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DFA-based algorithm for optimizing the accuracy and cost of shaft and hole assembly

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Abstract: Based on the idea of DFA, this study proposes an optimization algorithm for the shaft and hole assembly accuracy and cost. By simplifying and analysing the formula proposed by Abdel-Malek for the calculation of shaft and hole assembly throughput, a mathematical model is established to explore the selection of optimal values for machining cost, part machining tolerance and equipment positioning accuracy. And, the relationship between shaft and hole assembly clearance and spring wire diameter is analysed. The study shows that, within the constraints of cost, the optimum value is selected in eight cases, with different strategies. The maximum resistance to insertion is achieved when the spring wire diameter reaches a value of 0.8 times the difference between the average diameter of the hole and the spring. In addition, this study refines the traditional DFA design system by providing the appropriate algorithms to lay the foundation for software implementation.

1. Introduction

In the face of current fierce market competition, industries are striving to manufacture products as economically and easily as possible while ensuring that they are used as required. In streamlined manufacturing systems, process chains and integrated process control are important means of improving manufacturing accuracy and efficiency [1]. One of the more proven and successful methods is concurrent engineering. In concurrent engineering, almost all aspects of product design and manufacturing are analyzed and considered in an integrated and systematic way in parallel [2]. This approach emphasizes the inter-active consideration of all elements of the product development process, mainly customer requirements, product quality, manufacturing costs and production times, etc [3]. Most research in concurrent engineering currently focuses on integrating production considerations with product design issues [4]. In order to achieve better objectives, two indicators are important in the concurrent analysis of products: manufacturability and assemblability. DFA (Design For Assembly) technology has continued to develop under the impetus of concurrent engineering research, emphasizing the need to make full use of various technical tools such as analysis, evaluation, planning, and simulation in the product design process, to simplify complex models and reduce assembly time, to improve the product assembly structure under the condition of satisfying product performance and functionality, so that the designed product can be assembled and to reduce the assembly cost and total product cost as much as possible [5-7]. Most of the DFA research is based on the core idea of modularity, where the issue of improving assemblability is discussed at the system level of a functional module, a component, or even a whole product [8]. Through the accumulation of empirical data, the production time and production costs of individual components and whole products are estimated. Less research has been carried out on the relationship between the machining parameters of individual components and their assemblability.

The development of DFA will not only be limited to cost estimation but should also guide the machining of the parts with the idea of DFA so that the process parameters of the parts also meet the requirements of DFA. The study of assembly-oriented design for the shaft and hole assembly has had a long history of research since the 1980s [9]. Many theoretical results have been presented, but most of these studies have focused on the design of the part, which has been improved to reduce the insertion difficulty, increase the assemblability and reduce the assembly cost. For example, by increasing chamfers to improve assembly, the effect of the size and shape of the chamfer on insertion resistance has been studied [10]. Rationalization of the shape of the parts, avoidance of snapping, tangling and inconvenient access to the parts, improvement of the design of the parts, reduction of drag during assembly, and avoidance of blockage of the shaft bore during assembly, etc[11]. The goal of most studies is the total assembly time, which is used as the main, if not the only, indicator for estimating assembly costs. There is a lot of research on optimization methods for accuracy and efficiency in shaft-hole assembly, and Abhary et al [12] describe a genetic algorithm (GA) designed to optimize assembly sequence planning problems (ASPP), as well as large-scale and highly constrained combinatorial problems. Li et al [13] proposed an optimization algorithm for the automatic assembly of circular parts, which can help designers to select the optimal process parameters within budget constraints. Fang et al [14] proposed a new algorithm aimed at minimizing the number of remaining parts in a selective assembly. Gurusamy et al [15] proposed an arbitrary distribution algorithm for minimizing gap variation when performing selective assembly for the manufacture of assemblies with zero residual components and minimal gap variation with good results.

Each of these literature presents its own research in assembly relationships and provides enlightening help for related research. In this article, a mathematical model and corresponding calculation methods are introduced to quantitatively analyze the assemblability and incorporate the selection of process parameters into the DFA design system to optimize the part design parameters and the assembly process parameters accordingly. Using part machining tolerance, equipment positioning accuracy, and assembly clearance as optimization targets, the corresponding mathematical model is established, the relationship between the target parameters to be optimized and the assembly throughput of the shaft and hole is analyzed quantitatively, the optimization points for the target parameters are found using mathematical means such as function optimization and area optimization, and examples are given to illustrate the guiding role of this algorithm for design, manufacture, and assembly. In addition, the article is oriented towards automated assembly design and investigates the design of shaft hole assembly with elastic resistance and the optimization of its process parameters. The design of the upper and lower receiver of the floor fan, for example, uses the elastic force of the spring between the upper and lower receiver to make the adjustment of the upper receiver easier and thus facilitate the adjustment of the height of the floor fan; another example is the spring-loaded fruit knife, where the knife body is hidden in the sheath when not in use and pops out under the action of the spring when in use, with a compact structure, small size and easy to carry. In this article, the assembly of the spring and the hole is simplified by considering the spring as a shaft part, the assembly of the spring and the hole can be considered as a form of shaft-hole assembly, and the relationship between the spring wire diameter and the assembly clearance is studied.

2. Mathematical Modelling

The assembly process focuses on achieving a high throughput of the shaft into the hole, with constraints including assemblability and cost budget constraints. Based on these constraints, the problem is mathematically modeled. In this mathematical model, the shaft bore machining tolerances, the positioning accuracy of the assembly unit, the shaft bore assembly clearance, and the spring wire diameter are used as the final optimization target parameters to select the design parameters of the part and the machining process parameters and to select the appropriate assembly equipment. Abdel-Malek [16] proposed a formula for calculating the throughput rate of a simple shaft-hole assembly, which brings together in a single expression several parameters that affect the throughput rate. The expression is as follows:

$$P(I) = 2\phi(\frac{\sigma}{u}) - 1 \tag{1}$$

$$u = D_h - D_p \tag{2}$$

$$\sigma = \sqrt{\sigma_p^{\ 2} + \sigma_h^{\ 2} + \sigma_r^{\ 2}} \tag{3}$$

Among these, D_h, and D_p are the diameter of the hole and shaft respectively, u is the shaft-hole assembly clearance; σ_p is the machining tolerance of the shaft hole, σ_h is the positioning accuracy tolerance of the assembly device, σ_r is a comprehensive tolerance; $\phi(x)$ is a normal distribution function, it is a decreasing function in the range of values 0 to ∞ of the independent variable.

This equation allows us to conclude that the bore assembly clearance and the machining and positioning accuracy of the bore assembly unit influence the throughput rate. The greater the bore assembly clearance, the higher the machining and positioning accuracy, and the greater the throughput rate of the boring assembly.

Here we make a simplification of the expression proposed by Abdel-Malek, letting:

$$\sigma_0 = \sqrt{\sigma_p^{\ 2} + \sigma_h^{\ 2}} \tag{4}$$

Then

$$\sigma = \sqrt{\sigma_0^{\ 2} + \sigma_h^{\ 2}} \tag{5}$$

Since holes are more difficult to machine than shafts, the national standard stipulates that: in order to make holes and shafts equivalent in terms of process, the tolerance grade of holes is one level lower than that of shafts in fits of higher accuracy grades; in fits of lower accuracy grades, the same tolerance grade is used for holes and shafts [17]. Meanwhile, the tolerance grades of shaft holes and their corresponding tolerance values are listed as standard values in GB/T1800.3-1998, so if we have determined σ_0, we can determine the optimum tolerance grade in accordance with the national standard by means of a table search.

The smaller the tolerance value, the more difficult and costly the machining; the larger the tolerance value, the easier and less costly the machining. The higher the tolerance value, the easier the machining and the lower the machining cost. With high precision machining of the shaft and bore, a high assembly pass rate can be achieved; however, the production cost must be kept within a certain range, which creates a problem of optimization. For assemblies with different accuracy requirements, shafts and bores are given different tolerance levels, each with a defined tolerance value. For example, assuming the basic size of the shaft hole Φ40mm, for precision fits, the tolerance grade of the hole is IT6-8, corresponding to a tolerance value of 16-39μm, and the tolerance grade of the shaft is IT5-7, corresponding to a tolerance value of 11-25μm; for medium precision fits, the tolerance grade of the hole is IT9-10, corresponding to a tolerance value of 62-100μm, and the tolerance grade of the shaft is IT9-10, corresponding to a tolerance value of 62-100μm (Standard Tolerance). Given a shaft-hole assembly design, and has its own defined range of values, so also has a defined range of values, so the following equation:

$$L_0 \le \sigma_0 \le U_0 \tag{6}$$

Where L_r is the minimum positioning accuracy value and U_r is the maximum positioning accuracy value.

For each positioning accuracy value σ_r, there is an equipment cost W_r. The equipment cost corresponding to the minimum positioning accuracy value L_r is W_{Lr}. The equipment cost corresponding to the maximum positioning accuracy value U_r is W_{Lr}.

Let the cost budget value of the cost be W. Then we have:

$$W_0 + W_r \le W \tag{7}$$

Many studies have shown that the positioning accuracy of the equipment and the machining accuracy of the workpiece are positively related to the machining cost [18]. Therefore, the expressions for the machining cost and the equipment cost can be derived separately as follows:

$$W_0 = \frac{(W_{L0} - W_{U0})(U_0 - \sigma_0)}{U_0 - L_0} + W_{U0} \tag{8}$$

$$W_0 = \frac{(W_{Lr} - W_{Ur})(U_r - \sigma_r)}{U_r - L_r} + W_{Ur} \tag{9}$$

To simplify the calculation, let:

$$C_1 = \frac{W_{L0} - W_{U0}}{U_0 - L_0} \tag{10}$$

$$C_2 = \frac{W_{Lr} - W_{Ur}}{U_r - L_r} \tag{11}$$

Substituting Eqs. (8) (9) (10) (11) into Eq. (7), the collation gives:

$$C_1\sigma_0 + C_2\sigma_r \ge W_{U0} + W_{Ur} + C_1U_0 + C_2U_r - W \tag{12}$$

To simplify the calculation, let:

$$C = W_{U0} + W_{Ur} + C_1U_0 + C_2U_r - W \tag{13}$$

Then Eq. (13) simplifies to:

$$C_1\sigma_0 + C_2\sigma_r \ge C \tag{14}$$

Using σ_0 an σ_r as variables, determining their possible range of values and finding the optimum points within the range of values, the optimum points of the process parameters σ_0 and σ_r can be obtained.

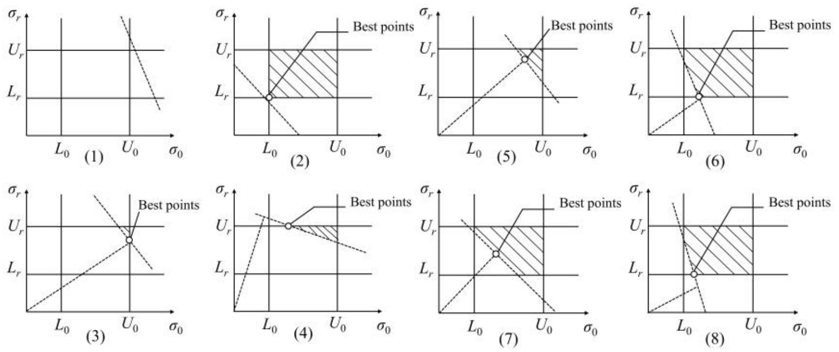

Figure 1: Selection of optimization points for process parameters σ_0 and σ_r

Make the straight-line $C_1\sigma_0+C_2\sigma_0=C$ (dashed line in Figure 1) and div ide the selection of the taking constraint region into eight cases depending on the different values of C_1, C_2, and C (as in Figure 1). In the space above the straight line $C_1\sigma_0+C_2\sigma_0=C$, the straight line and the region of possible fetch values of σ_0 an σ_r enclose a cost constraint region (the region filled by the diagonal line in Figure 1), and in this new region, the cost constraint is met. In addition to this, Eq. (1) shows that the value σ should be as small as possible to obtain a high pass rate. Eq. (5) shows that the distance from a point in the cost-constrained region to the origin represents the value of σ. Therefore, a vertical line (dotted line in Figure 1) is drawn from the origin to the line $C_1\sigma_0+C_2\sigma_0=C$. If the footfalls within the cost constraint region, the foot is the optimum point; if the footfalls outside the cost constraint region, the point on the line intersected by the cost constraint region and the line $C_1\sigma_0+C_2\sigma_0=C$ that is closest to the foot is the optimum point.

3. Optimisation of shaft bore assembly clearance and spring wire diameter

Elastic elements such as springs are often used in products to couple shafts and holes to achieve certain specific functions. In this article, the assembly of springs and holes is simplified so that further research can be done using the idea of shaft-hole assembly. The elastic coupling element in shaft-hole assembly, in the case of springs, is first fitted into the hole during the assembly process and only then the shaft is assembled. Due to the presence of elastic forces, the subsequent shaft assembly is then subject to elastic forces that may cause difficulties in assembly.

In the selection of springs, closed springs should be selected instead of open springs, open springs can become entangled and cause difficulties in automatic separation and pick-up in automated assembly, and the most prominent problem is that the dead ring of the spring may be stuck in the end face of the hole, causing assembly difficulties and even leading to assembly failure [19].

As shown in Figure 2, the following relationships are based on the assembly relationships:

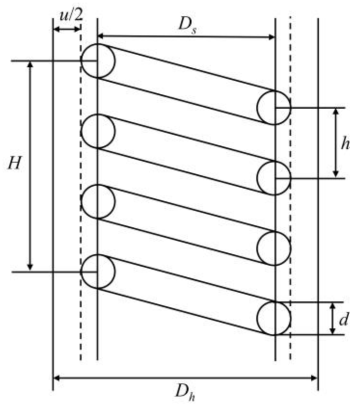

Figure 2: Spring assembly diagram

$$u = D_h - (D_s + d) \tag{15}$$

Where D_s is the average spring diameter, D_h is the bore diameter and d is the diameter of the spring wire.

To simplify the calculation, let $\lambda_2 = D_h - D_s$ and we have:

$$u = \lambda_2 - d \tag{16}$$

In practice, there is inevitably a certain offset between the centre line of the inserted shaft and the centre line of the spring, and as the shaft is inserted, the spring is not only deformed axially but also transversely, the magnitude of the deformation being related to the axial and transverse stiffness of the spring respectively. This article focuses on the transverse deformation of the spring, as this will produce a force *F* perpendicular to the direction of insertion, resulting in pressure on the end surfaces of the inserted shaft and the hole housing, which will result in a frictional resistance on the end surfaces of the shaft and the hole housing, which will make the assembly of the shaft and hole more difficult.

As can be seen from Figure 2, the maximum value of the transverse deformation of the spring is *u*. Due to the small shaft bore clearance, the transverse deformation of the spring will soon reach its maximum value with the insertion of the shaft, after which the spring will only produce axial deformation.

Here a mathematical model is developed to optimise the spring, the gap u between the holes, and the diameter *d* of the spring wire so that the force *F* is minimised. Due to the requirements of serviceability and pressure spring stability, elastic couplings such as springs are often secured by a spring seat at the lower end and free at the upper end. According to the sparse-loop spring theory [20], the following conclusions are drawn for the axial stiffness of the spring:

$$k' = \frac{3B}{H^3} \tag{17}$$

Where *k* is the lateral stiffness of the spring, *B* is the bending stiffness of the spring and *H* is the total height of the spring.

From the literature [21], the bending stiffness of the spring is:

$$B = \frac{2EIH}{S} \cdot \frac{1}{1 + E/2G} \tag{18}$$

Where *E* is the modulus of elasticity of the spring material, *G* is the shear modulus of the spring material, *I* is the axial moment of inertia of the spring, *H* is the total height of the spring and S is the spread length of the spring wire.

Based on knowledge of the mechanics of materials, there is:

$$I = \frac{\pi d^4}{64} \tag{19}$$

According to the relationship between the spring dimensions in Figure 2, it is possible to obtain:

$$H = n \cdot h = \pi n D_s \cdot \tan\alpha \tag{20}$$

$$S = \frac{H}{\sin\alpha} = \frac{\pi n D_s}{\cos\alpha} \tag{21}$$

Substituting Eqs. (17) (18) (19) (20) into Eq. (16), the final expression for the transverse stiffness is collated to give:

$$k' = \frac{\pi E \sin\alpha}{32(1+\frac{E}{2G})} \cdot d^4 \tag{22}$$

To simplify the calculation, let:

$$\lambda_1 = \frac{\pi E \sin\alpha}{32(1+\frac{E}{2G})} \tag{23}$$

Then there are:

$$k' = \lambda_1 \cdot d^4 \tag{24}$$

Force perpendicular to the direction of insertion:

$$F = k' \cdot u \tag{25}$$

Insertion resistance:

$$f = \mu \cdot F \tag{26}$$

Substituting Eqs. (22) (23) (24) (25) into Eq. (26), the final expression for the resistance to assembly is collated:

$$f = \mu\lambda_1\lambda_2 d^4 - \mu\lambda_1 d^5 \tag{27}$$

From the principle of monotonic increasing and decreasing functions and extremal maxima [22], we know that:

The resistance f takes its maximum value when d=4λ_2/5 so the spring wire diameter is chosen with care away from the value 4λ_2/5.

For example, in the assembly of the upper and lower receiver of a floor fan, set the inner diameter of the lower receiver ϕ40mm, according to the requirements of the use of the average diameter of the spring ϕ36mm, according to the spring manual spring wire diameter and winding ratio of the recommended value, the spring wire diameter should be selected 2.5-6mm, winding ratio of 4-10 series [23]. When selecting the spring wire diameter, care should be taken to avoid the value 3.2mm (4*x*(40-36)/5=3.2), which makes the assembly resistance far from the peak. Springs with larger spring wire diameters are less costly, and as the diameter of the spring wire has little effect on the performance of this product, a larger value is taken for the spring wire diameter given the cost savings. In this case, a spring with a spring wire diameter of 5mm and a winding ratio of 7 is suitable.

4. Conclusion

We Part machining tolerances, equipment positioning accuracy, and assembly clearances are three important factors that affect automated assembly. In this article, the influence of the three factors on the assembly throughput of the shaft bore is quantitatively analysed through the establishment of a mathematical model, and the following conclusions are drawn:

1) The selection of optimal values for part machining tolerances and equipment positioning accuracy is divided into eight cases with different strategies for the selection of values, depending on the total cost of machining and the individual cost constraints of machining. Cost is one of the most important factors influencing the selection of parts and equipment.

2) The selection of the spring wire diameter has a significant impact on the insertion resistance. When the diameter of the spring wire reaches a value of 0.8 times the difference between the average

diameter of the hole and the spring, the resistance to insertion is at its maximum. The spring wire diameter should be selected away from the value of 0.8 times the difference between the hole and the average diameter of the spring.

This study completes the traditional DFA design system with assembly time and cost as the ultimate optimisation objectives, incorporates the selection of process parameters into the DFA design system, identifies the theoretical basis for process parameter optimisation, provides the corresponding algorithms, lays the foundation for software implementation, and helps to combine DFA and CAPP.

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