

# *Various errors and correction methods affecting the positioning accuracy of satellite navigation*

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**Abstract:** This paper will briefly introduce the main methods to reduce or eliminate various errors at present. The main body of the paper is to introduce and discuss the correction methods of satellite ephemeris error and satellite clock difference, ionospheric delay, multipath effect, antenna phase center eccentricity correction, relativistic effect, tide correction and tropospheric path delay error which affect the accuracy of satellite navigation and positioning. In addition to the non-dispersive medium, the degree of change of tropospheric medium with time is relatively drastic, and it is difficult to use a simple and unified mathematical model to determine, especially in the long relative positioning of the baseline, the tropospheric delay between the two stations often shows a weak spatial correlation. Therefore, it is of great significance to establish a real-time tropospheric delay model in a certain region to improve the accuracy of satellite navigation.

## 1. Introduction

With the successful completion of the Beidou-3 satellite, China has become the fourth country or region with a global satellite navigation system in the world [1]. The completion of the three-step Beidou strategy has also greatly improved the accuracy and reliability of the Global Navigation Satellite System (GNSS). There are various errors in the global navigation satellite system (GNSS) when it is used to determine the three-dimensional coordinates of ground points by means of distance intersection. Due to the obvious regularity of systematic errors, some of these errors can be accurately estimated and corrected by strict mathematical models. In addition to the non-dispersive medium in the troposphere, the degree of change with time is relatively drastic, and it is difficult to use a simple and unified mathematical model to determine [2]. This study will focus on these two aspects, which is of great significance to improve the accuracy of satellite navigation and positioning.

## 2. Various kinds of errors and their correction methods

### 2.1 Satellite ephemeris error and satellite clock difference

Satellite ephemeris error is caused by the deviation of satellite position and velocity from the actual position and velocity in the process of satellite laser ranging. The satellite clock difference is the error caused by the difference between the time frequency provided by the satellite clock and the standard time frequency. The magnitude and accuracy of satellite ephemeris errors are affected by many factors, including the number and quality of orbital stations, and the extrapolation time of navigation messages used in satellite positioning systems. The number and quality of orbital stations (such as geographical location distribution, etc.) have an important impact on the accuracy of satellite ephemeris data, and higher quality trajectory data can provide more accurate ephemeris data. At the same time, the extrapolation time of navigation messages will also affect the error size of ephemeris data, because the density of satellite flight track is different, the time accuracy of extrapolation will be different. Therefore, the above factors should be considered comprehensively when using satellite ephemeris data for navigation and positioning to obtain more accurate and reliable position and time information. In order to reduce or eliminate the satellite ephemeris error, there are the following aspects: (1) Strengthen the management of the satellite clock. Because the satellite clock difference has the characteristic of cumulative error, when the satellite clock difference exceeds the reasonable range, the satellite clock difference is adjusted to make the satellite clock frequency converge to the standard frequency. At present, in order to solve the problem that the partial accuracy of ultra-fast ephemeris forecast in precision ephemeris is not enough to guarantee the precision single point positioning accuracy, IGS has set up a real-time working group to provide real-time products for precision single point positioning users, whose orbit accuracy can reach 0.05m, and satellite clock error accuracy can reach 0.3ns (2) Optimize Kalman filtering method. Currently, the most common method used to determine satellite orbit determination is the "decentralized" autonomous positioning method. Generally, it can be divided into two types: full-order distributed filtering and reduced order distributed filtering. Taking IREKF algorithm as an example, Wang Ziqi et al proposed that its orbit determination accuracy can reach less than 1m [3]. (3) Reduce the extrapolation time of navigation messages. Gou Changlong et al. proposed that currently commonly used navigation message extrapolation methods include Chebyshev polynomial extrapolation method and Lagrange polynomial extrapolation method. Taking the five-minute extrapolation interval as an example, the accuracy is shown in Table 1 and Table 2 [4].

Table 1: The navigation message extrapolation method is accurate at intervals of five minutes

Epoch(s)	Rank	$\Delta(cm)$	$\Delta Y(cm)$	$\Delta Z(cm)$
25530	7	2.920262	-5.035043	0.507901
	8	-0.701756	0.079484	1.048049
	9	-1.831871	-0.120223	-0.711757
	10	12.292416	-3.990597	-26.931163
25560	7	7.642960	-13.262661	1.600901
	8	-0.882345	0.183774	2.663719
	9	1.486034	-0.263657	-2.746377
	10	23.523591	-4.283230	-29.677144
25590	7	17.455512	4.222284	4.222284
	8	-1.438197	6.373023	6.373023
	9	8.802731	-8.311375	-8.311375

Table 2: The navigation message extrapolation method is accurate at intervals of five minutes

Epoch(s)	Rank	$\Delta(cm)$	$\Delta Y(cm)$	$\Delta Z(cm)$
254700	5	1.600000	-2.900000	2.400004
	6	-0.200060	-0.299998	2.100016
	7	-0.500060	-0.799999	1100009
	8	0.399970	4.200003	3.300034
255000	5	-2.800000	-8.200000	7.199997
	6	-4.900010	-6.099999	5.900011
	7	-5.199980	-6.600000	4.899994
	8	-5.199890	-6.600002	4.900000
255300	5	-24.500000	-32.200000	12.499998
	6	-26.900050	-30.599999	10.200013
	7	-27.200020	-31.100001	9.199999
	8	-27.200020	-31.100001	9.199999

## 2.2 Ionospheric delay

The ionosphere is the atmosphere in the range of 60-1000km, in this range, the neutral gas molecules will be ionized by the radiation of solar rays and high-energy particles, releasing a large number of electrons and positively charged ions, and the satellite signal along the propagation path through the ionosphere, its speed is slowed down at the same time, its propagation path will also appear a certain deflection. The extension of the propagation time caused by the above phenomenon is called the ionospheric delay. The size of the ionospheric delay is expressed as  $V(\text{ion})=-40.28/f^2$ , which obviously depends on the total charge in the direction of the signal path.

Methods to eliminate or weaken tropospheric delay (1) Establish an empirical correction model of ionospheric delay: as mentioned above, the size of the ionospheric delay depends on the total electron content in the direction of the propagation path, and the total electron content is related to the height, local time, season, solar activity degree and sunspot number. Therefore, the ionospheric delay can be simulated and corrected by measuring the above parameters and using appropriate mathematical formulas. The commonly used ionospheric delay correction models are Bentt model, international reference ionospheric model and Klobuchar model. In the early days when the US government implemented AS policy, it was difficult for unauthorized users to eliminate tropospheric delay by using dual-frequency correction method, so it was a mainstream method to weaken ionospheric delay by establishing delay empirical correction model. (2) Using GNSS dual-frequency observation data to establish the measured model: Using the dual-frequency observation data of global or regional GNSS base stations, the ionospheric delay of users in the world or the region is fitted by interpolation method. At present, the commonly used global VTEC models are: the global VTEC grid provided by IGS, CODE spherical harmonic function model. (3) Dual-frequency correction model. Because the ionospheric medium is dispersive, the transmission rate of satellite signals in the ionosphere is different, and this difference is only related to the frequency of satellite signals. Therefore, in theory, if we can accurately determine the time difference between signals of different frequencies to reach the phase center of the receiver antenna, we can accurately determine the ionospheric delay. The theoretical formula of ionospheric delay is as follows: by comparing the pseudo range measurements of L1 and L2 frequency carriers under the same observation conditions and calculating the time difference between them, the size of the ionospheric delay can be calculated.

$$P = P_1 + \frac{A}{F_1^2} \quad (1)$$

$$P = P_2 + \frac{A}{F_2^2} \quad (2)$$

With A 40.28 TEC, minus (2) by (1) type get  $\Delta P = P_1 - P_2 = C\Delta T = \frac{A}{F_1^2} \left( F_1^2 - \frac{F_2^2}{F_1^2} \right)$ . get  $C\Delta T = \nabla_{ion1} \left( \frac{F_1^2}{F_2^2} - 1 \right) = 0,6469 \nabla_{ion1}$ . backstepping to  $\nabla_{ion1} = 1.54573 \times C \times \Delta T$ . For the same reason  $\nabla_{ion2} = 2.54573 \times C \times \Delta T$ . It can be seen that in order to determine the ionospheric delay of signals of different frequencies, it is only necessary to know the propagation time difference. The dual-frequency observations can be expressed as  $P = 2.54573P_1 - 1.54573P_2$  Similarly, when you use carrier phase observations, your two-frequency observations are expressed as  $\eta = \left( \frac{F_1^2 \eta_1}{F_1^2} - F_2^2 \right) - \left( \frac{F_1 F_2 \eta_2}{F_1^2} - F_1^2 \right)$ .

### 2.3 Multipath effect

Because the satellite signal passes through the reflection source in the process of propagation, the interference phenomenon occurs when it reaches the phase center of the receiver antenna along different lines, and the point error caused by the interference phenomenon is called the multipath effect. At present, the main methods to reduce or eliminate the influence of multipath effect are: (1) The observation point should be selected to avoid the reflection source as much as possible, such as a huge lake. (2) A diameter suppression ring or a diameter suppression plate is arranged inside the receiver. (3) The configured receiving antenna can suppress reverse-polarized reflected waves well. (4) The observation period should be extended appropriately.

### 2.4 Antenna phase center eccentricity correction

It is known that GNSS is used to determine the coordinates of ground points by means of distance intersection. When measuring geodetic distance, we define the interval between the mean phase center of the receiver antenna and the mean phase center of the satellite antenna. This is the case with the eccentricity correction (PCV) caused by the difference between the instantaneous phase center of the day line and the mean phase center and the eccentricity correction when the antenna reference point does not coincide with the mean phase center [5] (PCO). Table 3 and Table 4 show the phase center shift and phase center change of the satellite antenna of Beidou satellite.

Table 3: ESA model BDS satellite IGSO/MEO PCO change value mm

frequency	X-PCO	Z-POC								
		IGSO-1	IGSO-2	IGSO-3	IGSO-4	IGSO-5	MEO-3	MEO-4	MEO-5	MEO-6
B1-B2	549.0	3049.0	3236.7	3842.6	3973.6	3882.1	2 069.5	2 313.5	2 201.8	2311.7
B1-B3	545.0	3509.5	4121.5	4710.2	5029.8	4935.1	2 214.2	2401.9	2 336.4	2 450.2

Table 4: Change value of IGSO/MEO PCV of BDS satellite in ESA model

Path	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°
IGSO	3.73	0.21	2.16	0.95	0.59	0.63	0.23	0.45	0.84	1.02	--	--	--	--
MEO	4.21	3.33	1.93	0.43	0.96	2.41	3.21	2.94	2.57	1.60	0.64	-1.10	-0.64	-2.69

## 2.5 Relativistic effect

Relativistic effects are divided into general relativistic effects and special relativistic effects. Generally speaking, relativistic effect is a relative deviation caused by space-time bending and relative velocity. The reasons for this deviation include: (1) Under the action of earth gravity, the relative motion of the satellite clock and the receiver clock causes space-time bending, resulting in their different time velocity, resulting in relative error; (2) In space, due to the difference in relative speed between the satellite clock and the receiver clock, relativistic effects will also occur, further affecting the accuracy of its clock.

The mathematical expression of the general relativistic effect is as follows:

$$\Delta f_2 = \frac{\mu}{c^2} \left( \frac{1}{R} - \frac{1}{r} \right) \cdot f \quad (3)$$

The mathematical expression for the effects of special relativity is as follows:

$$\Delta f_1 = f_3 - f = -\frac{v_s^2}{2c^2} \cdot f \quad (4)$$

The combined effects are as follows:

$$\Delta f = \frac{\mu}{c^2} \left( \frac{1}{R} - \frac{3}{2a} \right) \cdot f \quad (5)$$

Because we can easily obtain the speed and gravity position information of the fixed point and satellite on the earth. We get  $=4.443 \times 10^{-10}$ . Therefore, we only need to artificially lower the frequency of the satellite clock by  $4.443 \times 10^{-10}$  when producing it on the ground. In this way, when the satellite clock is launched into orbit with the satellite, its clock frequency will naturally align with the standard frequency.

## 2.6 Tidal correction

Tidal correction mainly includes ocean load tidal correction and earth tide correction. It is known that the total mass of the Earth is  $5.972 \times 10^{21}$  tons, and the total mass of the entire ocean is  $1.35 \times 10^{18}$  tons, equivalent to 1/4400 of the total mass of the Earth. [6] Therefore, the periodic fluctuation of sea water caused by the tidal force of the moon, the tidal force of other celestial bodies, and the rotation of the earth can not be ignored, and the effect on the position of the station in some coastal areas can reach the level of centimeter to decimeter. The effect of ocean load tides on a station can be expressed as follows:

$$\Delta j = \sum_{i=1}^N f_i A_j \cos(\omega_i t + x_i + u_i - \phi_j), j = 1, 2, 3 \quad (6)$$

In addition to the indirect precession of ocean load tides on the solid Earth, the position of the fixed point will also fluctuate periodically under the influence of solar and lunar gravitation and planetary gravitation, which can directly act on the surface of the solid Earth. The effect of earth tide on a station can be expressed as follows:

$$\Delta r = \sum_{j=2}^3 \frac{GM_j}{Gm} \cdot \frac{r^4}{R_j^3} \left\{ 3l_2 (\hat{R}_j \cdot \hat{r}) \cdot \hat{R}_j + \left[ 3 \cdot \left( \frac{h_2}{2} - l_2 \right) \cdot (\hat{R}_j \cdot \hat{r})^2 - \frac{h_2}{z} \right] \hat{r} \right\} + [-0.025m \cdot \sin \varphi \cos \varphi \sin(\theta_g + \lambda)] \cdot \hat{r} \quad (7)$$

## 2.7 Tropospheric path delay

Tropospheric propagation of electromagnetic waves will cause path bending and delay, resulting in an increase in the propagation distance of electromagnetic waves, which can be expressed as an

additional path length [7,8] :

$$\Delta L = L - G \quad (8)$$

$$L = \int n(s) ds \quad (9)$$

Formula: L is the path along the curve; G is the linear distance from the satellite to the GPS receiver. This extra path accounts for 0.1% of the geometric length (L-G) of the total path delay and is often ignored. It can be seen that signal block is the most important cause of path delay in the troposphere.

$$\Delta L = \int [n(s)^{-1}] ds = 10^{-6} \int n(s) ds \quad (10)$$

Where  $N=(n-1) \times 10^6$  is the refraction index of air. n is the atmospheric refraction coefficient. If its refractive index is spherical symmetric, it can be replaced by this formula. The refraction s in the atmosphere can be expressed by the r index (the diameter of an arc formed from the center of the Earth to any point in the atmosphere), which is generally expressed by empirical equations and is related to the thermodynamic parameters in the atmosphere.

### 3. Results

After the study of various errors, some of them can be accurately estimated and corrected by strict mathematical models due to the obvious regularity of systematic errors. They can be corrected by satellite calendar error and satellite clock difference, ionospheric delay correction, multipath effect correction, antenna phase center eccentricity correction, relativistic effect correction. Tidal correction and other means have greatly improved the accuracy of satellite navigation and positioning. In addition to the non-dispersive medium in the troposphere, the degree of change with time is relatively drastic, and it is difficult to use a simple and unified mathematical model to determine. The tropospheric path delay between two stations often shows a weak spatial correlation, so it is necessary to focus on the study of tropospheric path delay, which will play a very important role in satellite positioning.

### 4. Conclusions

In this paper, the characteristics of various errors affecting satellite navigation positioning and the methods of reducing and eliminating them are studied. It is found that most of the errors can be accurately estimated and corrected by using strict mathematical models. The tropospheric path delay between two stations often shows a weak spatial correlation, so it is necessary to establish a real-time model for tropospheric delay correction. At present, the study of tropospheric delay is a hot direction in the field of GNSS, especially the study of the wet component of tropospheric delay, because water vapor is the most active factor in the troposphere. Therefore, the use of more optimized mathematical and physical models for water vapor inversion is the main direction of future optimization of tropospheric delay model. In the future, a proper tropospheric delay correction model and projection function can be selected based on matlab programming language and meteorological data from local weather stations, and a tropospheric delay correction model along the direction of the propagation path can be established hourly. Improving the accuracy of tropospheric delay correction is an important step to improve the accuracy and reliability of GNSS. At the same time, this is also one of the topics that the author is interested in. In the future, the author hopes to solve this problem and contribute to the development of China's Beidou navigation satellite system.

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