

# *Research on Sliding Mode Observer Control for Missile Pitch System*

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**Abstract:** A new type of three sliding mode surface disturbance observer was design for the pitch channel control system of missile attack angle tracking. The error and disturbance of attack angle loop is approximate by the sliding mode type observer; and also the pitch speed signal loop was also designed by sliding mode observer with an integral sliding mode surface. And based on backstepping method, the pitch speed error loop was designed by sliding mode control method, and the whole system is stable according to Lyapunov stability theory with three sliding mode controller s and two sliding mdoe observers. At last, Numerical experiment was done to test the stability of the whole system with designed controller.

## 1. Introduction

With the increase of missile speed, the uncertainty of its pitch channel becomes more and more serious. These nonlinear uncertainties are often difficult to be directly measured, so they cannot be directly compensated in the controller<sup>[1-5]</sup>. This year, an idea based on disturbance observer has been applied well in many systems and achieved good control results. But the above disturbance observer uses high gain feedback to realize disturbance observation, which often requires high system gain and easily brings chatter to the system. Sliding mode control, especially integral sliding mode control, has the advantage of good rapidity, which can replace the effect of high gain feedback<sup>[6-9]</sup>. At the same time, the integral sliding mode has good stability<sup>[10-13]</sup>. Based on the above reasons, this paper combines the integral sliding mode controller and system disturbance observer, which improves the observation speed of the disturbance observer, reduces the gain of the observer, and improves the dynamic performance of the system. At the same time, for the angle of attack subsystem and the pitching angular rate subsystem, the inversion and sliding mode methods are used respectively, and the Lyapunov energy function is used as a whole to prove the stability of the system; Finally, the simulation results show the stability of the whole system.

## 2. Problem Description

A kind oflinear model of supersonic missile pitch channel with disturbance can be described as the following second order system:

$$\dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z + f_1 \quad (1)$$

$$\dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 \quad (2)$$

where  $a_{ij}$  is constant air dynamic coefficient of missile, which is defined by the out shape of missile,  $\alpha$  is called attack angle of missile,  $\omega_z$  is called the rotate speed of pitch angle,  $f_1$  and  $f_2$  are defined as outer disturbance.

The task of controller design is to design a controller  $\delta_z$  such that the attack angle  $\alpha$  can track the ideal angle command  $\alpha^d$ . Without loss of generality, we assume the command signal  $\alpha^d = 5/57.3$ , and the uncertain disturbance  $f_1$  and  $f_2$  can be observed and solved by the adaptive control law  $\delta_z$ .

### 3. Sliding Mode Disturbance Observer Design

For the first subsystem, we assume the following part  $-a_{35}\delta_z + f_1$  as the system disturbance  $D$ , then it satisfies

$$-a_{35}\delta_z + f_1 = D \quad (3)$$

And the above system can be changed as

$$\dot{\alpha} = \omega_z - a_{34}\alpha + D \quad (4)$$

Then we can choose a sliding mode surface of observer error as:

$$s_a = c_1(\hat{\alpha} - \alpha) + c_2 \int (\hat{\alpha} - \alpha) dt + c_3 \omega_z$$

And we construct a disturbance observer as

$$\begin{aligned} \dot{\hat{\alpha}} &= \omega_z - a_{34}\alpha + \hat{D} \\ \hat{D} &= -k_0 |\hat{\alpha} - \alpha|^{1/2} \frac{\hat{\alpha} - \alpha}{|\hat{\alpha} - \alpha| + |\omega_z| + |\alpha|} - k_0 \frac{s_a}{|s_a| + \varepsilon_1} + \hat{d} \\ \dot{\hat{d}} &= -k_1 \text{sign}(\hat{d} - \hat{D}) \end{aligned} \quad (5)$$

where  $\hat{d}$  and  $\hat{D}$  are two approximate value of disturbance  $D$ ,  $\hat{\alpha}$  is the approximation of state  $\alpha$ .

### 4. Sliding mode controller Design

Then, first step of desing is to define a new error variable as  $e_\alpha = \alpha - \alpha^d$ , and the first part of subsystem can be rewritten as

$$\dot{e}_\alpha = \omega_z - a_{34}\alpha + D - \dot{\alpha}^d \quad (6)$$

Then the expect value of  $\omega_z$  can be set as  $\omega_z^d$  as follows:

$$\omega_z^d = a_{34}\alpha - \hat{D} + \dot{\alpha}^d - k_{\alpha 1} e_\alpha - k_{\alpha 2} \frac{e_\alpha}{|e_\alpha| + \varepsilon_b} \quad (7)$$

And then the second step is to define a new variable as  $e_\omega = \omega - \omega_z^d$ , then the second part of subsystem is as follows

$$\dot{e}_\omega = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + f_2 - \dot{\omega}_z^d \quad (8)$$

And we design an integral sliding mode of observer error as

$$s_b = c_4(\hat{e}_\omega - e_\omega) + c_5 \int (\hat{e}_\omega - e_\omega) dt + c_6(\alpha - \alpha^d)$$

And we assume  $f_2 - \dot{\omega}_z^d$  as another disturbance of the system, and we define a new variable as  $W$ , then it has  $W = f_2 - \dot{\omega}_z^d$ , and we can use another disturbance observer to approximate it, then a disturbance observer can be designed as

$$\begin{aligned} \dot{\hat{e}}_\omega &= a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z + \hat{W} \\ \hat{W} &= -k_{w0} |\hat{e}_\omega - e_\omega|^{1/2} \frac{|\hat{e}_\omega - e_\omega|}{|\hat{e}_\omega - e_\omega| + |s_b| + \varepsilon_b} - k_{w1} s_b + \hat{w} \\ \dot{\hat{w}} &= -k_{w1} \text{sign}(\hat{w} - \hat{W}) \end{aligned} \quad (9)$$

Then we design a integral sliding mode surface of system error as:

$$s_c = c_7 e_\omega + c_8 \int e_\omega dt$$

Then the final control law is designed as follows

$$\delta_z = \frac{1}{a_{25}} \left\{ -a_{24}\alpha - a_{22}\omega_z - k_{a1} s_c - k_{a2} \frac{s_c}{|e_\omega| + \varepsilon_a} - \hat{W} \right\} \quad (10)$$

Then a Lyapunov function can be chosen as

$$V = \frac{1}{2} e_\alpha^2 + \frac{1}{2} s_c^2 \quad (11)$$

And its derivative can be solved as

$$\dot{V} = -k_{a1} e_\alpha^2 - k_{a2} \frac{e_\alpha^2}{|e_\alpha| + \varepsilon_b} - k_{a1} c_7 s_c^2 - k_{a2} c_7 \frac{s_c^2}{|e_\omega| + \varepsilon_a} + e_\alpha \tilde{D} + e_\omega \tilde{W} \quad (12)$$

Where the error variable of disturbance observer  $\tilde{D}$  and  $\tilde{W}$  can be described as

$$\tilde{D} = D - \hat{D}, \tilde{W} = W - \hat{W} \quad (13)$$

And the gain of the error of disturbance observer should be small enough, then it satisfies

$$\dot{V} \leq -k_{a1} e_\alpha^2 - k_{a1} s_c^2 \quad (14)$$

And according to the design of three order sliding mode observer theory, big gains can be desinged and then the error of observer is small enough. So according to the famous Lyapunov stability theorem, the all signals of system is stable, so  $\alpha$  can track the ideal command signal angle  $\alpha^d$ .

## 5. Numerical simulations

According to shape of missile, air coefficients of a type of missile can be set as constants as

$$a_{25} = -167.87; a_{35} = 0.243; a_{22} = -2.876; a_{24} = -193.65; a_{34} = 1.584$$

If we use the common observer control strategy, the control effect can see below figures 1-5.

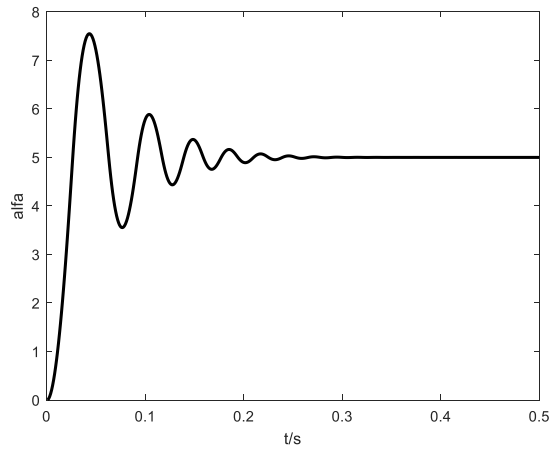


Figure 1: The attack angle of missile system

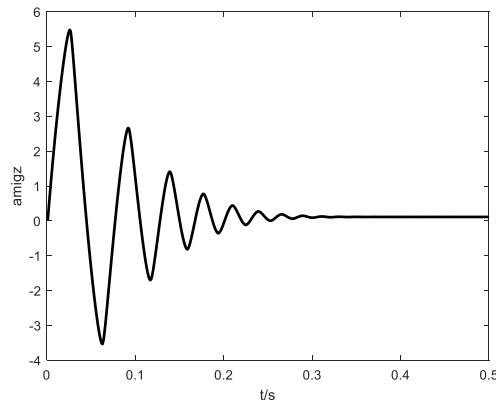


Figure 2: The curve of angle speed of missile system

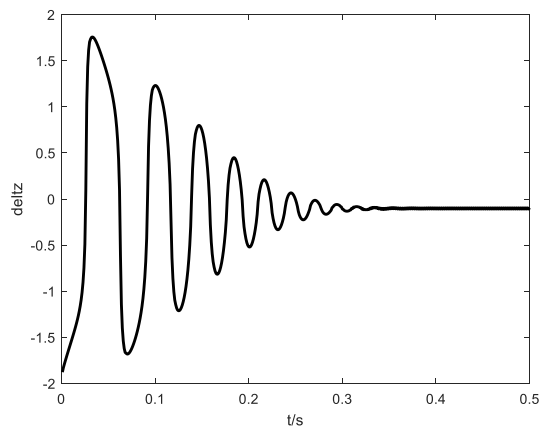


Figure 3: The figure of actuator of missile system

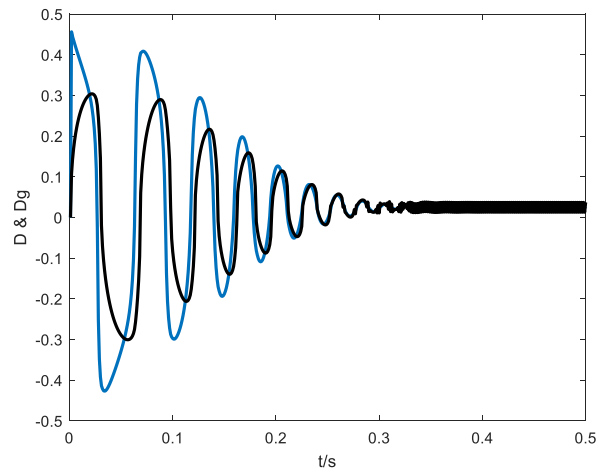


Figure 4: The estimation value of disturbance D

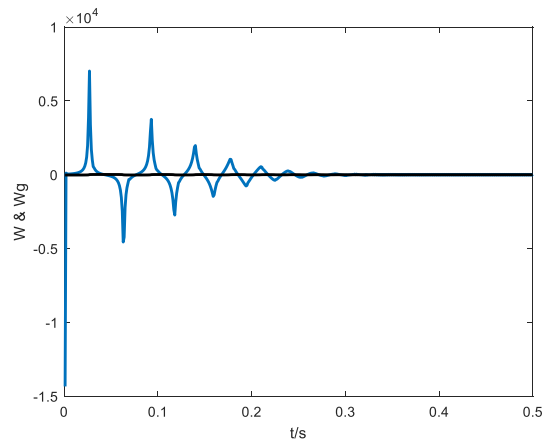


Figure 5: The estimation value of disturbance W

Set control parameters as

$$k_{\alpha 1} = 5; k_{\alpha 2} = 20; k_{\alpha 1} = 15; k_{\alpha 2} = 5;$$

Then simulation results can see below figures 6-10.

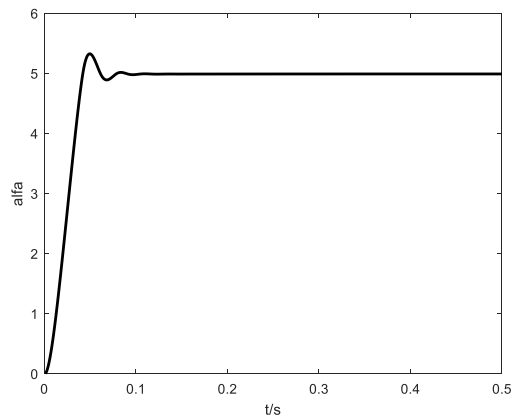


Figure 6: The attack angle of missile system

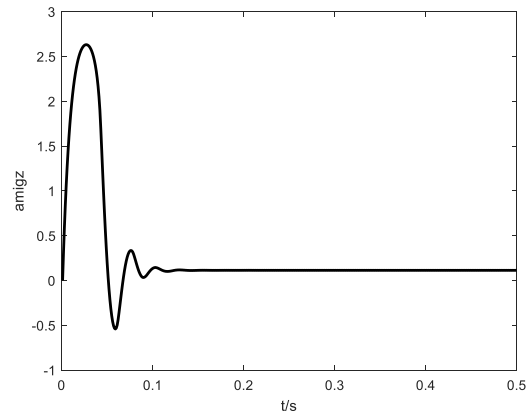


Figure 7: The curve of angle speed of missile system

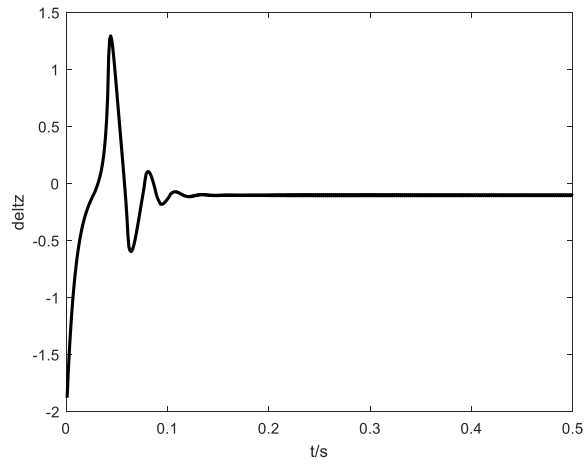


Figure 8: The curve of actuator of missile system

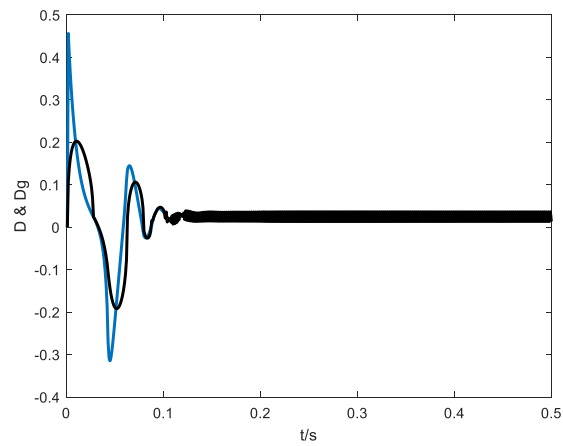


Figure 9: The estimation value of disturbance D

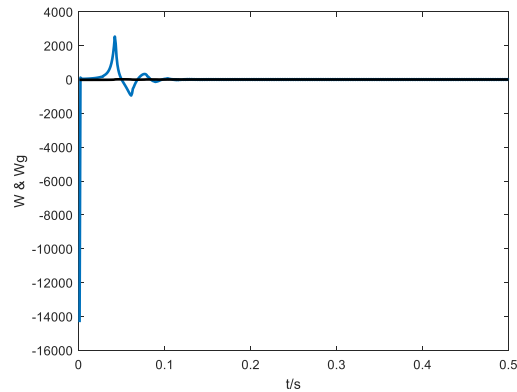


Figure 10: The estimation value of disturbance W

It can be seen from the comparison between Figure 1-5 and Figure 6-10 that the control effect of using ordinary observers has obvious oscillation; Its overshoot exceeds 60%; The observer control method with three integral sliding mode has the advantage of high speed; Its rise time is less than 0.1 s; Moreover, the regulating time is short and the overshoot is less than 5%; The dynamic performance of the whole response is significantly improved.

## 6. Conclusions

Aiming at the problem of angle of attack tracking in missile pitch channel, it is simplified into angle of attack tracking subsystem and pitch rate tracking subsystem; Then, an integral type sliding mode observer is constructed for each subsystem to solve the overall disturbance and uncertainty of the subsystem; Then, the subsystem is combined with inversion and integral sliding mode control to realize the tracking of missile attack angle; The stability of this method is proved by theory and simulation.

## References

- [1] Feigenbaum M J. Quantitative universality for a class of nonlinear transformations [J], *J. Stat. Phys.* 1978, 19:25-52.
- [2] Pecora L M and Carroll T L. Synchronization in chaotic systems [J], *Phys. Rev. Lett.* 1990, 64:821-824.
- [3] GE S S, Wang C, Lee T H. Adaptive backstepping control of a class of chaotic systems [J]. *Int J Bifurcation and chaos.* 2000, 10 (5): 1140-1156.
- [4] GE S S, Wang C, Adaptive control of uncertain chus's circuits [J]. *IEEE Trans Circuits System.* 2000, 47(9): 1397-1402.
- [5] Alexander L, Fradkov, Markov A Yu. Adaptive synchronization of chaotic systems based on speed gradient method and passification [J]. *IEEE Trans Circuits System* 1997, 44(10):905-912.
- [6] Dong X. Chen L. Adaptive control of the uncertain Duffing oscillator [J], *Int J Bifurcation and chaos.* 1997, 7(7):1651-1658.
- [7] Tao Yang, Chun-Mei Yang and Lin-Bao Yang, A Detailed Study of Adaptive Control of Chaotic Systems with Unknown Parameters [J]. *Dynamics and Control.* 1998, (8):255-267.
- [8] M.T. Yassen, Chaos control of chaotic dynamical systems using backstepping design, *Chaos Soliton Fract.* 27 (2006) 537-548.
- [9] Seung-Hwan Kim, Yoon-Sik Kim, Chanho Song, *Control engineering practice*, 12(2004) pp. 149-154
- [10] Hull, R.A., & Z. Qu, 1995. Design and evaluation of robust nonlinear missile autopilot from a performance perspective. *Proceedings of the ACC*, 189-193
- [11] Junwei Lei, Xinyu Wang, Yinhua Lei, *Physics Letters A*, Volume 373, Issue 14, 23 March 2009, Pages 1249-1256
- [12] Junwei Lei, Xinyu Wang, Yinhua Lei, *Communications in Nonlinear Science and Numerical Simulation*, Volume 14, Issue 8, August 2009, Pages 3439-3448
- [13] Xinyu Wang, Junwei Lei, Changpeng Pan. *Applied Mathematics and Computation*, 185 (2007) pp. 989-1002