation inertia, improve the line speed value of the palm, and increase the amount of reaction force.

According to our analysis, athletes are subjected to water resistance, thrust, their own gravity, buoyant force when swimming. If an athlete has less resistance and more thrust, he will swim faster.

In view of which of the common swimming postures is the fastest problem, we drew the various swimming positions of the effort, and according to the physical knowledge and practical experience of the force analysis.

The climbing of the arm is done in the air, and the density of water is equivalent to more than 800 times the density of the air, according to the formula, the arm in the air resistance is much less than the resistance in the water. Although climbing is not as much thrust by water as breaststroke, the effect of resistance on speed is much greater than that of thrust. The body of the climb has smaller ups and downs, and the middle axis of the body is very smooth compared to the dramatic body fluctuations of the breaststroke and butterfly. Combined with the above analysis, we conclude that climbing is the fastest of the common swimming positions.

The thrust of the stroke is related to the force of the legs against the water, and according to the torque $\times$ force arm, we conclude that the thrust of the stroke of the breaststroke is the largest.

## 3. Model Evaluation

We have carried out a comprehensive analysis of the force of athletes swimming, and closely combined with multiple models and physical knowledge and practical experience, the calculation effect is very significant. In addition, we use a variety of models and physical knowledge and practical experience, the model is logical, easy to understand, and the conclusion is more practical.

However, the whole force analysis is based on the two-dimensional image, and no threedimensional model is established for the image. On the other hand, this paper only considers one exercise cycle, and does not continuously analyze the swimming process. This paper does not do indepth development of the center of gravity and floating heart at the same point, the results are only valid for some athletes.

## 4. Model Improvement

As far as the actual situation is concerned, swimmers will inevitably be affected by the force, leg force, water mobility and other factors in the process of swimming. Based on the model, the improvement suggestions are given

Model 1: force model (including foot force and water thrust)
In the aspect of force analysis, the static analysis itself has its defects, here the establishment of the model can adopt more practical dynamic force analysis. Make full use of the knowledge of fluid mechanics to analyze the resistance of swimmers to water and the fluid force beyond the leg force.

Model 2: Bernoulli model
Bernoulli's theorem is a law in fluid mechanics. It only applies to the ideal fluid whose viscosity can be ignored and incompressible, that is, it is an idealized physical model. The environment in which we swimmers exercise can not be regarded as an ideal environment. We can only assume that its environment satisfies Bernoulli's principle.

Optimization of the model: for the defects of Bernoulli's principle, we can combine another famous theorem of fluid mechanics Euler motion to make necessary improvement, in order to reduce the error of the idealized model for the calculation results to a certain extent.

Model 3: fluid resistance model

The fluid resistance model mainly analyzes the resistance of the air to the arm when the swimmer's arm swings in the process of crawl. Although this part of the effort is very little or even can be ignored, in order to ensure that the data is as close to the reality as possible, we choose to keep it. For fluid resistance, we assume that the speed of the arm swing is constant. But in practice, it is difficult for an athlete to keep the consistency of his speed.

The improvement of the model: the dynamic analysis of the movement speed can be combined with the actual situation, and the mathematical model of the movement speed and the fluid resistance can be made.

## 5. Model Application

## 1, Application of Bernoulli model

(1) Airfoil lift

The streamline distribution of the air around the wing in flight refers to the asymmetry of the crosssection shape of the wing. The streamline above the wing is dense and the velocity is high, while the streamline below is sparse and the velocity is low. According to Bernoulli equation, the pressure above the wing is small, but the pressure below the wing is large.
(2) Banana balls

When the ball rotates, it will drive the surrounding air to rotate with it, so that the velocity of the air below the ball increases and the velocity above the ball decreases. The velocity below the ball is large and the pressure is small, while the velocity above the ball is small and the pressure is large. Compared with the non rotating ball, the rotating ball is forced downward because of rotation, and its flight path is bent downward.
(3) Ship suction effect

When the two ships are in parallel, because the water velocity between the two ships is faster and the pressure is lower, the velocity on the outer side is slower and the water pressure is relatively higher, the pressure difference between the port and starboard is formed, which promotes the ships to get close to each other.

2, Application of fluid resistance model
(1) Ships

Because the hull of a ship made of iron and steel is hollow, the volume of water discharged by the ship increases and the buoyancy is increased. At this time, the buoyancy of the ship is equal to its own gravity, so it can float on the water.
(2) Submarine

No matter how deep the submarine submerges, the volume of water discharged by the submerged submarine remains unchanged, so the buoyancy of the submarine remains unchanged. The buoyancy and sinking of the submarine are realized by controlling its own gravity by adjusting the amount of water in the tank with compressed air.
(3) Thermometers

Densitometer works by using the condition that the object floats on the liquid surface. When measuring the density of liquid with densitometer, its buoyancy is always equal to its gravity. Since the gravity of the densimeter is determined after it is made, the buoyancy it receives when it floats in different liquids is the same.

## 6. Conclusion

This paper takes the swimming environment as the ideal state, without considering the change of the swimmer's leg strength during the swimming process and the influence of other forces not considered in addition to this article, we can accurately solve the best posture when swimming, so that the swimmer swims as fast as possible.

First, we counted several swimming positions that are common in freestyle events. The subjects were determined to be climbing, backstroke, breaststroke and butterfly. Through the strength of different swimming positions in the water, and the distance between their body axis and the horizontal surface. Build a force model to determine which swimming position swims fastest and which swims the most stroke force.

Second, take climbing as an example. Through the analysis of the force relationship in the climbing process, the model of the swimmer's force analysis is constructed, and the relationship between the angle of the body axis and horizontal surface of the swimmer's climb can be clearly determined, so as to obtain the best angle to improve and improve the swimmer's speed.

Then, based on Bernoulli's law and fluid continuity theorists, it is not difficult to see that the water's force on both sides of the body is not the same at the same time during the swimmer's movement. That is, water gives swimmers a forward thrust.

Next, by combining Bernoulli's law and fluid continuity theorists, this paper establishes the thrust model, buoyant model and water resistance model of swimmers' swimming, and solves the relationship between the angle of the athlete's body axis and horizontal surface and the swimming speed change image, the law of time and speed change and the time, angle and velocity by using Matplotlib and Noumpy to analyze the data and draw the functional relationship image obtained by the model. In turn, the optimal swing amplitude of the athlete's body axis and horizontal surface is derived, which helps the coaches improve the athlete's swimming posture.

Finally, we conducted a comprehensive evaluation of the model, analyzed its advantages and disadvantages, as well as the limitations and errors in the processing and calculation methods of this article. At the same time, we think about the improvement and development of this model, and we analyze the application of different models in different aspects.

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