

Research on Path Planning and Trajectory Tracking of Autonomous Vehicle

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Abstract: With the rapid development of the world economy and science and technology, the intelligent era has kicked off, and the development and application of automatic driving technology has also attracted people's attention. Trajectory planning and tracking control, as the key technologies of autonomous driving, determine the safety and stability of autonomous vehicles during driving. Aiming at the problems of planning safety and control robustness, research on structured road obstacle avoidance trajectory planning and trajectory tracking control algorithm of autonomous vehicles can effectively improve the performance of the obstacle avoidance system of autonomous vehicles. In this paper, the obstacle avoidance path planning problem of autonomous vehicle is studied, and in order to reduce the difficulty of system modeling and improve the controller performance, the trajectory tracking control is decoupled into horizontal control and vertical control, and the research is carried out respectively.

1. Introduction

In recent years, with the rapid development of China's economy, the number of cars has increased year by year. According to the Ministry of Public Security, in March 2022, the number of motor vehicles in China reached 402 million, of which 307 million were automobiles [1]. However, at the same time, traffic accidents, traffic congestion and energy shortages are becoming more and more serious, which has become a key factor limiting the sustainable development of automobiles. In recent years, autonomous driving technology has developed rapidly with the strong support of national policies and the massive investment of enterprises and universities in this field [2]. According to the automatic driving classification standards issued by the National Highway Traffic Safety Administration of the United States, automatic driving technology is divided into five levels, and only at level 3 or above can automatic driving be realized [3]. At present, there are not many autonomous driving enterprises that can reach L3 level or above, and the autonomous driving system of most enterprises still stays at L2 level. It can be seen that the autonomous driving technology is not mature at present and still faces many challenges, so the research on autonomous driving technology has practical significance.

Autonomous vehicles work together through systems such as artificial intelligence, intelligent networking, visual radar and global positioning, and can operate the car safely and stably to reach

the destination without the operator. Automatic driving technology can be divided into four modules according to their functions: visual perception, behavioral decision making, path planning and tracking control [4]. Visual perception is used to perceive the dynamic driving environment around the vehicle in real time and transmit the environmental information to the behavioral decision module. The behavioral decision module issues various decision commands to the autonomous vehicle under the current driving environment, such as lane change to avoid obstacles, lane change to overtake and emergency braking. Path planning refers to planning a safe and comfortable path for the autonomous vehicle to reach the destination in advance, while tracking control needs to control the autonomous vehicle to drive accurately and stably according to the path planned by the path planning module. It can be seen that although each module has different functions, it is closely related to each other. Path planning and tracking control module is the key technology of automatic driving, and it is also a necessary condition for the active safety of vehicles. Its performance directly affects the performance of the entire automatic driving system. Therefore, the design of safe, efficient and stable path planning and tracking control strategies for autonomous vehicles has important theoretical research significance and application value for realizing autonomous driving and improving vehicle driving safety and comfort.

In this paper, the obstacle avoidance path planning and tracking control of autonomous vehicles on structured roads are studied. With the goal of improving vehicle driving safety, comfort and stability, the obstacle avoidance trajectory planning algorithm is designed based on road environment information. Aiming at improving the robustness and accuracy of the controller, the horizontal and vertical coordination controller is designed to realize the obstacle-avoiding trajectory planning and tracking control function of the autonomous vehicle.

2. Path planning based on Bezier curve

Local obstacle avoidance path planning of autonomous vehicles refers to the ability to plan a safe and easy to track path in time when the vehicle encounters obstacles ahead, which is one of the key technologies of autonomous driving technology [5]. When a vehicle is driving, it may encounter dynamic or static obstacles, which requires the trajectory planning algorithm to have the ability to continuously avoid dynamic and static obstacles, and the planned path considering the constraints of vehicle dynamics is more secure and comfortable. This paper considers the problem of local obstacle avoidance path planning in structured road scenarios. When the vehicle encounters stationary obstacles in front of it, an obstacle changing path is planned based on Bezier curve to meet the constraints of vehicle dynamics, real-time performance and safety. At the same time, considering that there may be dynamic vehicles in the target lane in the process of lane change, there is the possibility of collision with the car in the process of lane change, so speed planning is carried out based on Bezier curve and quadratic programming method to make further dynamic obstacle avoidance. Finally, the obstacle avoidance trajectory is obtained by combining the path and velocity curve.

The Bezier curve, invented in 1962 by French mathematician Pierre Bezier, is a mathematical parameter curve based on approximation, which can be expressed mathematically in any curve. It was originally used for car shape design. In recent years, Bezier curve has been widely used in the field of automatic driving path planning due to its excellent mathematical properties. In the process of path planning, the shape of the planned path can be arbitrarily changed by adjusting the control point of the Bezier curve, so as to plan the vehicle motion path that meets the constraints of the initial state and dynamics of the vehicle. $n + 1$ control points are usually defined to form an n -order Bessel curve, whose parametric equation is expressed as follows [6].

$$P(t) = \sum_{i=0}^n P_i / B_i^n(t), t \in [0, 1] \quad (1)$$

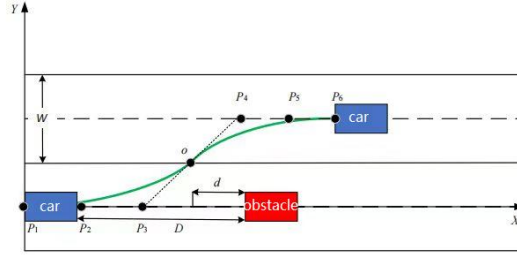


Figure 1: Lane change and obstacle avoidance path planning diagram

The blue rectangles in Figure 1 represent autonomous vehicles and the red rectangles represent stationary obstacles. P_1 - P_6 is the six control points that determine the obstacle avoidance lane change path, and point $O(x_0, y_0)$ is the imaginary trajectory symmetry point, which is located on the middle road line of the two-lane line. The coordinates of each control point can be determined according to the properties of Bessel curve. First of all, in order to meet the requirements of zero curvature of the starting point and target point, parallel to the center line of the lane and easy subsequent lane change, control points P_1, P_2, P_3 and P_4, P_5, P_6 are set on the same straight line respectively.

Then the control points P_1 and P_6 , that is, the initial and target points of the lane change of the self-contained vehicle, can be expressed as:

$$\begin{cases} P_1 = (x_1, y_1) = (0, 0) \\ P_6 = (x_6, y_6) = (2x_0, w) \end{cases} \quad (7)$$

Where, w is the lane width and its value is 4m.

Considering that in the process of lane change, there are various disturbances such as lateral wind, side slip and other uncertainties, in order to fully ensure the safety in the process of lane change, a lane change safety distance model is established and its mathematical equation is expressed as:

$$d = vt_1 + d_0 \quad (8)$$

Where, d_0 is the minimum safe distance, that is, the distance between the target vehicle emergency stop and the obstacle, generally taken as 2~3m; v is the vehicle speed and the obstacle hazard factor $t_1 \in [0, 1]$.

Based on this safety distance model, the coordinate of the hypothetical symmetry point O is set as:

$$O(x_0, y_0) = (D - d + a/2, w/2) \quad (9)$$

Where, a is the body length of the self-propelled vehicle; D is the initial distance between the target vehicle and the obstacle at the time of lane change, which is usually selected as 2~3v. Then the longitudinal coordinate position X_3 of point P_3 uniquely determines the shape of the lane change path, and then the path planning problem is transformed into finding the coordinates X_3 .

In order to further analyze the influence of the size of x_3 on the Bezier curve, different values were selected to generate curve clusters. By setting a multi-objective evaluation function, each curve was evaluated, and finally the optimal lane change curve satisfying the vehicle dynamics constraints and comfort constraints was selected.

First of all, in order to ensure the sideslip constraint of the vehicle, there should be the following constraint:

$$a_y = Kv^2 \leq ug \quad (10)$$

Where a_y is the lateral acceleration, v is the speed of the vehicle, u is the coefficient of friction and g is the acceleration of gravity. To ensure the safety of the planned path and to ensure that the shape of the planned path curve is within normal limits, the following constraint equation is set:

$$D - d + a/2 > x_3 > x_2 \quad (11)$$

In order to improve driving comfort and safety, it is important to reduce the curvature and path length as much as possible. In summary, a multi-objective evaluation function is established as follows:

$$J = w_1 k_{\max} + w_2 k_{\Delta k} + w_3 k_{\text{average}} + w_4 S \quad (12)$$

Where w_1 , w_2 , w_3 and w_4 are the weight coefficients of each optimisation indicator function respectively. The sub-optimisation indicator function and weighting parameters are designed to minimise the total optimisation function J , so that the optimal route change in the trajectory cluster can be selected to meet the vehicle dynamics constraints, comfort and safety. The first sub-optimisation indicator is k_{\max} , which represents the maximum curvature of the planned path. The maximum curvature of the path is positively related to the maximum lateral acceleration, i.e. the larger k_{\max} is, the higher the lateral acceleration of the vehicle, and the maximum curvature is too large to cause the vehicle to skid or even roll over. Therefore, this sub-optimisation indicator can filter out the paths with excessive curvature to ensure the safety of the vehicle. The second sub-optimisation indicator is k , which represents the sum of the curvature differences between adjacent discrete points on the planned path, and characterises the degree of curvature abruptness of the planned path, expressed in the form of an integral as:

$$k_{\Delta k} = \int_0^1 (\Delta k)^2 dt \quad (13)$$

Where Δk is the difference in curvature between adjacent discrete path points. The larger the sub-optimisation indicator, the greater the curvature variation of the planned path, which can lead to large changes in steering angle or sharp turns when the vehicle follows a path with curvature variation. Therefore, this sub-optimisation indicator can filter out the path without sudden changes to ensure the smoothness of the vehicle. The third sub-optimisation indicator is k_{average} , which represents the average curvature of the driving path and is used to reflect the overall lateral acceleration of the planned path, and can be expressed as follows:

$$k_{\text{average}} = \frac{1}{n} \int_0^1 (k)^2 dt \quad (14)$$

Where, k represents the curvature of all discrete points in the planned path, and n is the number of path points. The smaller the average curvature of the path, the smaller the overall lateral acceleration when the vehicle follows the path, the more stable the overall yaw performance of the vehicle, and also ensure the ride comfort. The last sub-optimization index s represents the total length of the planned lane change path and is used to represent the lane change time. Since the lane change process of the vehicle is a dangerous condition, the length of the vehicle lane change should be minimized, and the shorter lane change path also means low energy consumption. Therefore, the sub-optimization index function s is used to screen out the path with short time and low energy consumption.

$$S = \sum_{i=1}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \quad (15)$$

By adjusting the weight coefficients w_1 , w_2 , w_3 and w_4 of each optimization objective defined above, the optimal lane change path that meets the constraints of vehicle dynamics, real-time performance and safety can be selected, and the lane change obstacle avoidance path planning has been completed. Finally, the QP solving function Quadprog in MATLAB is used to solve the above

quadratic programming equation, and the optimized ST curve can be obtained, and the planned velocity and acceleration information can be obtained by derivation of it. It is worth noting that if the objective function of the above quadratic programming cannot be solved, that is, the constraints on the initial state, equality and inequality of the vehicle cannot be satisfied, it is proved that the obstacle avoidance effect of dynamic obstacles cannot be achieved only through speed planning. Then, the planned local lane change obstacle avoidance path needs to be replanned to ensure the vehicle's driving safety.

3. Path track tracking design

Stable and accurate tracking of reference trajectories is the basis for realizing lane change and obstacle avoidance of autonomous vehicles [8]. The trajectory tracking control layer mainly receives the path and time information planned by the trajectory planning layer, and enables the vehicle to change lanes safely, stably and accurately to avoid obstacles by controlling the steering wheel, throttle and brake of the vehicle. In this paper, a sliding mode control algorithm with strong robustness is used to design a horizontal controller for tracking the desired path. In the longitudinal aspect, in order to improve the control accuracy, hierarchical design method is adopted to design the upper controller based on the double closed-loop PID algorithm, and the lower controller based on the inverse longitudinal dynamics model, so as to achieve the purpose of tracking the expected speed. Finally, the trajectory tracking of the autonomous vehicle is realized through horizontal and longitudinal coordinated control.

3.1 Design of equivalent sliding mode controller

For the convenience of description, consider writing the vehicle tracking error model as follows.

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = A + Bu \\ \dot{e}_3 = e_4 \\ \dot{e}_4 = C + Du \\ y = [e_1, e_3] \end{cases} \quad (16)$$

Where $A = \frac{C_f + C_r}{mv_x} \dot{e}_d - \frac{C_f + C_r}{m} e_\varphi + \frac{C_{fl} + C_{rl}}{mv_x} \dot{e}_\varphi + (\frac{C_{fl} + C_{rl}}{mv_x} - v_x) \phi_r$; $B = -\frac{C_f}{m}$; $C = \frac{C_{fl} + C_{rl}}{I_z v_x} \dot{e}_d - \frac{C_{fl} + C_{rl}}{I_z} e_\varphi + \frac{C_{fl}^2 + C_{rl}^2}{I_z v_x} \dot{e}_\varphi + \frac{C_{fl}^2 + C_{rl}^2}{I_z v_x} \phi_r$; $D = -C_{fl}/I_z$; e_1, e_2, e_3, e_4 are transverse error e_d and its rate of change \dot{e}_d , heading error e_φ and its rate of change \dot{e}_φ , respectively. Define the error state vector as $e = [e_1, e_2, e_3, e_4]^T$; y is the system output vector; u is for control output, where is the front wheel Angle δ_f .

The system described in Equation (16) is a single-input multi-output system. Obviously, a single input cannot satisfy two governing equations at the same time, so the system is regarded as two first-order subsystems composed of $[e_1, e_2]^T$ and $[e_3, e_4]^T$ state vectors respectively, which are defined as the lateral error subsystem and the heading error subsystem respectively. For this system, the sliding mode surfaces of two subsystems are designed as follows:

$$\begin{cases} S_1 = c_1 e_1 + e_2, c_1 > 0 \\ S_2 = c_2 e_3 + e_4, c_2 > 0 \end{cases} \quad (17)$$

Where, s_1 and s_2 are the sliding mode surface of the lateral error subsystem and the sliding mode surface of the heading error subsystem respectively, and c_1 and c_2 are the control coefficients of the sliding mode surface s_1 and s_2 respectively. At this time, the control objective is to approximate the

lateral error and the heading error to zero with a single input. Based on the above ideas, the design of the total sliding surface that can approximate two sub-sliding surface at the same time is as follows:

$$s = c(c_1 e_1 + e_2) + (1 - c)(c_2 e_3 + e_4) \quad (18)$$

Where, s is the total sliding mode surface, c is the control coefficient of the sliding mode surface s , $c \in (0, 1)$. When the state of the system reaches the total sliding mode surface after proportional weighting, the two subsystems can also reach the designed sliding mode surface, that is, the precise tracking control of lateral and heading is realized at the same time.

After determining the sliding mode surface, it is necessary to design a suitable sliding mode control law to make the system state quickly reach and stably maintain on the sliding mode surface. In this paper, the equivalent sliding mode control method is adopted, and its control law consists of equivalent control item u_{eq} and switching robust control item u_{sw} . The equivalent control can ensure that the system state moves to the sliding mode surface, and the switching robust control can ensure that the system state does not leave the sliding mode surface.

Taking the derivative of the sliding mode surface s , we have:

$$\dot{s}(t) = c(c_1 e_2 + A + Bu) + (1 - c)(c_2 e_4 + C + Du) \quad (19)$$

In the case of ignoring external interference and uncertainty, taking $\dot{s} = 0$, the equivalent control term can be obtained:

$$u_{eq} = -\frac{c(c_1 e_2 + A)}{(1-c)D + cB} - \frac{(1-c)(c_2 e_4 + C)}{(1-c)D + cB} \quad (20)$$

When there is external interference and instability in the system, it is necessary to design a switching robust control item. To ensure that the arrival condition of the sliding mode is established, that is, $s\dot{s} < 0$, based on the exponential reaching law mentioned above, design the switching robust control:

$$u_{sw} = \frac{-\varepsilon \tanh(s) - ks}{(1-c)D + cB} \quad (21)$$

Where, $\varepsilon < 0, k > 0$. Here, the original sign function $\text{sgn}(s)$ in the iso-speed reaching term of the exponential reaching law is replaced by the hyperbolic tangent function $\tanh(s)$, because the hyperbolic tangent function is smoother than the sign function, and can effectively reduce the chattering caused by the control discontinuous switching characteristics.

The sliding mode control law can be expressed as the sum of equivalent controls and switching robust controls [9].

$$u = u_{eq} + u_{sw} \quad (22)$$

Lyapunov function can be used to prove the stability of the designed sliding mode controller [10].

3.2 Design of vertical controller based on double closed-loop PID

Longitudinal control of autonomous vehicles refers to the control of vehicle acceleration and braking actuators, so that the current state of the vehicle (such as position, speed, acceleration, etc.) and the longitudinal information contained in the desired waypoint is consistent. The overall framework of the longitudinal tracking controller system is shown in Figure 2, which is composed of the upper controller and the lower controller. The upper controller obtains the path point information needed to be tracked by the current time stamp according to the upper planning module,

calculates the acceleration required by the longitudinal tracking control through the double closed-loop PID controller and transmits it to the lower controller. The lower controller is mainly to convert the expected acceleration into the execution amount of the vehicle acceleration and deceleration actuator, and update the control amount according to the real-time feedback of the vehicle status information to achieve the automatic control of the vehicle in the longitudinal.

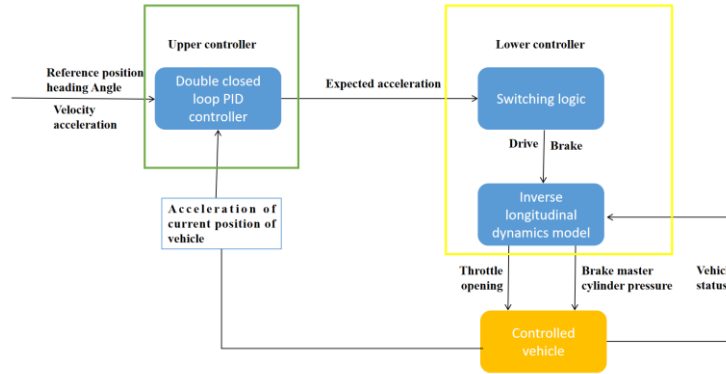


Figure 2: Overall frame of vertical controller

4. Conclusion

Aiming at the problem of obstacle avoidance trajectory planning and trajectory tracking control of autonomous vehicles on structured roads, this paper studies the path planning algorithm of Bessel curve for obstacle avoidance based on multi-objective evaluation and the speed planning algorithm of Bessel curve for obstacle avoidance based on quadratic programming. The horizontal and longitudinal trajectory tracking controllers based on sliding mode control algorithm and double closed-loop PID algorithm are studied, and the obstacle-avoiding trajectory planning and tracking control functions are realized. It provides a good reference for the comprehensive performance simulation verification of trajectory planning and tracking control algorithms.

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