Modeling and optimal design of heliostat field based on particle swarm optimization algorithm

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Keywords: Tower Solar Power Plant, Particle Swarm Optimization, Heliostat Field

Abstract: Building a new type of power system with new energy as the main body is the goal of achieving "carbon peak" and "carbon neutrality" in China. An important measure of the target. Tower solar thermal power generation is a new type of clean energy technology with low carbon and environmental protection. The annual average optical efficiency of the fixed sun station is affected by shadow occlusion, cosine efficiency, atmospheric transmittance, collector truncation efficiency, mirror reflectance, size layout of heliostat and so on. In this paper, the heliostat field is modeled based on reflection theorem, solar cone theory and solar motion law, and optimized based on particle swarm optimization algorithm to calculate the annual average optical efficiency, annual average output thermal power, and annual average output thermal power per unit mirror area of heliostat field. To solve this problem, The paper need to consider the height and Angle of the sun, the blocking of sunlight, and the cone model of sunlight and other factors, first calculate the sun's height Angle, azimuth Angle and cosine loss, and then calculate the shadow blocking efficiency through the tower shadow blocking, and then calculate the collector truncation efficiency according to the sun light cone theory, etc. The mathematical model of a single heliostat can be modeled, and the average optical efficiency and output thermal power of the heliostat field can be calculated by traversing the average. Then, this paper adopts single objective optimization model and particle swarm optimization algorithm. Firstly, an optimization model is established with the rated power reaching 60MW as the constraint condition, and the annual average output thermal power per unit mirror area is as large as possible to optimize the target. Then particle swarm optimization algorithm is used to find the maximum output thermal power and the final particle convergence, indicating the rationality of the mathematical model.

1. Introduction

With the increasingly prominent energy and environmental problems, the construction of a new power system with new energy as the main body is an important measure to achieve the goal of "carbon peak" and "carbon neutrality" in China [1]. Solar thermal power generation, a new type of solar energy utilization technology, has become the focus of attention in the field of new energy in recent years due to its outstanding advantages and considerable development prospects. Heliostat is
one of the main devices of the tower solar thermal power station, which is composed of a concentrating device composed of a mirror, a frame and a base, a tracking transmission system, a control system, etc., for tracking and receiving and gathering reflected solar light into the collector located at the top of the receiving tower\textsuperscript{[2]}. So it is of great significance to study the layout of heliostat field and optimize its design. In this paper, the average annual optical efficiency, average annual thermal output power and average annual thermal output power per unit mirror area of the heliostat field are calculated through reasonable modeling. At the same time, the heliostat field is optimized to make the average annual thermal output power per unit mirror area as large as possible under the condition that the heliostat field reaches the rated power. All the data in this paper are from question A of "2023 National Mathematical Modeling Competition for College Students".

2. Model establishment and analysis

Firstly, a heliostatic field coordinate system is defined. The origin of the coordinate system is located in the center of the circular heliostatic field region. The due east direction is the positive direction of the x axis, the due north direction is the positive direction of the y axis, and the direction perpendicular to the horizontal face is the positive direction of the z axis. The following formulas are used to calculate the sun's altitude Angle\textsuperscript{[3]} and the Sun's azimuth Angle\textsuperscript{[4]}:

\[
\alpha_s = \arcsin(\cos \delta \cos \varphi \cos \omega + \sin \delta \sin \varphi)
\]

\[
\gamma_s = \text{sgn}(\omega) \arccos\left(\frac{\sin \delta - \sin \alpha_s \sin \varphi}{\cos \alpha_s \cos \varphi}\right)
\]

Where, $\varphi$ is the local latitude, $\delta$ is the solar declination Angle, and $\omega$ is the solar hour Angle; sgn is a symbolic function, and the units of solar altitude Angle $\alpha_s$ and solar azimuth Angle $\gamma_s$ are both rad, and have:

\[
\begin{align*}
\omega &= \frac{\pi}{12} (ST - 12) \\
\delta &= \arcsin\left(\sin \frac{2\pi D}{365} \sin \left(\frac{2\pi}{360} 23.45\right)\right)
\end{align*}
\]

Where ST is the local time and D is the number of days counting from the vernal equinox as the 0th day.

Under the known conditions such as longitude and latitude, only the results of five time points on the 21st day of 12 months need to be calculated, and the sun's altitude Angle $\alpha_s$ and solar azimuth Angle $\gamma_s$ can be calculated using formulas (1), (2) and (3) to determine the exact position of the sun at a certain moment\textsuperscript{[5]}. The output thermal power of heliostat field is

\[
E_{\text{field}} = \text{DNI} \sum_i A_i \eta_i
\]

DNI is the irradiance of normal direct radiation; N is the total number of heliostats (unit: face); $A_i$ is the lighting area of the I-side heliostat (unit: m$^2$); $\eta_i$ is the optical efficiency of the I-th mirror.
DNI = \( G_0 \left[ a + b \exp \left( -\frac{c}{\sin \alpha_r} \right) \right] \)  
\[ (5) \]

\[
\begin{align*}
a &= 0.4237 - 0.00821(6 - H)^2 \\
b &= 0.5055 + 0.00595(6.5 - H)^2 \\
c &= 0.2711 + 0.01858(2.5 - H)^2
\end{align*}
\]
\[ (6) \]

Where \( G_0 \) is the solar constant, whose value is 1.366 \( kW/m^2 \), and \( H \) is the altitude of the place, which is 3km. The optical efficiency of heliostat \( [6] \) \( \eta \) is:

\[
\eta = \eta_{\text{sh}} \eta_{\text{cos}} \eta_{\text{trunc}} \eta_{\text{ref}}
\]
\[ (7) \]

Among them, shadow occlusion efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency and specular reflection rate \( [7] \):

\[
\begin{align*}
\eta_{\text{sh}} &= 1 - \text{Shadow occlusion loss} \\
\eta_{\text{cos}} &= 1 - \text{Cosine loss} \\
\eta_{\text{at}} &= \begin{cases} 0.99321 - 0.0001176 d_{\text{HR}} + 1.97 \times 10^{-8} d_{\text{HR}}^2, & \text{if } d_{\text{HR}} \leq 1000 \\ 1, & \text{if } d_{\text{HR}} > 1000 \end{cases}
\eta_{\text{trunc}} &= \frac{\text{The collector receives energy}}{\text{The mirror completely reflects energy}} - \text{Shading loses energy}
\eta_{\text{ref}} &= 0.92 \text{ (constant)}
\end{align*}
\]
\[ (8) \]

\[ (9) \]
\[ (10) \]

\[ (11) \]

\[ (12) \]

To calculate the cosine loss:

\[
\eta_{\text{cos}} = \cos \left( \theta_{\text{cos}} \right)
\]
\[
= \frac{1}{2} \arccos \left( -\frac{x}{d_{\text{HR}}} \sin \gamma \cos \alpha_s + \frac{y}{d_{\text{HR}}} \cos \gamma \cos \alpha_s + \frac{\sin \alpha_s}{d_{\text{HR}}} (H - h) \right)
\]
\[ (13) \]

For shadow occlusion loss, considering the shielding effect of the absorbing tower body on sunlight, especially in the range of the heliostatic field radius of 350 meters, the heliostatic field is not installed within the range of 100 meters of the center of the circle. Among them, \( H \) represents the altitude of the mirror field, \( d \) represents the diameter of the cylindrical collector, and \( h \) represents the height of the cylindrical collector. Therefore, the paper can calculate the effective heliostat arrangement area, as follows:

\[
\eta_{\text{at}} = 1 - \frac{S_{\text{shadow}}}{S}
\]
\[ (14) \]
\[
\begin{align*}
S &= \pi \left( R^2 - r^2 \right) \\
H_{\text{shadow}} &= \frac{H}{\tan(\alpha_s)} \\
S_{\text{shadow}} &= H_{\text{shadow}} d
\end{align*}
\]  

(15)

Regarding the collector truncation efficiency, the energy loss and beam Angle are analyzed by using the cone model of sunlight. The beam Angle is \( \phi = 9.3 \text{rad} \). First, the column collector is equivalent to the ball collector, which is helpful for subsequent metering. Calculate the absorption area of the beam.

\[
\pi r_{\text{collect}}^2 = h d
\]  

(16)

The light cone model [8] is shown in Fig 1, the beam reflected by a flat mirror is actually prismatic, and the length and width of the base are calculated as follows:

\[
\begin{align*}
L &= L + \frac{2d_{HR} \tan \left( \frac{\alpha}{2} \right)}{\sqrt{2}} \\
W' &= W + \frac{2d_{HR} \tan \left( \frac{\alpha}{2} \right)}{\sqrt{2}}
\end{align*}
\]  

(17)

At the same time, the rectangular illumination area at the bottom of the prism beam is equivalent to the circular illumination area of the conical beam, as follows:

\[
\pi r_{\text{rad}}^2 = W' L
\]  

(18)

Energy flux density:

\[
f(\rho) = \frac{2}{r_{\text{rad}}} - \frac{2}{r_{\text{rad}}} \rho, \rho \in [0, r_{\text{rad}}]
\]  

(19)

Finally, the paper can calculate the truncation efficiency of the collector:

\[
\eta_{\text{trunc}} = \int_{0}^{r_{\text{collect}}} f(\rho) d\rho = \frac{2r_{\text{collect}}}{r_{\text{rad}}} - \left( \frac{r_{\text{collect}}}{r_{\text{rad}}} \right)^2
\]  

(20)
3. Model solving

According to the optical efficiency model of single heliostat established above, it is only necessary to bring in data for traversing average, which calculates the annual average optical efficiency, annual average thermal output power, and annual average thermal output power per unit mirror area of the heliostat field. The results are visualized as follows:

First of all, the model needs to determine the sun's azimuth angle and altitude angle. In this paper, Fig 2 of the result is drawn and displayed, reflecting the position changes of the sun at 12*5 time points in a year.

![Figure 2: The elevation Angle and azimuth Angle of the sun change with time](image)

According to formula (5), normal irradiance DNI can be calculated, and its change over time is shown in Fig 3.
Figure 3: DNI variation diagram of normal radiation intensity

The optical efficiency of each heliostat at every moment, including shadow blocking efficiency $\eta_{sb}$, cosine efficiency $\eta_{cos}$, atmospheric transmittance $\eta_{at}$, collector truncation efficiency $\eta_{trunc}$, mirror reflectance $\eta_{ref}$ (constant 0.92), is obtained by traversing the whole heliostat. As shown in Fig 4. Obviously, the optical efficiency of each heliostat changes with the position of the sun, and its size also changes.

Figure 4: Optical efficiency

4. Particle swarm optimization model

In this paper, particle swarm optimization is adopted to further study the processing optimization. If the position distribution of heliostat is random, it can be regarded as a single objective optimization problem. In this paper, particle swarm optimization algorithm is used to find the optimal solution\cite{9}. Aiming at the annual average output thermal power, the heliostat distribution is determined by iterative algorithm. The height angle, azimuth angle and DNI diagram are unchanged. Particle swarm optimization (PSO) is an algorithm inspired by the simulation of the predation behavior of birds\cite{10}. The basic idea is that in the search process, each particle is in its own search space and constantly updates its position and speed in order to better find the optimal solution. Here, the heliostat
parameters of each side are taken as particles, with the maximum output thermal power as the optimization goal, and solved through programming. The results are as follows:

The optimal heliofield position obtained by the particle swarm optimization algorithm is shown in Fig 5, where the coordinates of the absorption tower are (0, -44.3).

![Figure 5: Position distribution of the optimal solar field](image)

As shown in Fig 6, after 23 generations, the average annual thermal output power has reached the optimal level, meeting the restriction condition that the rated annual average thermal output power (hereafter referred to as rated power) of the heliostatic mirror field in the title is 60 MW. In subsequent iterations, the power curve tends to be smooth, and the maximum power reaches 67MW, which is the optimal.

![Figure 6: PSO iteration diagram](image)

In the iterative process, this paper calculates the average of each efficiency of the whole field at each moment, including shadow blocking efficiency, cosine efficiency, atmospheric transmittance, collector truncation efficiency and mirror reflectance. The value of mirror reflectance can be 0.92. The optical efficiency is shown in Fig 7.
5. Conclusions

In this paper, the position distribution of absorber and heliostat in heliostat field is studied by establishing a reasonable geometric model with the aim of maximizing the output thermal power. The model established in this paper fully considers the occlusion factor of heliostatic mirror, and incorporates the occlusion problem into the modeling process. The model can solve the basic problem with excellent accuracy and obtain the answer through efficient calculation. Secondly, in order to deal with the occlusion problem of heliostatic mirror, the model adopts the exquisite analytic geometry method, which transforms the complex geometry problem into the algebraic calculation problem. This process not only lays a solid mathematical foundation, but also makes the problem solving clear and efficient. In addition, in the process of building the model, more basic mathematical operation tools are used as far as possible. This helps to reduce the complexity and difficulty of the model, making it easier to be widely used and popularized. However, in the case of sufficient computational resources, it is necessary to build more complex and detailed models. Such a model can fully consider other factors that may affect efficiency. Future studies can further explore the effects of these complex factors to improve the accuracy and comprehensiveness of the model. This research provides a theoretical basis for the construction of new energy power system, and has the potential of practical application.

References