Design and analysis of "figure-8" trajectory flapping wing mechanism based on crank rocker

Shan Tang
College of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China

Keywords: crank rocker, flapping wing mechanism, figure-8 trajectory

Abstract: In order to improve the flight performance of flapping-wing aircraft, a new type of flapping-wing mechanism based on a crank-rocker mechanism was designed by the flight motion characteristics of small and medium-sized birds. Firstly, the kinematics model of the flapping-wing aircraft drive mechanism is established through kinematics analysis. Then, the simulation analysis model of the flapping-wing mechanism is established in the ADAMS simulation software to verify the theoretical analysis. The results show that the designed driving mechanism can realize flapping and twisting movements through a single degree of freedom drive. The upper maximum flapping angle is 28.07°, the lower maximum flapping angle is 25.02°, and the maximum torsion angle is 2.40°. The output "8" shape trajectory is the same as the wingtip trajectory when the creature is flying, and has good aerodynamic performance; the kinematic parameters obtained from the simulation are consistent with the theoretical calculation, which verifies the correctness of the theoretical calculation.

1. Introduction

The flapping-wing aircraft is a small aircraft based on the principle of bionics. Flapping-wing aircraft has good aerodynamic performance, strong flight flexibility and anti-interference ability. It has become a research hotspot at home and abroad, and has a wide range of application prospects, such as low-altitude reconnaissance, environmental monitoring and so on.

The current flapping mechanisms can be divided into two categories. One of them is an innovative single-degree-of-freedom mechanism based on the crank-rocker mechanism. For example, Pornsin-Sirirak [1] et al. designed a miniature bird-like aircraft based on a single-crank and double-rocker mechanism., Xu Yicun [2] et al. constructed a kind of miniature bird imitation aircraft based on space crank rocker flapping mechanism that can effectively improve the symmetry. The other type is a multi-degree-of-freedom mechanism that realizes the flapping, twisting and folding of the wings through the combination of multiple sets of links. For example, M. Sitti [3] designed a flapping wing driven by piezoelectric bimorphs. Ruan Longhuan [4] designed a flapping-torsion mechanism which has two degrees of freedom similar to the flapping of a hummingbird. The multi-degree-of-freedom flapping-wing drive mechanism is mostly a space mechanism, which can realize a complex combination of various motions, but the mechanism is relatively large and complex, can not be well applied to micro and small flapping-wing aircraft.

The flapping mechanism is often designed with the goal of realizing the flapping process of the
birds or insects. Among them, the most representative wing flapping trajectory is the "figure-8" trajectory. The "figure-8" trajectory can make the flapping airfoil have a positive angle of attack in the whole flapping cycle, and reuse the energy of the previous flapping well [5]. Zhang Hongmei [6] et al. found that the "figure-8" trajectory can significantly increase the lift force. Therefore, it is of great significance to design a flapping mechanism which can realize the "figure-8" trajectory.

To sum up, the flapping mechanism with a single degree of freedom and capable of realizing the "8" trajectory is very practical and easy to implement. Based on the crank-slider mechanism and the crank-rocker mechanism, this paper designs a crank-rocker mechanism that can realize the "8"-shaped wingtip trajectory. The simulation proves that this mechanism has practical use value.

2. Institutional comparative analysis

2.1 Analysis of flapping mechanism

Zhu Baoli et al. [7] proposed a flapping wing mechanism based on the seven-bar and eight-hinge mechanism, as shown in Figure 1. The mechanism has two degrees of freedom, which enables the flapping wing to twist around the spanwise axis, and can generate a 8-shaped or banana-shaped trajectory at the C point. But its structure is much complex, and the energy utilization rate is low. Jiang Sen [8] proposed a mechanism that couples the space RSSR mechanism and the space RRSSR mechanism, as shown in Figure 2, which realizes the twist and flapping of the wingtip at the same time, and can form an "8"-shaped motion trajectory. But it requires two driving device so it is not conducive to miniaturization. Therefore, in order to reduce the complexity of the flapping mechanism, this paper adopts a single input drive to realize the "figure-8" movement.

![Figure 1 Motion diagram of multi-degree-of-freedom flapping wing mechanism based on seven-bar and eight-hinge mechanism](image)

![Figure 2 Space linkage mechanism](image)

Jiang Jingqiang et al. [9] used a gear linkage mechanism to achieve an "8"-shaped trajectory, as shown in Figure 3. But there are gear pairs in the mechanism, which reduces the transmission efficiency and increases the weight and volume.
To sum up, in order to reduce the volume and mass of the driving mechanism of the flapping-wing aircraft and realize the 8"-shaped motion trajectory, based on the crank-slider mechanism and the crank-rocker mechanism, this paper designed a flapping wing mechanism with planer drive, whose tip of wing has a pointed "8"-shaped trajectory. Figure 4 shows the motion diagram of the mechanism.

2.2 Degree of freedom calculation

To determine whether the mechanism has a definite motion, the degrees of freedom of the flapper drive mechanism should be calculated. According to the calculation formula of the degree of freedom of the planar mechanism:

\[ F = 3n - 2P_l - P_h \]  

(1)

In the formula, \( n \) is the number of active components, \( P_l \) is the number of lower pairs, \( P_h \) is the number of higher pairs.

In the flapping wing drive mechanism, the number of active components is 5, the number of revolute pairs is 5, the number of sliding pairs is 2. So the number of lower pairs is 7, the number of higher pairs is 0. The degree of freedom of the flapping drive mechanism of this aircraft \( F \) is:

\[ F = 3 \times 5 - 2 \times 1 \times 0 = 1 \]  

(2)

That is, the designed flapping-wing drive mechanism is a single-degree-of-freedom mechanism, and the drive mechanism has a definite motion.

3. Kinematic analysis of flapping wing mechanism

3.1 Parameter setting

In order to facilitate the study of the flapping wing drive mechanism, taking the crank’s endpoint \( O_1 \) as the origin of coordinates, the \( O_1O_2 \) direction as the x-axis, and the direction perpendicular to \( O_1O_2 \) as the y-axis to establish a plane rectangular coordinate system, which is shown in Figure 5. Let
the crank $O_A$ length be $l_1$, $O_B$ length be $l_2$, rocker $BC$ length be $l_3$, distance between two fixed hinges be $e$, distance between the slider center of mass and the right hinge be $f$, crank angle be $\theta_1$, crank angular velocity be $\omega_1$, rocker swing angle be $\theta_2$, rocker angular velocity be $\omega_2$, flutter angle of the mechanism be $\varphi$, twist angle be $\psi$, flutter angular velocity be $\mu$, torsional angular velocity be $\nu$.

![Figure 5 Mechanism coordinate system and rod length definition](image)

According to the coordinate system established in Figure 5, the coordinates of each point can be obtained as: $A(l_1 \cos \theta_1, l_1 \sin \theta_1)$, $B(l_2 \cos \theta_1, l_2 \sin \theta_1)$, $O_2(l_3 \cos \theta_1, l_3 \sin \theta_1)$.

In order to make the flapping trajectory closer to the "8" shape, reasonable parameters should be set for the length and relative position of each component. In this paper, choose: $l_1 = 10\text{mm}$, $l_2 = 30\text{mm}$, $l_3 = 60\text{mm}$, $e = 58\text{mm}$, $\omega_1 = 30\text{rad/s}$.

### 3.2 Calculation of flutter displacement

According to the schematic diagram of the crank-rocker mechanism shown in the figure 5, combined with the geometric relationship between the various components, the algebraic relationship between $\theta_2$ and $\theta_1$ is obtained as:

$$\tan \theta_2 = -\frac{l_1 \sin \theta_1}{e - l_1 \cos \theta_1}$$  \hspace{1cm} (3)

Solve (3), and the rocker angle $\theta_2$ is obtained as:

$$\theta_2 = -\arctan\left(\frac{l_1 \sin \theta_1}{e - l_1 \cos \theta_1}\right)$$  \hspace{1cm} (4)

According to the geometric constraints of the flapping mechanism, the constraint equation of the rocker $C$ is obtained as:

$$l_{BC} = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = l_3^2$$  \hspace{1cm} (5)

Further, substitute the coordinates of point $B$ into equation (5) for expansion and sorting, the distance between the rocking block and the right hinge $f$ is solved as:

$$f = \frac{-a + \sqrt{a^2 - 4b}}{2}$$  \hspace{1cm} (6)

In the formula, $a$ and $b$ are the following formulas respectively:
\[
\begin{align*}
  a &= 2\left[ e \cos \theta_2 - l_2 \cos(\theta_1 - \theta_2) \right] \\
  b &= e^2 + l_2^2 - 2el_1 \cos \theta_1 - l_1^2
\end{align*}
\]

Combining the above formulas, it can be solved that the displacements of point C on the x-axis and y-axis are:

\[
\begin{align*}
  x_c &= e + f \cos \theta_2 \\
  y_c &= f \sin \theta_2
\end{align*}
\]

(7)  
(8)  

Bring the geometric parameters of the mechanism into the displacement formulas (7) and (8), and use matlab to solve the trajectory curve of point C in one cycle, as shown in Figure 6. It can be seen from the figure that the motion trajectory is a "8" shape symmetrical about the x-axis, and the shape of the upper half is slightly fuller than the lower half. According to the above theoretical formula and Figure 6, it can be obtained that, the upper limit point of trajectory is \( x_{c_{\text{max}}} = 90 \), the lower limit point is \( x_{c_{\text{min}}} = 30 \), the right limit point \( y_{c_{\text{max}}} = 2.6789 \), the left limit point \( y_{c_{\text{min}}} = -2.6789 \).

![Figure 6 Rocker trajectory curve](image)

![Figure 7 Installation drawing of flapping wing mechanism](image)

### 3.3 Calculation of flutter angle and twist angle

In order to make the flapping-wing drive mechanism more suitable for the flapping-wing aircraft, based on the characteristics of its 8"-shaped trajectory, the installation method shown in Figure 7 can be used. Since the wings on both sides of the flapper are symmetrical in flight, this article only describes the installation of one side of the wings. Assuming that, the crank, rocker, connecting rod, and guide rail are all in a straight line and the state is vertically arranged when the mechanism is stationary. The thickness of each component is ignored. A spherical spherical plain bearing is fixed at
point Q at the distance m from the lower hinge. The line connecting the center of the bearing and the center of the lower hinge is horizontal and vertical to the flapping mechanism. Pass the wing through the spherical bearing and connect one end of it with a ball hinge to the center of the slider surface. After the installation is completed, the other end of the wing can generate a spatial figure-of-eight trajectory when the crank rotates.

In order to make the wing tip trajectory closer to the wing flapping trajectory of birds in flight, the distance m that between the slider and the spherical bearing and the length of the wing are very critical. Refering to [10], select m=60mm, the lenth of the wing is 200mm.

As mentioned above, calculated by matlab, it can be obtained that \( x_{c_{\text{max}}} = 90 \), \( x_{c_{\text{min}}} = 30 \). The abscissa of the intersection point of the figure-eight trajectory is \( x_c = 58 \). Based on the spatial geometric constraints, it can be deduced that the relationships between the flutter angle, the torsion angle and the position of the rocker are as follows:

\[
\begin{align*}
\varphi &= \begin{cases} 
\arctan \frac{\sqrt{(x_c - 58)^2 + y_c^2}}{m} & (x_c > 58) \\
-\arctan \frac{\sqrt{(x_c - 58)^2 + y_c^2}}{m} & (x_c \leq 58)
\end{cases} \\
\psi &= \arccot \frac{\sqrt{(x_c - 58)^2 + m^2}}{y_c}
\end{align*}
\]

Substitute formulas (7) and (8) into the above angle formulas, and use matlab to solve the changes of the flutter angle and torsion angle in one cycle, as shown in Figure 8 and Figure 9 respectively:

![Figure 8](image1)

**Figure 8** Diagram of the relationship between the flutter angle and the input angle

![Figure 9](image2)

**Figure 9** Diagram of the relationship between the twist angle and the input angle
From the curve in Figure 8, it can be concluded that within one cycle, the maximum upward flutter angle of the mechanism is \( \varphi_{\text{up}} = 28.07^\circ \), the maximum downward flutter angle is \( \varphi_{\text{down}} = 25.02^\circ \). From the curve in Figure 9, it can be concluded that the left and right maximum torsion angles are both \( \psi_{\text{max}} = 2.40^\circ \).

### 3.4 Calculation of Flutter angular velocity and torsion angular velocity

Take the time derivative of angle formula (4), the relationship between the rocker angular velocity and the crank input angle can be obtained:

\[
\omega_z = -\frac{l_1 \omega_{el} \cos \theta_1 - l_2^2 \omega_2}{e^2 + l_1^2 - 2e l_1 \cos \theta_1}
\]  

(11)

In order to obtain the relationship between the flutter torsional angular velocity and the crank angle, and to facilitate the later calculation of the derivation, we can first derive the displacement formula (7) and (8) with respect to time respectively. Due to the complexity of the formula, let:

\[
\begin{align*}
\alpha &= 2[-e \omega_2 \sin \theta_1 + l_1 (\omega_1 - \omega_2) \sin(\theta_1 - \theta_2)] \\
\beta &= 4(-e^2 \omega_2 \sin(2\theta_2) - l_1^2 (\omega_1 - \omega_2) \sin 2(\theta_1 - \theta_2) - 2e l_1 [\omega_2 \sin(\theta_1 - 2\theta_2) \\
&\quad - \omega_1 \sin(\theta_1 - \theta_2) \cos \theta_2 + 4\omega_1 \sin \theta_1])
\end{align*}
\]

After simplification, the derivatives of the coordinates of point C on the x-axis and y-axis can be obtained as:

\[
x_c' = \frac{1}{2} \cos \theta_2 (-\alpha + \frac{\beta}{2 \sqrt{a^2 - 4b}}) - \omega_2 f \sin \theta_2
\]

(12)

\[
y_c' = \frac{1}{2} \sin \theta_2 (-\alpha + \frac{\beta}{2 \sqrt{a^2 - 4b}}) + \omega_2 f \cos \theta_2
\]

(13)

The above two derivatives represent the speed of point C in the x and y-axis directions. Use matlab to solve the variation range of the two derivatives in a cycle, as shown in Figures 10 and 11 respectively:

---

Figure 10 x-axis velocity curve
Figure 11 y-axis velocity curve

From the theoretical formula and the speed curve, we can find that the maximum value of the upward and downward flutter velocity of the block are both $943.9053 \text{mm/s}$, the maximum value of the upward flutter velocity is $v_{up} = 157.78 \text{mm/s}$, the maximum value of the upward flutter velocity is $v_{down} = 200 \text{mm/s}$.

Deriving the flutter angle and twist angle formulas (9) and (10) with respect to time respectively, the expressions of the flutter angular velocity and the torsional angular velocity are obtained as:

$$
\mu = \left\{ \begin{array}{ll}
\frac{m}{(x_c - 58)^2 + y_c^2 + m^2} \cdot \frac{(x_c - 58)x_c' + y_c y_c'}{(x_c - 58)^2 + y_c^2} & (x_c > 58) \\
-\frac{m}{(x_c - 58)^2 + y_c^2 + m^2} \cdot \frac{(x_c - 58)x_c' + y_c y_c'}{(x_c - 58)^2 + y_c^2} & (x_c \leq 58)
\end{array} \right.
$$

(14)

$$
\nu = \frac{-y_c x_c' (x_c - 58) [(x_c - 58)^2 + y_c^2]^{1/2} - y_c' [(x_c - 58)^2 + y_c^2]^{1/2}}{(x_c - 58)^2 + y_c^2 + m^2}
$$

(15)

Substitute the geometric parameters of the mechanism into formulas (14) and (15) respectively, and use matlab to solve the flutter angular velocity and torsional angular velocity curves of the wingtip in one cycle, as shown in Figures 12 and 13 respectively:

Figure 12 The relationship between flutter angular velocity and input angle

Figure 13 Torsional angular velocity curve
It can be concluded from Figure 12 that the maximum value of the flutter angular velocity in one cycle is \( \mu_{\text{max}} = 15.65 \text{rad/s} \), and the minimum value is \( \mu_{\text{min}} = -15.65 \text{rad/s} \). It can be concluded from Figure 13 that the maximum value of the torsional angular velocity is \( \nu_{\text{max}} = 2.63 \text{rad/s} \) and the minimum value is \( \nu_{\text{min}} = -2.9412 \text{rad/s} \).

4. Simulation analysis

In order to verify the rationality of the theoretical analysis of the driving mechanism, a simulation model of the crank-slider drive mechanism was established according to the designed size parameters of the flapping wing driving mechanism. Enter the initial coordinates of each point in ADAMS/View, establish the three-dimensional model of flapping wing structure, including crank, rocker, connecting rod, guide rail and slider. The model diagram is shown in Figure 14. After establishing the model, add the appropriate motion pair. One rotating pair is set up between the yellow crank and the ground, and another rotating pair is set between the end of the rocker and the slide block. The red link and the black rocker are respectively connected with the crank with rotating pairs. One moving pair is set up between the slider and the guide rail, and another moving pair is set up between the rocker and the guide rail. A rotating pair is established between the guide rail and the ground at a distance of 58mm from the crank rotation center. The model is ideal that avoid the influence of friction and air resistance.

After the motion simulation is performed in ADAMS, measure the speed of the slider by establishing a reference point, and compare with the theoretical calculation, which is shown in Figures 16 and 17. The trajectory of the output slider is shown in Figure 15.
It can be seen from the Figure 15 which shows the output trajectory of the slide in one cycle: the movement trajectory of the wingtip of the driving mechanism is a space "8" shape, which proves that the designed driving mechanism can achieve the expected flapping motion. In addition, it can be seen from the output track that the output track of the output rod is continuous under the drive of the motor, and the mechanism transmission is continuous during the whole movement process, which has good transmission performance.

From the comparison between the ADAMS simulation output speed of the output rod and the theoretical calculation angle in one movement cycle in Figures 16 and 17, it can be seen that the output angle obtained by the ADAMS simulation results is the same as the mechanism theoretical calculation result. The correctness of the mechanism kinematics analysis theory is verified.
5. Conclusion

This paper designs a flapping wing mechanism based on a plane crank rocker mechanism, which makes the rocker generate a figure-8 trajectory. The kinematics model of the mechanism is established by geometric relationship and analytical method, and the values of parameters are set according to the aerodynamic performance requirements of mechanism transmission and flapping flight. The following conclusions are drawn:

1) During the flapping process of the flapping wing, the upper maximum flapping angle is 28.07°, the lower maximum flapping angle is 25.02°, and the maximum torsion angle is 2.40°. The flapping wing has good aerodynamic performance.

2) The driving mechanism can output the same space "8" trajectory as biological flight.

3) ADAMS is used to simulate the flapping wing mechanism, and the curve of ADAMS simulation analysis is consistent with the curve of MATLAB theoretical analysis, which verifies the correctness of theoretical analysis.

References