

Risk Assessment of Disease Transmission

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Abstract: The plague is one of the most feared disasters for mankind since ancient times. In recent years, the new type of coronavirus pneumonia has raged on a global scale, and it has aroused international attention. Human beings have invested in the prevention of the epidemic, and have invested considerable research during this period to allow all mankind to overcome this difficulty. This article focuses on the spread of infectious diseases and uses the Wells-Riley equation to evaluate the spread of infectious diseases in different forms of transmission. Finally, it is given how to protect yourself when an epidemic occurs, so as to minimize the risk of infection as much as possible.

1. Introduction

In recent years, with the continuous emergence of epidemics, people and scientific researchers have paid more and more attention to the transmission mechanism of epidemics. Earlier people's understanding of acquired infections was that it only spreads through contaminated droplets, and there is no other way of transmission. But after the SARS outbreak in 2003, researchers retrospectively studied airflow patterns and found that airborne transmission plays an important role in infectious diseases. Although related researches have been increasing in recent years, there are few researches on air transmission in confined indoor spaces and most of them are based on the analytical expressions of the Wells-Riley equation summarized by the research of Wells (1955) and Riley et al (1978).

However, because the air in the above model is assumed to be uniformly mixed, and it does not take into account the distance between the infected and the susceptible and the distribution of people in a confined space, therefore, it has certain limitations for some small-scale problems. Based on this problem, this article establishes a random formula and combines it with the simple contact equation of the spatial distribution of susceptible and infected persons. Solve some of the limitations of the Wells-Riley equation, so that the improved model can evaluate the risk status in a small area considering the influence of airflow and the distance from the source of infection.

2. Models

2.1 Basic Model: Droplet propagation model

2.1.1 Model analysis

The essence of droplet propagation is actually an aerosol composed of a two-phase mixture of gas and tiny droplets, and after leaving the human body at a certain initial velocity, it comes in contact with the ambient gas and mixes with the ambient gas to slow down and then spread to further places. The water vapor on the surface of the aerosol particles sprayed from the mouth will gradually evaporate under the action of the gas phase of the air. Therefore, the larger particles in the aerosol will fall in a parabolic trajectory due to the force of gravity greater than the buoyancy force; but most of the smaller particles with a diameter of 5~10 μm will first settle down under the action of gravity, and the smaller the diameter, the greater the impact of air resistance and the slower the settling speed. Most of the tiny particles, due to the settling action of the gas phase far greater than the gravity, form an aerosol cloud as shown in Figure 1.

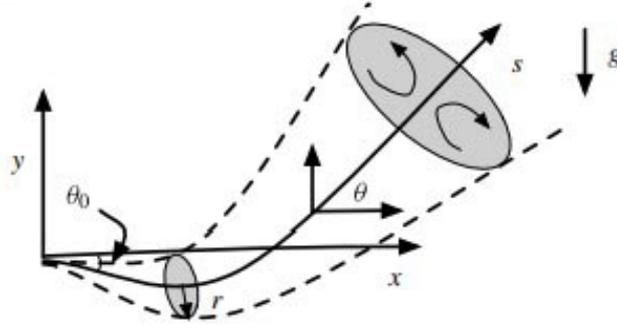


Figure 1: Trajectory of aerosol cloud

2.1.2 Model establishment

In 2014, Bourouiba et al ^[6] studied the trajectory of droplets forming an aerosol cloud and gave the corresponding analytical solution of the differential equation. The mathematical expression is:

$$\left[Z_p + \frac{\eta \rho_f s^4}{4} \right] = \frac{I_0^2}{2B_0} \left\{ \frac{\tan \theta}{\cos \theta} + \ln \left[\tan \theta + \frac{1}{\cos \theta} \right] \right\};$$

$$\theta = \arctan \left[\frac{B_0 t}{I_0} \right];$$

$$Z_p = V_0 (\rho_f - \rho_a) + \sum_{n=1} n v_p (\rho_p - \rho_f)$$

In the formula, I_0 is the initial momentum of the aerosol; B_0 is the resultant force in the vertical direction, that is, the resultant force of gravity and buoyancy; θ is the angle between the velocity and the horizontal plane, and other parameters are described in detail in the literature ^[6].

After solving the stable distribution of the aerosol in a closed space, compare its gas phase distribution with the research results of using ellipsoid to describe the shape of aerosol cloud formed by the experiment of Scorer ^[7] in 1957. The result is shown in Figure 2. This shows that the stable aerosol spatial distribution is in line with the experimental results of Scorer.

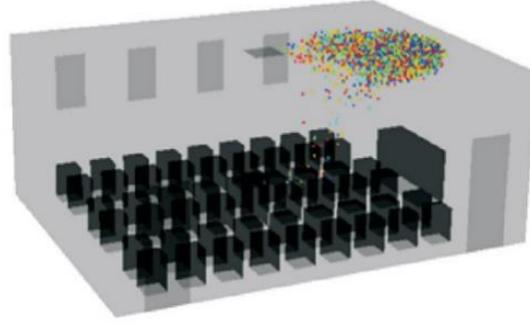


Figure 2: Spatial distribution of stabilized aerosol containing pathogens

Then, use the Stokes sedimentation formula to calculate the spatial distribution of the aerosol density when the aerosol cloud falls to the same plane as the human nose and mouth, and use the formula:

$$q = \frac{\rho_c \eta \alpha^3 s^3}{m_c}$$

Calculate the corresponding "quanta" value in the space and bring it into the revised Wells-Riley equation to calculate the probability of infection of droplet-borne diseases.

2.1.3 Model solution

Under the condition of spraying using open source simulation software CAD, the cross-sectional plane distribution of droplet propagation with distance is shown in Figure 3. Set the room temperature condition to 20°C, the relative humidity to 90%, the initial droplet velocity to 40km/h, and the initial “quanta” value to be sprayed to $q_0 = 800$ and the room is closed without considering the dilution effect of outdoor air on the room.

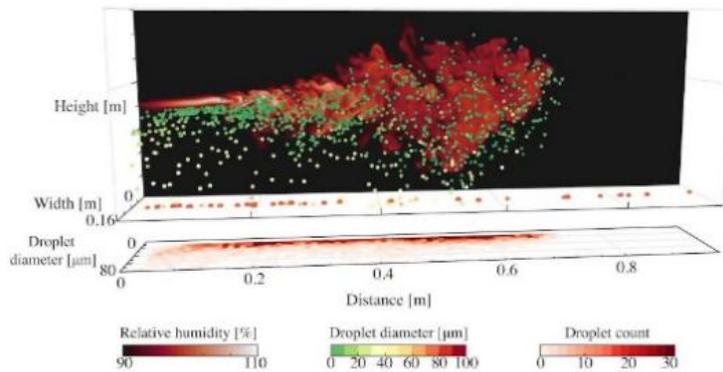


Figure 3: Section plane distribution of droplet propagation with distance

2.2 Improved Model

2.2.1 Model analysis

Considering the impact of the turbulent flow of air in enclosed environments such as large stadiums and cabins on air and droplet propagation, we chose to add a Realizable model to the original model to simulate the impact of indoor airflow on the spread of pathogens.

2.2.2 Model establishment

Consider the method of coupling the continuous phase air flow field and the aerosol cloud to carry out the numerical model, and the mathematical expression of the gas phase governing equation is as follows:

$$\frac{\partial(\rho_f N)}{\partial t} + \nabla \cdot (\rho_f v_f N) = \nabla \cdot (D \nabla N) + S$$

The numerical calculation of the particle phase adopts the discrete particle simulation (DPM) based on the Euler-Lagrangian method. When used to study the trajectory of particles spread by virus particles, it is necessary to write its motion equation. According to Newton's second law, the particle motion equation can be written as a mathematical expression in the following form ^[3]:

$$m(dv_p/dt) = F_D + mg(\rho_p - \rho_a)/\rho_p + F_i$$

$$F_D = m(18\mu/\rho_p d_p^2) \left(\frac{C_D Re}{24} \right) (v - v_p)$$

According to the research results of the literature ^[6], the Reynolds number of cough and sneeze is $1e^4 \sim 4e^4$, both of the order of 10^4 .

2.2.3 Model solution

Set the initial environmental conditions to a cube indoor environment with width, length and height of 6m, 8m, and 3m respectively, and the personnel are uniformly distributed in the space with a step length of 1m and set the crowd to the spreading object as the capture state, and set the vent outlet as the boundary condition of escape. Bring the default parameters, such as the initial quanta, ambient ventilation, ventilation frequency, etc. into the above formula and use the revised Wells-Riley equation to calculate the risk of infection. Finally, according to the calculation results, the probability of illness of people at different locations is evaluated.

3. Conclusion and Reflection

The model adopted in this article aims to simplify the complex problems in life and systematize the abstract problems, and it has strong universality or adaptability. However, in terms of data processing, due to the lack of important experimental data and many objective restrictions, the specific probability of infection has not been solved. This is our shortcoming in this task. In the future work, we will try our best to try to transform the problem in other more ingenious directions, so that the conditions for solving the problem are no longer a headache for most ordinary students.

References

- [1] Qian Hua, Zheng Xiaohong, Zhang Xuejun. Prediction Model of Infection Probability of Airborne Respiratory Infectious Diseases [J]. Journal of Southeast University (Natural Science Edition), 2012, 42(03): 468-472.
- [2] Noakes, C. J., & Sleight, P. A. (2008). Applying the Wells-Riley equation to the risk of airborne infection in hospital environments: The importance of stochastic and proximity effects. Indoor Air 2008.
- [3] Luan Yigang, Yin Yue, Li Yongxi, Fu Hao, Ma Zhengwei. Study on the diffusion characteristics of aerosol in public spaces [J]. Journal of South China Normal University (Natural Science Edition), 2021, 53(05): 1-9.
- [4] Scientific research migrant workers. Numerical solution of differential equations-finite difference method [DB/OL]. <https://zhuanlan.zhihu.com/p/411798670>, 2021-9-18
- [5] Aerosol-the real culprit behind the smog [J]. Great Science and Technology (Science Mystery), 2013(7).
- [6] Bourouiba L, Dehandschoewercker E, & Bush J. (2014). Violent expiratory events: on coughing and sneezing[j]. Journal of Fluid Mechanics, 2014, 745: 537-563.

[7] Scorer, R. S. *Experiments on convection of isolated masses of buoyant fluid [J]. Journal of Fluid Mechanics, 1957, 2(06): 583.*

[8] Lin Jiaquan, Sun Fengshan, Li Yachong. *Transmission mechanism and infection risk assessment of respiratory pathogens in the cabin [J]. Chinese Journal of Safety Science, 2020, 30(2): 6.*