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# *Unleash Your Energy with Nonlinear Programming*

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***Keywords:*** Optimization; Kinetics, Constraints nonlinearity, practicability

***Abstract:*** Cycling race track is long, at the same time, the road conditions are complex and changeable. For a cyclist, a good strategy can help him get better results on their original basis. To help riders complete the cycling race faster, we established parameters by collecting basic information about the athletes and the impact of the environment on the race. The cycling track optimization model was constructed through kinetic models and constrained nonlinear programming; considering the characteristics of human movement and the lactate level of exercise, and verified by the data of the Tokyo Olympic Games, the results are basically in line with the level of players.In addition, our model is easy to implement and extend. By changing the rider's parameters in the code, we can help the rider to find the optimal strategy in many cases.

## 1. Introduction

There are many types of bicycle road races including a criterium, a team time trial, and an individual time trial. A rider's chance of success can vary for these contests depending on the type of event, the course, and the rider's abilities. In an individual time trial, each individual cyclist is expected to ride a fixed course alone, and the winner is the rider who does so in the least amount of time.

An individual rider can produce different levels of power for different lengths of time, and the amount of power and how long a given amount of power a rider can produce varies greatly between riders. A rider's power curve indicates how long a rider can produce a given amount of power. Generally, the more power a rider produces, the less time the rider can maintain that power before having to reduce the amount of power and recover. Thus, Riders are always looking to minimize the time required to cover a given distance. Considering the background information and restricted conditions identified in the problem statement, we need develop a model that can be applied to any type of rider that determines the relationship between the rider's position on the course and the power the rider applies.

## 2. The strategic model and application of cycling road races

### 2.1 Model preparation

The full name of $FTP$ is called Functional Threshold Power. The origin of "functionality" here is from the lecture of $FTP$ inventor Dr. Stannard Stephen R. As the intensity of our activity increases, the more energy production increases, and the body's ability to discharge acid increases. When the intensity continues to rise, the amount of lactic acid exceeds the limit of acid discharge, lactic acid will accumulate in the blood, at this point we can no longer continue the movement.

There are four very important values, which are *FTP*, $TP_0$, $TP_1$, $TP_2$, where, $TP_1$ means sprint abilities in 5 seconds, $TP_0$ means anaerobic capacity described by 1 minute maximum power, $TP_2$ tells us about *VO2* max capability and 60 min to describe our *FTP*.

## 2.2 The Establishment of Model

To clearly describe the rider's power profile under the influence of wind factors and energy consumption (body fatigue). We present Figure 1 for rider stress during exercise.

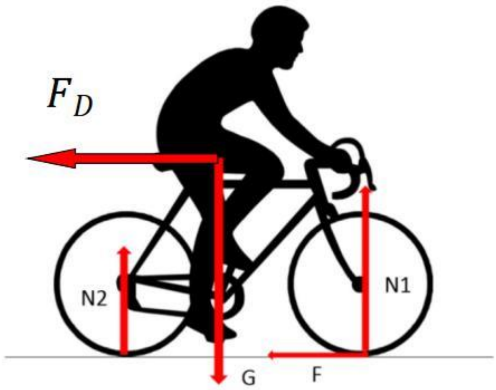


Figure 1: Force analysis of the rider sustained during exercise

Since our model aims to address the relationship between the rider's position on the track and the force exerted by the rider. We divide the whole track according to different road conditions, which is divided into four types: uphill, downhill, turn, and straight track. Different output work models are obtained by analyzing the forces of the riders under different types of road conditions. We build a constraint relationship between output work models and *FTP*. Using a nonlinear programming with constraints, we get the optimal allocation between time and energy. Finally, the shortest time to complete the competition was obtained from the computational analysis described above. So we get the following calculation formula:

$$W_{ALL} = \sum \left( W_{fi} + \gamma_i \right) \tag{1}$$

Where, $\gamma_i$ is the power limit caused by the rider on the section $i$+ 1 journey due to fatigue during cycling. $W_{fi}$ indicates the value of the output work by the rider through section $i$. $W_{All}$ is the maximum energy (output work).

## 2.3 The Solution of Model

(1) The power limit of the rider on the next ride due to fatigue $\gamma_i$:

$$\gamma_i = \varepsilon t_i \tag{2}$$

Where, $\varepsilon$ indicates the fatigue factor of the rider during riding [1],

$$\varepsilon = \alpha \frac{TP_1}{FTP} \tag{3}$$

Where, $TP_1$ is the average power output from the unit quality rider sprint phase. *FTP* is the maximum average power output of one hour by the unit quality rider. The ratio of $TP_1$ and *FTP* should be positively correlated with the recovery time required after the rider sprint. α is the fatigue correlation [2].

(2) The maximum energy (output work) $W_{All}$

$$W_{ALL} = FTP\sum_{k=0}^{i-1} t_k \tag{4}$$

The product of $FTP$ and the sum of time t of the rider takes through the track with different lengths, which means what the rider can output throughout the race.

(3) The value of the output work by the rider through a section of the track $W_{fi}$

$$W_{fi} = \sum_{k=0}^{i-1} P_{all0k} t_k \tag{5}$$

Due to constraints, we define a unit of mass average power $P_{all0}$:

$$\begin{cases} F_D = \tau v^2 \\ F_G = mg\sin\theta \\ v = \dfrac{d}{t} \\ \tau = \dfrac{1}{2} C_\rho S \end{cases} \tag{6}$$

According to Xu Wensheng's formula:

$$\begin{aligned} S_{\text{man}} &= 0.0057h + 0.0121w + 0.0882 \\ S_{\text{woman}} &= 0.0073h + 0.0127w - 0.2106 \end{aligned} \tag{7}$$

Where, $S_{man}$ is male body surface area. $S_{woman}$ is female body surface area. $h$ indicates height, $w$ indicates weight. We can get

$$S = \frac{1}{2} S_{\text{man}} \sin\beta \text{ or } S = \frac{1}{2} S_{\text{woman}} \sin\beta \tag{8}$$

Where, $\beta$ indicates the wind Angle during riding.

Thus we obtain the calculation formula for the final $P_{all0}$:

$$P_{all0} = \frac{P_{all}}{m} = \frac{\tau d^3}{mt^3} + g\sin\theta\frac{d}{t} \tag{9}$$

We define that a sharp turn with a turn angle greater than 90 ° affects the rider's riding speed, To ensure that the rider makes a sharp turn smoothly, The rider must receive less centripetal force than the static friction in the curve:

$$\frac{v_s^2}{R} \le \mu g \tag{10}$$

Where, $R$ is the radius of the curve, $\mu$ is tatic friction factor in curves (By comparing the static friction factor of the curves of different road cycling lanes, we chose =0.6)

By observing the players' handling skills of sharp turns during the competition, we defined the rider as only producing linear changes in speed at 50m before and after the turn [3]. Therefore, the average speed of the curve distance is

$$v_b = \frac{v + v_s}{2} \tag{11}$$

We define $q$ for the different road conditions of the whole track, and track road conditions are curves when $q$ = 1. In summary, based on kinetic and constrained nonlinear models we can derive cycling race strategy models:

$$\min z = \mathrm{sum}(t) \tag{12}$$

$$\text{s.t.} \begin{cases} \sum_{k=0}^{i-1} P_{all0k} t_k - FTP \sum_{k=0}^{i-1} t_k \le 0 \\ P_{all0i} - TP_2 + \varepsilon \sum_{k=0}^{i-1} t_k \le 0 \quad (i = 1, 2, \cdots, n) \\ q(i) v_{i-1} + q(1) v_s - 2 v_i = 0 \\ t_i > 0 \end{cases} \tag{13}$$

## 2.4 Model application for example analysis

(1) Track overview

2021 Olympic Time Trial course in Tokyo, Japan used the Fuji International Raceway. It is a racing track in southwest Tokyo, situated in the shadow of the majestic Mount Fuji. Figure 2 shows the track map of the individual time trial at the 2021 Olympics in Tokyo, Japan. Where the women's individual time trial track is 22.1km, and the men's individual time trial track is 44.2km. Figure 3 represents respectively the time trial circuit profile of men's individual time trial, Figure 4 represents the time trial circuit profile of women's individual time trial.

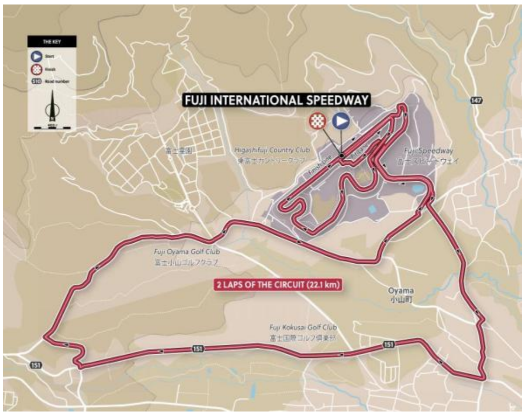


Figure 2: Map of the track for the individual time trial at the 2021 Olympics in Tokyo, Japan

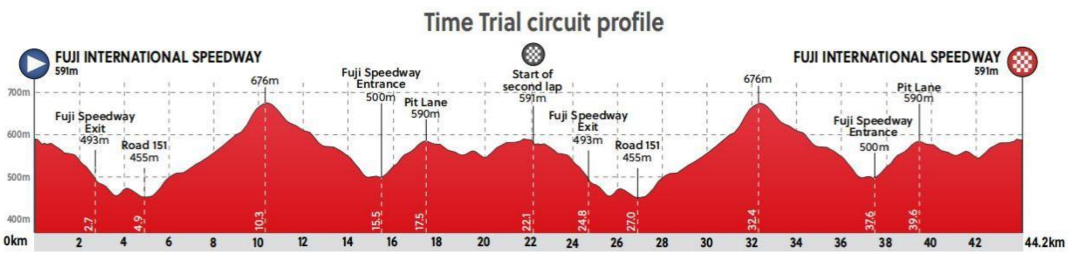


Figure 3: Time Trial circuit profile of men's individual time trial

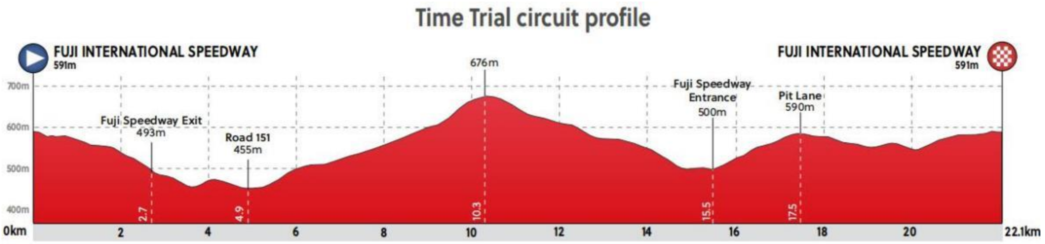


Figure 4: Time Trial circuit profile of women's individual time trial

(2) Data processing and results

Due to the complexity of road conditions, in order to simplify the data processing process, we divided the track into straight lanes and curves according to the different road conditions of the track. It is known to complete the acceleration and deceleration within the specified curve length (50m), the final curve speed is the average speed. The straight and curve segmentation into a series of segmented tracks. The time and average speed used by the men's time trial rider through the straight and curve were calculated respectively, and the results are shown in 错误!未找到引用源。 Similarly, the results of women are shown in Table 2 .

Table 1 The men's individual time trial event at the 2020 Tokyo Olympics

| **Men Rouleur** | | | | **Men Puncheur** | | | |
|---|---|---|---|---|---|---|---|
| Total Time(sec) | | Average Speed(m/s) | | Total Time(sec) | | Average Speed(m/s) | |
| straightaway | curve | straightaway | curve | straightaway | curve | straightaway | curve |
| 340.7858 | 18.0818 | 13.7917 | 11.0608 | 305.6775 | 16.8735 | 15.3756 | 11.8528 |
| 391.3416 | 4.5452 | 13.6709 | 11.0005 | 348.2094 | 4.2204 | 15.3643 | 11.8471 |
| 375.8289 | 4.5386 | 13.7030 | 11.0165 | 335.1259 | 4.2198 | 15.3673 | 11.8486 |
| 102.0072 | 7.6994 | 17.6458 | 12.9879 | 114.8606 | 8.3329 | 15.6711 | 12.0005 |
| 317.2937 | 9.0101 | 13.8673 | 11.0986 | 286.0353 | 8.4342 | 15.3827 | 11.8563 |
| 340.7859 | 18.0818 | 13.7917 | 11.0608 | 325.0557 | 17.5522 | 14.4590 | 11.3945 |
| 392.3416 | 4.5452 | 13.6709 | 11.0005 | 412.7449 | 4.6966 | 12.9620 | 10.6460 |
| 375.8288 | 4.5386 | 13.7030 | 11.0165 | 447.2531 | 5.0391 | 11.5147 | 9.9223 |
| 102.0073 | 7.6994 | 17.6458 | 12.9879 | 120.1904 | 8.5813 | 14.9762 | 11.6531 |
| 317.2937 | 9.0101 | 13.8673 | 11.0986 | 531.0815 | 12.0373 | 8.2849 | 8.2074 |
| The best time: $3.1423 \times 10^3$ | | | | The best time: $3.3162 \times 10^3$ | | | |

Table 2: The women's individual time trial event at the 2020 Tokyo Olympics

| **Women Rouleur** | | | | **Women Puncheur** | | | |
|---|---|---|---|---|---|---|---|
| Total Time(sec) | | Average Speed(m/s) | | Total Time(sec) | | Average Speed(m/s) | |
| straightaway | curve | straightaway | curve | straightaway | curve | straightaway | curve |
| 380.5251 | 19.3410 | 12.3513 | 10.3406 | 398.4742 | 16.8735 | 15.3756 | 11.8528 |
| 436.5157 | 4.8576 | 12.2561 | 10.2930 | 457.2839 | 4.2204 | 15.3643 | 11.8471 |
| 419.3300 | 4.8516 | 12.2814 | 10.3057 | 439.2350 | 4.2198 | 15.3673 | 11.8486 |
| 117.5338 | 8.4585 | 15.3147 | 11.8223 | 114.8606 | 8.3329 | 15.6711 | 12.0005 |
| 354.5272 | 9.6427 | 12.4108 | 10.3704 | 286.0353 | 8.4342 | 15.3827 | 11.8563 |
| The best time: $1.7556 \times 10^3$ | | | | The best time: $1.8362 \times 10^3$ | | | |

## 3. Effect of the weather conditions on the model

Because the weather changes are too complicated. In order to simplify the model, we ignore the impact of extreme weather on the race, and focus on the impact of the direction of the wind on the track and the size of the wind on the rider's speed. In addition, we consider the impact of light rain on the players.

### 3.1 Influence of wind direction and wind force

As the direction of the wind varies, we consider when the rider rides in the wind. It may be downwind or upwind. Based on the strategy model of bicycle road race, considering the influence of wind, we established and optimized the following model:

$$\min z = \text{sum}(t) \tag{14}$$

$$\text{s.t.} \begin{cases} \sum_{k=0}^{i-1} \tau v_{rk}^2 v_k t_k - FTP \sum_{k=0}^{i-1} t_k \le 0 \\ \tau v_{ri}^2 v_i - TP_2 + \varepsilon \sum_{k=0}^{i-1} t_k \le 0 \qquad (i = 1, 2, \cdots, n) \\ t_i > 0 \end{cases} \tag{15}$$

Where, $v_r$ is the rider's relative speed to the front of the wind.

$$v_r = (v + v_w)\cos\delta \tag{16}$$

We define $v_w$ representing the wind speed and the relative speed of the rider. The value range of the $v_w$ is $-10m/s$~$10m/s$. $\delta$ is the angle between the direction of the wind and the track.

According to the model we established above, we use the track data of the men's time trial of the 2020 Tokyo Olympic Games to analyze the impact of the north wind on the model. We divide the Fuji international track into 10 parts. Bring the data of each part of the track into the model to obtain the change of the time taken by the rider to pass through each part of the track. Figure 5 shows the three-dimensional diagram of the time required for the rider to pass through each part of the track under the influence of different wind speeds.

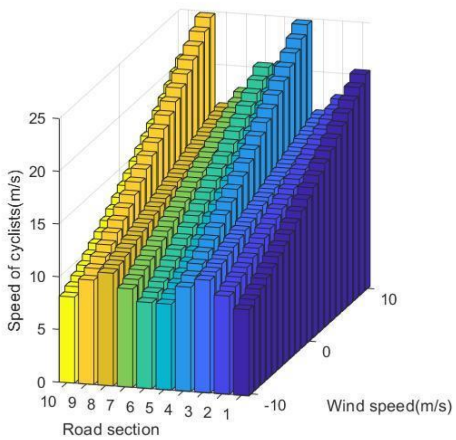


Figure 5: The three-dimensional diagram of the time required for the rider

## 3.2 Effect of small amount of precipitation on Model

According to the data, as long as the road state changes (from dry to wet), it will have a significant impact on the friction coefficient. When the water film thickness is 3mm, the friction coefficient of wet pavement decreases compared with wet pavement, and the maximum decrease is about 37.2%. The friction coefficient of wet pavement is significantly lower than that of dry pavement, and the maximum decrease is about 42.1%.

Therefore, in such weather conditions, players should slow down when crossing corners to avoid rollover.[2]

## 4. The development of model

According to the literature, in the case of following riding, except for the first rider, the resistance coefficient of other riders decreases by 40%[4].

Therefore, our basic model is extended. Assume that six players are composed of different types of players, the first journey by the team a contestant number as the first rider, and follow the cycling speed with the same velocity, the first journey first contestants wind resistance coefficient is constant, and other players drag coefficient reduced by 40%, for following the first ride, the size of the energy is less than the value of the individual time trial, Therefore, the fatigue factor of these 5 players was reduced by 40%. After the completion of the first section of the track, the runner who consumes the least energy will be the first runner in the second section of the track. It is stipulated that n=1 represents the first runner, and n=0 represents the following cyclist. According to the value of n, whether the runner is a wind breaker is determined. We took the total time t of the team after the completion of the competition as the total result of the team competition, and obtained the following extended model:

$$\begin{gathered}\sum\left(\frac{40\%n\tau v_p^2+F_G v_p}{m}+40\%n\varepsilon t\right)-FTP\sum t\le 0\\ n=\left[\min\left(\frac{40\%n_0\tau v_p^2+F_G v_p}{m_i}+40\%n_0\varepsilon t\right)/\ \text{others}\right]:[1/0]\end{gathered}\tag{17}$$

Where $n_0$ indicates whether the player is a wind breaker in the last stage. $v_p$ represents the time spent by the team following the uniform average speed of the windbreaker team in the upper stage, $t_p$ represents the time spent by the team in the upper stage. Formula 17 says that if the rider has expended the least amount of energy on the previous stretch, then the rider will be the first rider on the next stretch.

## 5. Conclusion

We integrated cycling race strategy models based on kinetic and constrained nonlinear models. We brought the time trial data at the 2021 Tokyo, Japan, we obtained the results for the time and average speed of the men's and women's time trial riders through the straights and corners Therefore, the accuracy of our model is demonstrated.

We considered the effect of the weather conditions on the model. We focused on the effect of the wind direction and the intensity of the wind on the rider speed. In addition, we analyze the bias of the results with the target results through our model when the rider deviations from the target power distribution. Our model was further optimized. The anti-interference ability had also been improved.

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