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The Teaching Design of Triple Integral and Its Application in College Students' Mathematics Competition

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Abstract: Triple integral is a difficult point in the teaching of higher mathematics. How to teach a triple integral course and learn to apply it is particularly important. Through the teaching content design of triple integral and its application in college students ' mathematics competition, this paper aims to make students better understand the content through clear teaching framework and explanation ideas, simple and easy to understand teaching methods and concise expression methods, which has certain reference value for teachers ' teaching, guiding students to participate in the competition and participating students.

1. Introduction

Triple integral is an important concept in higher mathematics and a method of integrating functions in a certain area of three-dimensional space. Triple integral in rectangular coordinate system is a very important tool, which can be used to solve many practical problems, such as physics, engineering, computer graphics and other fields. Through in-depth study and understanding of triple integral, we can better grasp the nature of functions in space and provide more effective mathematical tools for solving practical problems. The triple integral is a difficult knowledge point in the double integral, especially the calculation of the triple integral, many students generally find it difficult to do the problem, based on this, this paper combs the teaching content of the triple integral, and combines the test points of the triple integral in the national college students ' mathematics competition over the years, makes a general summary, and lists the knowledge points according to the teaching content, which is concise, clear and easy to understand.

2. Teaching Design Ideas

Firstly, the concept of triple integral is introduced through an example, and then the nature and geometric significance of triple integral are briefly introduced. Then, the calculation of triple integral is mainly introduced, and the specific conversion formulas in rectangular coordinate system, cylindrical coordinate system and spherical coordinate system are introduced. Finally, the content summary, classroom practice, homework assignment and classroom extension are carried out. The teaching content frame is shown in Figure 1 [1-6].

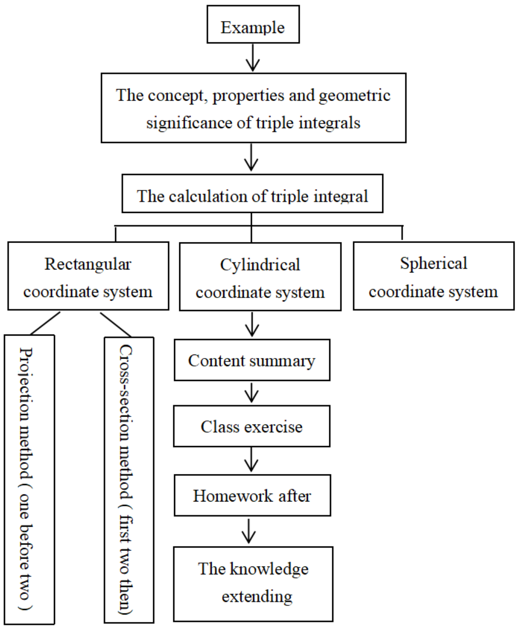

Figure 1: Teaching Content Frame Diagram

3. Teaching Content Design

In order to effectively use the limited time of the classroom and better introduce the content of the triple integral, the teaching content is summarized below, and the classroom teaching of the triple integral is realized according to the design scheme.

3.1. Example

The mass of uniformly distributed object: M = volume density×volume (volume density is constant)

The non-uniform distribution of the object, known volume density $\rho(x,y,z)$, find its mass M. (volume density is a function)

The solution: ' large into small, constant generation, approximate and, limit '

(1) Segmentation: the object is divided into n pieces of Δv_i $(i=1,2,\cdots,n)$;

(2) Approximate summation: $M \approx \sum_{i=1}^{n} \rho(\xi_i,\eta_i,\zeta_i)\Delta v_i$;

(3) Take the limit: $M = \lim_{\lambda\to 0}\sum_{i=1}^{n} \rho(\xi_i,\eta_i,\zeta_i)\Delta v_i$, where diameter $\lambda = \max_{1\le i\le n}\{\Delta v_i\}$.

3.2. Concept of triple integrals

Let $f(x,y,z)$ be a bounded function on the space bounded closed region Ω.

Segmentation: The closed region Ω is arbitrarily divided into n small closed regions

$\Delta v_1, \Delta v_2, \cdots, \Delta v_n$, where Δv_i represents the i th small region and its volume.

Approximate summation: take a point Δv_i on each (ξ_i, η_i, ζ_i), and make and $\sum_{i=1}^{n} f(\xi_i, \eta_i, \zeta_i) \Delta v_i$.

Take the limit: $\lim_{\lambda \to 0} \sum_{i=1}^{n} f(\xi_i, \eta_i, \zeta_i) \Delta v_i$ (λ is the maximum diameter of each small closed region)

If the limit of the sum exists, the limit is called the triple integral of the function $f(x, y, z)$ on the closed region Ω, denoted by $\iiint_{\Omega} f(x, y, z) dv$. Quasi

$$\iiint_{\Omega} f(x, y, z) dv = \lim_{\lambda \to 0} \sum_{i=1}^{n} f(\xi_i, \eta_i, \zeta_i) \Delta v_i$$

dv is the volume element, $dv = dxdydz$ in the rectangular coordinate system, and the triple integral can be expressed as $\iiint_{\Omega} f(x, y, z) dv = \iiint_{\Omega} f(x, y, z) dxdydz$ in the rectangular coordinate system.

3.3. Properties of Triple Integrals

The property of triple integral is similar to that of double integral, and it is no longer repeated. Only one property of simplifying the calculation of triple integral is introduced.

If the integral region Ω is symmetric with respect to the plane $x = 0$, then

$$\iiint_{\Omega} f(x, y, z) dv = \begin{cases} \iiint_{\Omega_1} f(x, y, z) dv, & f \text{ is an even function about } x. \\ 0, & f \text{ is an odd function about } x. \end{cases}$$

Ω_1 is the region of $x \geq 0$ in region Ω.

Similarly, if the integral region Ω is symmetrical about the plane $y = 0$ or $z = 0$, there is a similar conclusion.

3.4. Geometric Meaning of Triple Integral

The mass $M = \iiint_{\Omega} f(x, y, z) dv$ of the space object with $f(x, y, z)$ as the volume density in the space finite closed region Ω is given.

When $f(x, y, z) = 1$, $\iiint_{\Omega} dv = V_{\Omega}$.

Three analogy:

The mass of the rod $M = \int_a^b f(x) dx$, linear density: $f(x)$

The mass of the flake $M = \iint_{D} f(x, y) d\sigma$, surface density: $f(x, y)$

The mass of the space object $M = \iiint\limits_{\Omega} f(x,y,z)dv$, volume density: $f(x,y,z)$

3.5. Calculation of Triple Integral [7-12]

(1) Rectangular coordinate system
Method 1: Projection method (one before two)

By projecting the closed region Ω to the xOy plane, a plane region D_{xy} is obtained, and the boundary of D_{xy} is used as the reference line to make the cylindrical surface parallel to the z axis.This cylinder divides the boundary surface S of Ω into two parts, $S_1 : z = z_1(x,y)$ and $S_2 : z = z_2(x,y)$, then the closed region Ω can be expressed as

$\Omega = \{(x,y,z) | z_1(x,y) \le z \le z_2(x,y), (x,y) \in D_{xy}\}$, The calculation formula of triple integral is:

$$\iiint\limits_{\Omega} f(x,y,z)dv = \iint\limits_{D_{xy}} dxdy \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)dz = \int_a^b dx \int_{y_1(x)}^{y_2(x)} dy \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)dz$$

or

$$\iiint\limits_{\Omega} f(x,y,z)dv = \iint\limits_{D_{xy}} dxdy \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)dz = \int_c^d dy \int_{x_1(y)}^{x_2(y)} dx \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)dz$$

Example 1 Calculates the triple integral $\iiint\limits_{\Omega} xdxdydz$, where Ω is a closed region surrounded by plane $x = 0, y = 0, z = 0$ and plane $x + y + z = 1$.

Answer: Projection method (one before two).

$$\iiint\limits_{\Omega} xdxdydz = \iint\limits_{D_{xy}} xdxdy \int_0^{1-x-y} dz = \iint\limits_{D_{xy}} x(1-x-y)dxdy = \int_0^1 xdx \int_0^{1-x} (1-x-y)dy$$

$$= \frac{1}{2}\int_0^1 (x - 2x^2 + x^3)dx = \frac{1}{2}[\frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4]_0^1 = \frac{1}{24}$$

Method 2: Cross-section method (first two then one).

Assume that the region Ω is located between two planes $z = a$ and $z = b$ $(a < b)$ perpendicular to the z axis, the cross section perpendicular to the x axis is denoted as D_z, and the cross section is projected onto the xOy plane, then the closed region Ω can be expressed as $\Omega = \{(x,y,z) | (x,y) \in D_z, a \le z \le b\}$, and the triple integral can be transformed into

$$\iiint\limits_{\Omega} f(x,y,z)dv = \int_{c_1}^{c_2} dz \iint\limits_{D_z} f(x,y,z)dxdy$$

When the integrand contains only z functions, then

$$\iiint\limits_{\Omega} f(x,y,z)dv = \int_{c_1}^{c_2} dz \iint\limits_{D_z} f(z)dxdy = \int_{c_1}^{c_2} f(z)dz \iint\limits_{D_z} dxdy = \int_{c_1}^{c_2} f(z)\sigma(z)dz$$

When $f(x,y,z)=1$, $\iint\limits_{D_z} dxdy=\sigma(z)$, $\sigma(z)$ is the cross-sectional area.

Applicable conditions: (1) the integrand contains only the function of z; (2) The section D_z perpendicular to the z axis should be regular and easy to calculate the area.

Example 2 Calculates the triple integral $\iiint\limits_{\Omega} zdxdydz$, where Ω is a closed region surrounded by the surface $z=x^2+y^2$ and the plane $z=4$.

Answer: Section method (first two then one).

$$\iiint\limits_{\Omega} zdxdydz=\int_0^4 zdz\iint\limits_{D_z} dxdy=\int_0^4 z\cdot\pi zdz=\frac{\pi}{3}z^3\Big|_0^4=\frac{64}{3}\pi$$

(2) Cylindrical coordinate system

Let $M(x,y,z)$ be a point in space, and let the polar coordinate of the projection P of point M on the xOy plane be r,θ, then such three numbers r,θ,z are called the cylindrical coordinates of point M.

The formula $\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases}$ for the transformation of rectangular coordinates to cylindrical coordinates, where $0\le r<+\infty, 0\le\theta\le 2\pi, -\infty<z<+\infty$

Three sets of coordinate surfaces, when r is a constant, represent the cylindrical surface; when θ is a constant, it represents the half plane; when z is a constant, it represents the plane.

The volume element dv in the cylindrical coordinates is surrounded by six coordinate planes: (1) the half plane with the angle of θ and $\theta+d\theta$; (2) Cylindrical surfaces with radius r and $r+dr$; (3) Planes of height z and $z+dz$.

So the triple integral calculation formula is

$$\iiint\limits_{\Omega} f(x,y,z)dv=\iiint\limits_{\Omega} f(r\cos\theta, r\sin\theta, z)rdrd\theta dz$$

Example 3 Calculates the triple integral $\iiint\limits_{\Omega} zdxdydz$, where Ω is a closed region surrounded by the surface $z=x^2+y^2$ and the plane $z=4$.

Answer: Using cylindrical coordinate system to calculate triple integral

$$\iiint\limits_{\Omega} zdv=\iiint\limits_{\Omega} zrdrd\theta dz=\int_0^{2\pi}d\theta\int_0^2 rdr\int_{r^2}^4 zdz \quad =2\pi\cdot\frac{1}{2}\int_0^2 r(16-r^4)dr=\pi[8r^2-\frac{1}{6}r^6]_0^2=\frac{64}{3}\pi$$

Let $M(x,y,z)$ be a point in space, then point M can be determined by three ordered numbers, where r is the distance between the origin O and point M, φ is the angle between the directional line segment OM and the positive axis, and θ is the angle from the positive z axis meter to the directional line segment OP in the counterclockwise direction, where P is the projection of point M on the xOy plane. Such three numbers r,φ,θ are called the spherical coordinates of point M.

The formula $\begin{cases} x = r\sin\varphi\cos\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\varphi \end{cases}$ for converting rectangular coordinates to spherical coordinates, where $0 \le r < +\infty, 0 \le \theta \le 2\pi, -\infty < z < +\infty$

Three sets of coordinate surfaces, when r is a constant, represent the sphere; when φ is a constant, it represents a cone; when θ is a constant, it represents the half plane.

The volume element dv in spherical coordinates is surrounded by six coordinate surfaces: (1) half plane θ and $\theta + d\theta$; (2) Conical surface φ and $\varphi + d\varphi$; (3) Spheres with radius r and $r + dr$.

So the triple integral calculation formula is:

$$\iiint_{\Omega} f(x,y,z)dv = \iiint_{\Omega} f(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\varphi)r^2\sin\varphi drd\varphi d\theta$$

Example 4 Calculates the triple integral $\iiint_{\Omega} z^2 dxdydz$, where Ω is calculated by the surface

$$\Omega = \{(x,y,z) | x^2 + y^2 + z^2 \le 1\}$$

Answer: Using spherical coordinate system to calculate triple integral:

$$\iiint_{\Omega} z^2 dxdydz = \iiint_{\Omega} r^2\cos^2\varphi \cdot r^2\sin\varphi drd\varphi d\theta = \int_0^{2\pi} d\theta \int_0^{\pi} \sin\varphi\cos^2\varphi d\varphi \int_0^1 r^4 dr$$

$$= \frac{2\pi}{5}\int_0^{\pi}\cos^2\varphi d\cos\varphi = \frac{2\pi}{15}\cos^3\varphi\Big|_0^{\pi} = \frac{14}{15}\pi$$

3.6. Summary of the Content

(1) Calculation of triple integral, select the coordinate system is the key, from the characteristics of the integrand and integral region to consider:

1) When the integral region Ω is cuboid, tetrahedron or arbitrary shape, choose the rectangular coordinate system → projection method (first one, then two).

2) When the integrand function is $f(x,y,z) = g(z)$, choose the rectangular coordinate system → cross section method (first two then one);

3) When the integral region Ω is a rotating conical surface, a rotating parabolic surface, and a spherical surface, the cylindrical coordinate system is selected;

4) When the integral region Ω is spherical, the spherical coordinate system is selected.

Table 1: Summary of Calculation of Triple Integral

Coordinate system	volume element	application situation
Rectangular coordinate system	$dv = dxdydz$	The integral region is mostly surrounded by coordinate planes.
Cylindrical coordinate system	$dv = rdrd\theta dz$	The integrand form is simple, or the dependent variable can be separated.
Spherical coordinate system	$dv = r^2\sin\varphi drd\varphi d\theta$	

(2) Under different coordinate systems, the conversion formula of volume elements and its application (Table 1):

3.7. Classroom Exercises

1) Calculate the triple integral $\iiint\limits_{\Omega} zdxdydz$, where Ω is the closed region surrounded by plane $x=0, y=0, z=0$ and plane $x+y+z=1$.

Tip: Select the rectangular coordinate system, method one: projection method (first one after two); method 2: Cross-section method (first two then one).

2) Calculate the triple integral $\iiint\limits_{\Omega} (x^2+y^2+z^2)dxdydz$, where Ω is the cube surrounded by a cone $z=\sqrt{x^2+y^2}$ and a sphere $x^2+y^2+z^2=R^2$.

Tip: Select the spherical coordinate system.

3.8. After-school Homework

Textbook after-school exercises

3.9. Knowledge Extension

In order to stimulate students ' interest in learning mathematics and cultivate students ' ability to analyze and solve problems, after teaching the content of triple integral, this paper gives several typical real problems of college students ' mathematics competition over the years, expands students ' knowledge, and exercises their mathematical thinking and problem solving ability. The specific topics are as follows.

1) (2016 Eighth National College Students ' Mathematics Competition Preliminary Questions) The space area where an object is located is $\Omega: x^2+y^2+2z^2 \le x+y+2z$, the density function is $x^2+y^2+z^2$, and the mass $M=\iiint\limits_{\Omega}(x^2+y^2+z^2)dxdydz$ is calculated.

Analysis: Steps of triple integral calculation:

Step 1: Draw the figure of the integral area, investigate whether the integral area has symmetry, the symmetry and rotation symmetry of the coordinate surface, the non-standard integral area, and the conversion to the standard integral area.

Step 2: Determine the coordinate system and construct the cubic definite integral expression

Step 3: Calculate three definite integrals

Answer: $\Omega: x^2-x+y^2-y+2(z^2-z)\le 0$

$$\Rightarrow \Omega:(x-\frac{1}{2})^2-\frac{1}{4}+(y-\frac{1}{2})^2-\frac{1}{4}+2[(z-\frac{1}{2})^2-\frac{1}{4}]\le 0$$

$$\Rightarrow \Omega:(x-\frac{1}{2})^2+(y-\frac{1}{2})^2+2(z-\frac{1}{2})^2\le 1.$$

The center point $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ is in the ellipsoid domain.

Ream $u = x - \frac{1}{2}, v = y - \frac{1}{2}, w = \sqrt{2}(z - \frac{1}{2})$

Scilicet $x = u + \frac{1}{2}, y = v + \frac{1}{2}, z = \frac{w}{\sqrt{2}} + \frac{1}{2}$ $\Rightarrow \Omega_{uvw} : u^2 + v^2 + w^2 \leq 1$

$$M = \iiint_{\Omega} (x^2 + y^2 + z^2) dxdydz = \iiint_{\Omega_{uvw}} F(u,v,w) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| dudvdw$$

$$F(u,v,w) = (u + \frac{1}{2})^2 + (v + \frac{1}{2})^2 + (\frac{w}{\sqrt{2}} + \frac{1}{2})^2 = u^2 + u + v^2 + v + \frac{w^2}{2} + \frac{w}{\sqrt{2}} + \frac{3}{4}$$

$$\left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| = \left\| \begin{matrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{matrix} \right\| = \left\| \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{matrix} \right\| = \frac{1}{\sqrt{2}}, \quad \frac{\partial(x,y,z)}{\partial(u,v,w)} = \frac{1}{\frac{\partial(u,v,w)}{\partial(x,y,z)}}$$

$\Omega_{uvw} : u^2 + v^2 + w^2 \leq 1$ On the symmetry of three coordinate planes, it has rotational symmetry.

$$M = \iiint_{\Omega_{uvw}} (u^2 + u + v^2 + v + \frac{w^2}{2} + \frac{w}{\sqrt{2}} + \frac{3}{4}) dudvdw$$

$$= \frac{1}{\sqrt{2}} \iiint_{\Omega_{uvw}} (u^2 + v^2 + \frac{w^2}{2}) dudvdw - \frac{3}{4\sqrt{2}} \iiint_{\Omega_{uvw}} dudvdw = \frac{3}{4\sqrt{2}} \frac{4\pi}{3} = \frac{\pi}{\sqrt{2}}$$

$$\iiint_{\Omega_{uvw}} u^2 dudvdw = \iiint_{\Omega_{uvw}} v^2 dudvdw = \iiint_{\Omega_{uvw}} w^2 dudvdw = \frac{1}{\sqrt{2}} \frac{5}{2} \iiint_{\Omega_{uvw}} u^2 dudvdw = \frac{5}{6\sqrt{2}} \iiint_{\Omega_{uvw}} (u^2 + v^2 + w^2) dudvdw$$

$$\Omega_{uvw} : u^2 + v^2 + w^2 \leq 1, \quad I = \iiint_{\Omega_{uvw}} (u^2 + v^2 + w^2) dudvdw, \quad M = \frac{5}{6\sqrt{2}} I + \frac{\pi}{\sqrt{2}}$$

The spherical coordinate calculation method of triple integral:

$$\Omega_{\theta\varphi r} = \{(\theta, \varphi, r) | 0 \leq \theta \leq 2\pi, 0 \leq \varphi \leq \pi, 0 \leq r \leq 1\}$$

$$I = \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \int_0^1 r^2 r^2 \sin\varphi dr = \frac{4\pi}{5} \quad \Rightarrow M = \frac{5}{6\sqrt{2}} \frac{4\pi}{5} + \frac{\pi}{\sqrt{2}} = \frac{5\sqrt{2}\pi}{6}$$

2) (2017 Ninth National College Students ' Mathematics Competition Preliminary Competition)

Let the space region surrounded by the surface $z^2 = x^2 + y^2$ and $z = \sqrt{4 - x^2 - y^2}$ be V, and calculate the triple integral $\iiint_V zdxdydz$.

Analysis: The general steps of triple integral calculation:

Step 1: Draw the graphics of the integral region

Step 2: Simplify and transform the integral model by means of symmetry.

Step 3: Determine the coordinate system of the conversion of repeated integration
Step 4: Construct the expression of cumulative integral.
Step 5: Calculate three definite integrals
Four methods:
Spherical coordinate system
Cylindrical coordinate system
Rectangular space coordinate system (projection method ' first one after two ')
Spatial rectangular coordinate system (section method ' first two then one ')
Method 1: spherical coordinate system

Answer: $\iiint\limits_{\Omega} f(x,y,z)dV = \iiint\limits_{\Omega} f(r\sin\varphi\cos\theta, r\sin\varphi\sin\theta, r\cos\varphi)r^2\sin\varphi d\theta d\varphi dr$

$$\Gamma:\begin{cases} z^2 = x^2 + y^2 \\ z = \sqrt{4-x^2-y^2} \end{cases} \Rightarrow \quad \Gamma:\begin{cases} x^2 + y^2 \le 2 \\ z = 0 \end{cases}$$

$$\Rightarrow \ V: 0 \le \theta \le 2\pi, \ 0 \le \varphi \le \frac{\pi}{4}, \ 0 \le r \le 2$$

$$\iiint\limits_{V} zdxdydz = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} d\varphi \int_0^2 r\cos\varphi \cdot r^2 \sin\varphi dr = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} \sin\varphi\cos\varphi d\varphi \int_0^2 r^3 dr$$

$$= 8\pi \int_0^{\frac{\pi}{4}} \sin\varphi d\sin\varphi = 8\pi \cdot \frac{1}{4} = 2\pi$$

Method 2: Cylindrical coordinate system
Answer: from the meaning of the problem, get

$$D_{\theta\rho z} = \{(\theta,\rho,z) | 0 \le \theta \le 2\pi, 0 \le \rho \le \sqrt{2}, \rho \le z \le \sqrt{4-\rho^2}\}$$

$$\iiint\limits_{V} zdxdydz = \int_0^{2\pi} d\theta \int_0^{\sqrt{2}} \rho d\rho \int_{\rho}^{\sqrt{4-\rho^2}} zdz$$

$$= 2\pi \int_0^{\sqrt{2}} (2-\rho^2)\rho d\rho = 2\pi \int_0^{\sqrt{2}} (2\rho - \rho^3) d\rho = 2\pi$$

Method 3: space rectangular coordinate system (projection method' first one after two ')

Answer: $V = \{(x,y,z) | \sqrt{x^2+y^2} \le z \le \sqrt{4-x^2-y^2}, (x,y) \in D_{xy}\}$

$$D_{\theta\rho} = \{(\theta,\rho) | 0 \le \theta \le 2\pi, 0 \le \rho \le \sqrt{2}\}$$

$$\iiint\limits_{V} zdxdydz = \iint\limits_{D_{xy}} dxdy \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} zdz = \iint\limits_{D_{xy}} (2 - x^2 - y^2) dxdy$$

$$= \int_0^{2\pi} d\theta \int_0^{\sqrt{2}} (2-\rho^2)\rho d\rho = 2\pi \int_0^{\sqrt{2}} (2\rho - \rho^3) d\rho = 2\pi$$

Method 4: space rectangular coordinate system (section method ' first two after one ')

Answer: $\Gamma:\begin{cases} z^2 = x^2+y^2 \\ z=\sqrt{4-x^2-y^2} \end{cases}$, $z=\sqrt{2}$ is obtained by eliminating x,y variable.

$$V_{上}:\sqrt{2}\le z\le 2,\ D(z):x^2+y^2\le 4-z^2,\ V_{下}:0\le z\le\sqrt{2},\ D(z):x^2+y^2\le z^2$$

$$\iiint\limits_{V_{上}} zdxdydz=\int_{\sqrt{2}}^{2} zdz\iint\limits_{D(z)}dxdy=\int_{\sqrt{2}}^{2} z[\pi(4-z^2)]dz=\int_{\sqrt{2}}^{2}(4\pi z-\pi z^3)dz=\pi$$

$$\iiint\limits_{V_{下}} zdxdydz=\int_0^{\sqrt{2}} zdz\iint\limits_{D(z)}dxdy=\int_0^{\sqrt{2}}\pi z^3dz=\pi$$

$$\iiint\limits_{V} zdxdydz=\iiint\limits_{V_{上}} zdxdydz+\iiint\limits_{V_{下}} zdxdydz=\pi+\pi=2\pi$$

3. (In 2019, the 11th National College Students ' Mathematics Competition Preliminary Questions)

The triple integral $\iiint\limits_{\Omega}\frac{xyz}{x^2+y^2}dxdydz$ is calculated, where Ω is the area surrounded by the surface $(x^2+y^2+z^2)^2=2xy$ in the first quadrant.

Analysis: The first step is to determine and construct the coordinate system for calculating the triple integral. By analyzing the graphic features, it can be known from the surface $(x^2+y^2+z^2)^2=2xy$ expression that the spherical coordinate system should be established. θ is the angle between the projection line $|ON|$ of the positive half axis of x rotating counterclockwise on the xOy coordinate plane to the point on the surface and the connection line $|OP|$ of the origin on the xOy coordinate plane. φ is the angle between the vector $\overrightarrow{OP}$ and the positive direction of the z axis from the origin to the point on the surface; r is the distance from the point on the surface to the origin.

Ball coordinates: $(\theta,\varphi,r)=\begin{cases} x=r\sin\varphi\cos\theta \\ y=r\sin\varphi\sin\theta \\ z=r\cos\varphi \end{cases}$, General $\begin{cases} 0\le\theta\le 2\pi \\ 0\le\varphi\le\pi \\ r\ge 0 \end{cases}$

The second step: the boundary surface equation and the integrand function are used to describe the surface equation with the corresponding coordinate variables, and the spherical coordinates are substituted into the surface $(x^2+y^2+z^2)^2=2xy$ equation to obtain $r^4=2r^2\sin^2\varphi\cos\theta\sin\theta$

simplified to $r^2=\sin^2\varphi\sin 2\theta$. Because $r=r(\theta,\varphi)$, $r\ge 0$, so $r=\sin\varphi\sqrt{\sin 2\theta}$, to make the equation meaningful, need the same sign, because $r\ge 0$, so $\sqrt{\sin 2\theta}\ge 0$, farther $\sin\varphi>0$ in the first quadrant, so $0\le\theta\le\frac{\pi}{2}$, the equation does not mention the value of z, so $0\le\varphi\le\frac{\pi}{2}$.

Integrand $\frac{xyz}{x^2+y^2}=\frac{r\sin\varphi\cos\theta\cdot r\sin\varphi\sin\theta\cdot r\cos\varphi}{r^2\sin^2\varphi\cos^2\theta+r^2\sin^2\varphi\sin^2\theta}=r\cos\varphi\sin\theta\cos\theta$

Step 3: Determine the scope of the integral variable and construct the expression of the cumulative integral.

Step 4: Calculate the cumulative integral to get the integral result.

Answer: $\Omega: 0 \le \theta \le \dfrac{\pi}{2}, 0 \le \varphi \le \dfrac{\pi}{2}, 0 \le r \le \sin\varphi\sqrt{\sin 2\theta}$

$$\iiint_{\Omega} \frac{xyz}{x^2+y^2} dxdydz = \int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\pi}{2}} d\varphi \int_0^{\sin\varphi\sqrt{\sin 2\theta}} r\cos\varphi\sin\theta\cos\theta \cdot r^2 \sin\varphi dr$$

$$= \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta \int_0^{\frac{\pi}{2}} \cos\varphi\sin\varphi d\varphi \int_0^{\sin\varphi\sqrt{\sin 2\theta}} r^3 dr = \frac{1}{2}\int_0^{\frac{\pi}{2}} \sin 2\theta d\theta \int_0^{\frac{\pi}{2}} \cos\varphi\sin\varphi \frac{1}{4}\sin^4\varphi \sin^2 2\theta d\varphi$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} \sin 2\theta \sin^2 2\theta d\theta \int_0^{\frac{\pi}{2}} \sin^5\varphi d\sin\varphi = -\frac{1}{96}\int_0^{\frac{\pi}{2}} (1-\cos^2 2\theta) d\cos 2\theta$$

$$= -\frac{1}{96}[\cos 2\theta - \frac{1}{3}\cos^3 2\theta]_0^{\frac{\pi}{2}} = \frac{1}{72}$$

4. Conclusion

By summarizing the teaching design of triple integral and its application in college students ' mathematics competition, it shows a clear teaching framework and explanation ideas, simple and easy to understand teaching methods, concise and comprehensive expression, so that students can better grasp the content of triple integral, especially the calculation methods and skills of triple integral [13-15]. At the same time, it is also helpful to improve students ' ability to analyze and solve logical thinking, stimulate students ' interest in learning, improve teachers ' classroom teaching effect, and have certain reference value for teachers ' teaching, guiding students to participate in competitions and participating students[16-17].

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