# Rational predictive analysis of fire rescue problems based on optimal path algorithm

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*Keywords:* Fire rescue, Gray prediction model, Optimal path algorithm, Linear regression

*Abstract:* In this paper, based on the relevant data of fire rescue in five years in a certain place, through the establishment of mathematical model, aiming at the prediction of the number of fire officers and the reasonable planning of the fire station. And use Matlab and other software to establish the model for solving analysis. First of all, on the basis of the data in the first four years of fire rescue, in the month, using the classic grey prediction model to establish the forecast model of the fire rescue response times, and in the fifth year of data validation data set as a model, the accuracy and stability of the evaluation model, on this basis, for the future predict fire rescue response times of the year; Finally, according to the existing data, Dijkstra algorithm in the optimal path algorithm is used to solve the optimal distance of linear programming. By establishing the linear programming mathematical model to calculate the total cost forecast of fire station, the reasonable sequence of fire station construction area in the next 9 years is obtained. The results can provide reference for the construction planning of fire station.

## **1. Introduction**

In all stages of fire rescue work, it is necessary to combine the actual situation for reasonable layout planning, such as the deployment of fire rescue teams, the police situation, the location and construction of fire stations, etc. By establishing a mathematical model, the prediction of the number of fire rescue calls and the construction planning of fire stations can be realized to ensure the smooth implementation of fire rescue work<sup>[1].</sup>

This paper uses mathematical methods to achieve reasonable prediction and analysis of fire rescue problems based on fire rescue data over a five-year period. Firstly, by analyzing the data, a prediction model of the number of fire rescue calls is established, and on this basis the established model is used to predict the number of fire rescue calls in the coming year. Then, based on the relevant information and considering various factors, the reasonable location for building fire stations in the future is determined by establishing a mathematical model.

## 2. Model building and solving

#### 2.1 Prediction of the number of police calls based on gray time series prediction model

## 2.1.1 Principle of the model

The data of fire rescue work in a place in five years (2016-2020) were selected to establish a suitable mathematical model by analyzing the number of police calls from 2016 to 2019, in months, for a total of 48 months. And based on the data of a whole year in 2020 to judge the accuracy and stability of the model, in the analysis of the data, the average value of each indicator for the current year was calculated using Excel to facilitate the prediction of each indicator and the calculation of valuation indicators<sup>[2]</sup>. Therefore, this paper uses the gray time series forecasting model and establishes the GM (1,1) model respectively. the forecasting principle of the GM (1,1) model is: a new set of data series with obvious trend is generated for a certain data series by accumulation, and a model is built for forecasting according to the growth trend of the new data series, and then the forecasting results<sup>[3]</sup>. The gray time series forecasting, using the observed five-year data series of the trend of the predicted object to construct a gray forecasting model, predicts the number of fire rescue calls in each month in the future 2021.

## 2.1.2 Establishment of Model

The gray time series prediction model is built in the following steps.

**Step 1:** Testing and processing of data (1,2,...,48 represent each month from 2016-2019)

In order to ensure the feasibility of the GM (1,1) modeling approach, the necessary validation processing of the data is required. With the original time series

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(48))$$
(1)

48 is the number of data, and thex<sup>(0)</sup> accumulation in order to weaken the volatility and randomness of the random sequence and obtain the new series as

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(48))$$
(2)

Among them 
$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, ..., n$$

Find the level ratio  $\lambda(k)$ , where  $\lambda(k) = \frac{x^{(0)}(k-1)}{x^{(0)}(k)}$ , by checking whether the level ratio falls

within the tolerable coverage interval  $X = \left(e^{\frac{-2}{n+1}}, e^{\frac{2}{n+1}}\right)$ . So our data column can be built with GM(1,1)

model for gray prediction. If there are data that are not in the data column, appropriate transformations are made to the data, such as translation transformations, so that all the data are within the tolerable coverage.

**Step 2:** Build GM(1,1) model

The model with  $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(48))$  as the data column is

 $x^{(0)}(k) + az^{(1)}(k) = b$  (3)

By regression analysis we solve the estimates a, b, and the corresponding whitening models as

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$$
(4)

The solution is

$$x^{(1)} = (x^{(0)}(1) - \frac{b}{a})e^{-a(t-1)} + \frac{b}{a}$$
 (5)

The predicted value is then

$$x^{(1)} (k+1) = (x^{(0)}(1) - \frac{b}{a})e^{-ak} + \frac{b}{a}, k = 1, 2, ..., n-1$$
(6)

Step 3: Generate B with a vector of constant terms by doing the mean on the cumulative generated  $dataY_n$ 

$$B = \begin{bmatrix} -z^{(1)}(2) & \dots & 1 \\ -z^{(1)}(3) & \dots & 1 \\ \dots & \dots & \dots \\ -z^{(1)}(48) & \dots & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & \dots & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & \dots & 1 \\ \dots & \dots & \dots \\ -\frac{1}{2}(x^{(1)}(47) + x^{(1)}(48)) & \dots & 1 \end{bmatrix}$$
(7)  
$$Y_{n} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \dots \\ x^{(0)}(48) \end{bmatrix}$$
(8)

Among them  $z^{(1)}(k) = 0.5x^{(1)}(k-1) + 0.5x^{(1)}(k), k = 2,3,...,n$ Step 4: Using least squares to solve for the gray parameter  $\hat{a}$ ,  $\hat{a} = (B^T B)^{-1} B^T Y_n$ 

**Step 5:** Substitute the gray parameter  $\hat{a}$  into  $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$  and solve for  $\frac{dx^{(1)}}{dt} + ax^{(1)} = b$  to obtain

$$\begin{array}{c} \overset{\wedge^{(1)}}{x} & \overset{\wedge^{(0)}}{x} \\ x & (k+1) = (x & (1) - \frac{b}{a})e^{-at} + \frac{b}{a} \\ \end{array} \tag{9}$$

Step 6: Reduce the above results to get the predicted value

$$\begin{array}{ccc}
\wedge^{(0)} & \wedge^{(1)} & \wedge^{(1)} \\
x & (t+1) = x & (t+1) - x & (t) \\
\end{array} (10)$$

Step 7: Use the model to make predictions

$$\begin{array}{c} \wedge \quad \wedge^{(0)} \quad \wedge^{(0)} \quad \wedge^{(0)} \quad \wedge^{(0)} \\ x = x \quad (1), x \quad (2), \dots, x \quad (48), x \quad (49), \dots, x \quad (48+m) \end{array}$$
(11)

The original data are presented before group 48, and the predicted data are presented afterwards.

## Step 8:

(1) Accuracy check of the established gray model

Residuals. 
$$E(k) = x^{(0)}(k) - x^{(0)}(k)$$

Variance. 
$$Q(k) = \frac{E^{(0)}(k)}{x^{(0)}(k)}, k = 2, 3, ..., n$$
 (12)

(2) Comparison of prediction accuracy levels

From top to bottom, the order is good, pass, barely pass, and fail.

$$\begin{cases} P > 0.95C < 0.35 \\ P > 0.80C < 0.45 \\ P > 0.70C < 0.50 \\ P <= 0.70C >= 0.65 \end{cases}$$
(13)

## **2.1.3 Calculation results**

(1) Accuracy and stability evaluation of the gray prediction model

p=1; c=0.4; predicted grade: pass.

From the operation results, for linear data using GM(1,1) prediction, its prediction effect still tends to be stable, and with this model can predict the 10 months of 2020 data and the real data comparison passed a higher precision test. The gray forecasting model is suitable for medium and long-term forecasting, and it has to be tested to determine whether it is reasonable or not. It is more reasonable to combine the models, using two or more models together, to achieve better accuracy and stability.

(2) Solution of the gray prediction model

According to the model, the number of fire rescue calls for each month in 2021 can be predicted, and the results are shown in Figure 1: the horizontal coordinate of the model is the month, and the vertical coordinate is the number of calls for each month. By constructing a linear model of the number of police calls for each month from 2016 to 2020, the number of police calls for each month in 2021 is then predicted, and the accuracy of the prediction model is verified.



Figure 1: Predicted results

According to the above steps, the GM(1,1) model was implemented by Matlab software, and the calculated prediction results are shown in Table 1.

Month	Predictive value / times		
January 2021	28		
February 2021	33		
March 2021	35		
April 2021	36		
May 2021	38		
June 2021	39		
July 2021	40		
August 2021	42		
September 2021	43		
October 2021	44		
November 2021	46		
December 2021	47		

Table 1: Monthly forecast results

## 2.2 Fire station construction planning based on linear programming model

## 2.2.1 Selection of the model

In the process of fire station siting planning, several factors need to be considered to ensure the rationality of construction planning, a new fire station is planned to be built every three years in the next nine years in a place studied in this paper, in order to make full use of resources and the lowest cost of police attendance, a linear programming model (decision variable is region X, objective function is the lowest total cost of police attendance) and linear weight analysis method (the degree of influence of various factors is not The constraints to be considered in this model are the spatial distribution of each type of incident density in each region and the relationship between the density of each type of incident and the population density (the requirements of different categories and time on how much police force is not considered), and the optimal distance from the fire station to each region (the optimal distance is calculated with the shortest-circuit algorithm, such as Dijkstra and Floyd's algorithm, etc.); finally, the optimal construction site plan of the fire station is obtained<sup>[4]</sup>.

#### 2.2.2 Modeling

In this paper, a fire station is selected as the object of study. Among the 15 regions (numbered A-P respectively) in the area, two fire stations are already located in region J and region N. It is necessary to calculate the shortest distance of the other regions from the two regions respectively, so as to make the shortest distance to the police and reach the incident location most quickly. Here Dijkstra algorithm is used to solve the shortest distance of each region from region J and region N. Dijkstra (Dijkstra) algorithm is a typical single-source shortest path algorithm for calculating the shortest path from one node to all other nodes<sup>[5]</sup>. The main feature is to expand outward in layers with the starting point as the center until the expansion reaches the end point.

The main idea of the algorithm is: let G=(V,E) be a directed graph with weights, divide the set V of vertices in the graph into two groups, the first group is the set of vertices for which the shortest path has been found (denoted by S. Initially, there is only one source point in S. Later, every time a

shortest path is found, it will be added to the set S until all the vertices are added to S, and the algorithm ends), the second group is the set of vertices for which the shortest path has not been determined (denoted by U), and the vertices of the second group are added to S in increasing order of the shortest path length. During the joining process, the shortest path length from the source point v to each vertex in S is always kept no greater than the shortest path length from the source point v to any vertex in U. In addition, each vertex corresponds to a distance, and the distance of a vertex in S is the shortest path length from v to this vertex, and the distance of a vertex in U is the current shortest path length from v to this vertex including only the vertices in S as intermediate vertices.

According to the algorithm principle, the shortest distance between each region and region J and region N was obtained by programming and calculation on Matlab software, as shown in Table 2.

	Area J / km	Area N / km	
Area A	17.8	21.4	
Area B	28.9	22.8	
Area C	20.4	17.7	
Area D	12.7	10	
Area E	25.5	18.6	
Area F	18.1	11.2	
Area G	12.9	10.6	
Area H	26.2	33.1	
Area I	35.2	42.1	
Area J	0	6.9	
Area K	9.5	16.4	
Area L	13.9	20.8	
Area M	9.6	16.5	
Area N	6.9	0	
Area P	4.2	5.9	

Table 2 The shortest distance between each region and region J and N

For the siting of fire stations, we used research methods including: principal component analysis, hierarchical analysis, kernel density analysis, location-configuration model and network analysis<sup>[6]</sup>. Firstly, we construct the event risk assessment index system, determine the weights of factors by using principal component analysis and hierarchical analysis, and overlay each index according to its weight size by using kernel density analysis and superposition analysis of GIS to discern the spatial distribution of event risk in the study area. As shown in Table 3.

First-level indicators	First-level indicators weight	Second-level indicators	Second-level indicators weight
The event type	0.5	1	0.1
		2	0.2
		3	0.2
		4	0.1
		5	0.1
		6	0.2
		7	0.1
The population density	0.2		
The optimal distance	0.3		

Table 3 Event risk assessment indicators and weight allocation

On the basis of the regional network model, the layout strategy of fire stations is studied using the zone-configuration model and GIS network analysis technology. The model established is as follows.

(1) Model for minimizing fire station call-out costs

The model operation principle is the minimum cost of fire station to each area to respond to the police, which is used to calculate the minimum distance of each area of the required fire station, so that the cost of responding to the police is minimized.

$$\min \sum y_i$$
s.t. 
$$\begin{cases} \sum_{d_{ij} \leq D} x_{ij} \geq 1, \forall i \in I \\ y_i \in (0,1), \forall i \in J \\ \end{cases}$$
(14)

where: i represents the area point, I represents the set of area points,  $I = \{i | i = 1, 2, ..., 15\}$ ; J represents the set of candidate fire station addresses,  $J = \{j | j = 1, 2, 3\}d_{ij}$  represents the minimum distance between area i and candidate fire station  $j, i \in I, j \in J$ ; D represents the maximum distance allowed between an area and a fire station. $x_{ij}$  represents area i being served by candidate fire station  $j.y_j = 0$  represents the demand point selected at  $j.y_j = 1$  represents the demand point selected at other location.

(2) Model for maximizing coverage

The model operates on the principle of covering the maximum number and range of demand points within a threshold, and is used to study the calculation of the maximum range of coverage under the condition that the fire stations are determined, and the minimum number of fire stations under the condition that the range of coverage is determined. On the result of minimizing the number of fire stations model, the location of fire stations is optimally adjusted so that the fire stations reach the maximum value of coverage.

$$s.t.\begin{cases} \sum_{d_{ij} \leq D} y_i - x_i \geq 0, \forall i \in I\\ \sum_{j \in I} y_i = p\\ y_i \in (0,1), \forall j \in J \end{cases}$$
(15)

Where: i represents the area point, I represents the set of area points,  $I = \{i | i = 1, 2, ..., 15\}$ ; J represents the set of candidate fire stations,  $J = \{j | j = 1, 2, 3\}d_{ij}$  represents the minimum distance between area point i and candidate fire station  $j, i \in I, j \in J$ ; D represents the maximum distance allowed between area point and fire station; p represents the number of candidate fire stations. $y_i = 0$  represents the fire station selected at j, and  $y_i = 1$  represents the demand point selected at other locations. $y_i$  represents the service capacity of candidate facility point j. $w_i$  represents the weight of demand point i.

## 2.3 Solving of the model

The results can be obtained by solving the above two models. After comprehensive consideration of various factors, the establishment sequence of the fire station is finally determined as E area first, H area last, and B area last.

### **3.** Conclusion

This paper uses mathematical methods to make reasonable predictions and analysis of fire rescue problems. The characteristics of the gray system prediction model used in the fire rescue prediction are that in this paper a large number of data samples are not needed, the sample data is relatively small, only the short-term prediction of each month in 2021 is good, and the gray model can predict the data more accurately; the linear programming model method used in the fire station construction planning The linear programming model method used in fire station construction planning is simple and easy to implement, and the operation process is relatively simple and easy to apply to real life.

However, in the process of model building, because there are many indicators reflecting the construction of ecological civilization, different researchers have different understanding and awareness, and the selection of indicators and the setting of indicator weights also have their own focus, and often these differences will produce different evaluation results. In the subsequent research, the influence of each evaluation index on the results can be considered in combination to make the results more accurate.

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