

Geometric analysis of the bronze solar wheel and the half-gold mask based on the curve equations

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Abstract: The problems of drawing the pattern of the bronze sun wheel and estimating the surface area and volume of the half gold mask are difficult to resolve. In this paper, we first use the theory of the polar coordinate system and the curve fitting method to construct the curve equation model. Secondly, the Python and Excel software are used to solve the problems by drawing and programming comprehensively so as to derive the drawing law of different rays of the bronze sun wheel and estimate the surface area and volume of the complete gold mask. To solve the first problem, we first used the theory of polar coordinate system to calculate the coordinates of the five inner "rays", constructed a mathematical model of the five inner rays, then derived the general equation of the inner arcs, and then drew the bronze sun wheel patterns of four, six, eight and twelve rays according to the general equation. To solve the second problem, we first adjusted the picture of the half-golden mold in the title to a width of 23 cm², used the nose tip of the gold mask as the coordinate origin to measure the discrete data points of the "two-eye side" and "two-ear side", then fitted and plotted the curves. The equations of the corresponding curves of "both eyes" and "both ears" in the gold mask and the surface area of the ears are calculated, and finally the surface area of the complete gold mask is estimated to be 721.54 cm², the mass is 524.13 g. Finally, the model is tested and analyzed, the advantages and disadvantages of the model are evaluated objectively, and the model is extended horizontally and vertically.

1. Introduction

Ancient Shu civilization, represented by Sanxingdui, is an important historical witness to the diversity, tolerance and extension of Chinese civilization^[1]. The bronze wheel was a symbol of the Sun God cult, with a small hole in the center of the round, convex hub and in the wheel's halo for fixed use. Since May 2022, six newly discovered ritual pits have been excavated at the Sanxingdui site, yielding a total of 13,000 cataloged artifacts^[2]. Half a mysterious gold mask is a gold vessel unearthed in the No.5 sacrificial pit of Sanxingdui Site. Masks exhibit structural asymmetry compared to other highly refined objects^[3]. Mask square face shape, wide ears, hollow big eyes^[4].

The bronze wheel is 0.85 m in diameter, centered on a circular raised hub surrounded by five radiating spokes with a one-week rim outside the spokes. The half-face of the gold mask measures

approximately 23 centimeters in width and 28 centimeters in height. It contains approximately 85% gold and 13% to 14% silver^[5]. The thinnest point is 0.2 mm and the thickest point is 0.4 mm. However, most devices are copper-lead-tin alloys with varying tin and lead content^[6]. The density of gold is 19.32 g/cm³ and the density of silver is 10.53 g/cm³. Taking the center of the bronze wheel as the coordinate origin, calculate the five inner "rays" corresponding to the curve equations of the bronze wheel and the arc of the circle, establish a mathematical model, derive the general equations of the inner arc of the circle, and draw the bronze wheel pattern with four, six, eight and twelve rays according to the general equations. Secondly, the half-mask is reduced to a complete mask, and the coordinate system is established with the tip of the nose of the golden mask as the coordinate origin. Calculate the equations of the curves corresponding to the "edges of the eyes" and the "ears" in the gold mask, and estimate the surface area and mass of the complete gold mask.

Through the study of the Sanxingdui, we know that the bronze wheel is a circle with five equal internal "rays" and equal angles between the five rays. Other bronze wheels have six, eight and twelve internal "rays". The objective is to calculate the equations of the bronze sun wheel curve and the five internal "rays" corresponding to the arc of the circle, to develop a mathematical model, to derive a general equation for the arc of a circle, and to plot a bronze sun wheel pattern with four, six, eight, and twelve rays based on the general equation.

Secondly, half of the gold mask is about 23 centimeters wide and 28 centimeters high. Our goal is to recover this half of the mask and establish a coordinate system using the tip of the nose of the golden mask as the origin of the coordinates. The equations of the curves corresponding to the "edge of the eye" and the "ear" in the gold mask were calculated, and the surface area and mass of the complete gold mask were estimated. Based on half of the gold mask, shrink the other half by symmetry to make the complete gold mask. To solve this problem, the image of the gold mask was adjusted to 23 cm wide, discrete data points for the "edge of the eye" and the "ear" were measured, and then curves were fitted and plotted^[7]. Its surface area needs to be subtracted from the surface area of the eye and the surface area of the ear. Once the surface areas of the eyes and ears are found, the mass can be derived from the thickness and density.

2. Model hypothesis

Table 1: Symbol description

symbol	Meaning	unit
d	diameter of a circle	cm
r	radius of a circle	cm
O	central angle	angle
x	abscissa	cm
Y	ordinate	cm
n	Number of internal rays	twig
S	superficial area	cm ²
V	volume	cm ³
P	density	kg/m ³
H	thickness	cm
M	quality	g

The following assumptions are made for this modeling:

- (1) Suppose that the interior of the bronze solar wheel is a positive n border;
- (2) Suppose that the outer edge of the bronze sun wheel is a circle;
- (3) Hypothesis $n \geq 3$;
- (4) Suppose that the internal ray width of the bronze solar wheel is excluded;

- (5) Suppose that the main body of the golden mask is a cuboid shape;
 - (6) Suppose that the impurities of the half-gold mask are ignored;
- The symbols required for the model are shown in Table 1

3. Model building and solution

3.1 Question 1

Using the center of the bronze wheel as the origin of the coordinates, one ray of the sun wheel is placed on the Y-axis and the rest of the rays are obtained by rotation. The interior is a square pentagon, and the angle between the five interior rays and the central angle O is equal to 72°. As shown in the Fig. 1:

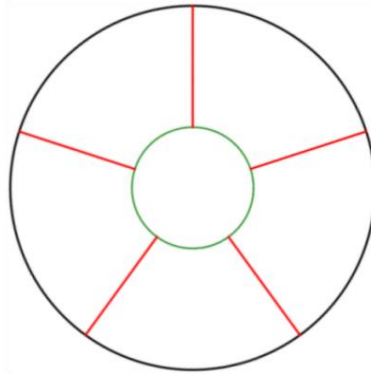


Figure 1: Five-ray bronze sun wheels

Based on the diameter of 85 cm, the coordinate of edge n1 on the X axis and the bronze wheel's cut point a1 is (42.5, 0). In other words, it can be expressed as $(R \cos 0, R \sin 0)$. The angle between side n2 and angle is 72 degrees. By applying the cosine theorem, we can determine that the coordinates of intersection between edge n2 and bronze wheel circumference, namely cutoff point a2, are $(R \cos 72, R \sin 72)$. Using this method, we can ascertain that coordinates of a3, a4 and a5 are $(R \cos 144, R \sin 144)$, $(R \cos 216, R \sin 216)$, and $(R \cos 298, R \sin 298)$. The remaining edges can be obtained by successively rotating by 72 degrees counterclockwise. Find five coordinates, introduce this bronze sun wheel curve equation:

$$x = R \cdot \cos(\theta) = R \cdot \cos(360^\circ k / 5) \quad (1)$$

$$y = R \cdot \sin(\theta) = R \cdot \sin(360^\circ k / 5) \quad (2)$$

$$(k = 1, 2, \dots, n)(k \in Z)$$

The general equation for the internal arc is

$$x = R \cdot \cos(\theta) = R \cdot \cos(360^\circ k / n) \quad (3)$$

$$y = R \cdot \sin(\theta) = R \cdot \sin(360^\circ k / n) \quad (4)$$

$$(k = 1, 2, \dots, n)(k \in Z)$$

According to the general equation of four, six, eight, twelve rays, the bronze sun wheel pattern is shown in Figure 2.

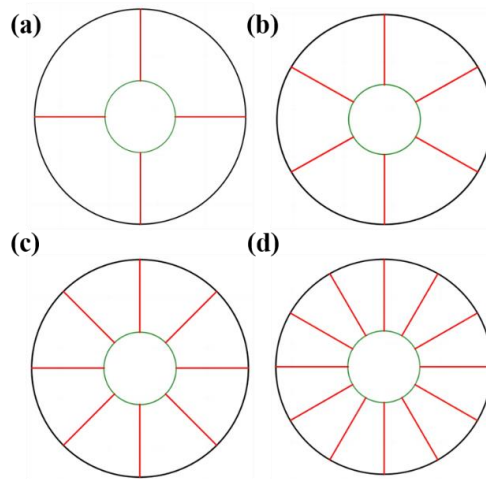


Figure 2: Four-ray (a), six-ray (b), eight-ray (c), twelve rays (d) bronze of sun wheels

3.2 Question 2

A complete gold mask: Only half of the gold mask is shown in the picture. We can restore the other half through symmetry and make it a complete gold mask.as illustrated in following Fig. 3:



Figure 3: Complete gold mask

Adjust the picture of the half-gold mold in Fig. 3 in the title to 23 cm wide, and use the nose tip of the golden mask as the coordinate origin to measure the discrete data points of "double eye edge" and "double ear". The table 2 is listed as follows:

Table 2: The coordinates of the original table.

Under the eye	On the eye	Next ear	Ear side	On the ear
(±9.5,6)	(±13.2,12.2)	(±16,-8)	(±18.5,-6.8)	(±23,14)
(±7.3,6.6)	(±10.9,13.6)	(±17.5,-7.4)	(±19.7,0)	(±22.6,15.4)
(±5.8,8.6)	(±7.5,13)	(±18.5,-6.8)	(±21,4)	(±20.4,16.4)
(±13.2,12.2)			(±22,9.6)	(±17.5,16)
			(±23,14)	

Then fit to draw the curve.as illustrated in following figure 4, figure 5, figure 6, figure 7 and figure 8:

It is known that the lower eye side x takes the range (5.8, 13.2). Y takes the range (5.6,12.2) from which the corresponding curve equation is derived:

$$y = 0.01x^4 + 0.15x^2 - 4.1x + 2.61, x \in (5.8,13.2) \quad (5)$$

Knowing the upper eye edge, the range of values of x (7.5, 13.2), the range of values of y (12.2, 13.6). This leads to the equation of the corresponding curve:

$$y = -0.12x^2 + 2.43x + 1.46, x \in (7.5, 13.2) \quad (6)$$

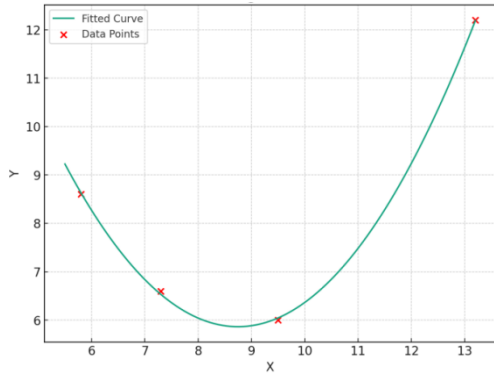


Figure 4: The curve of under the eye

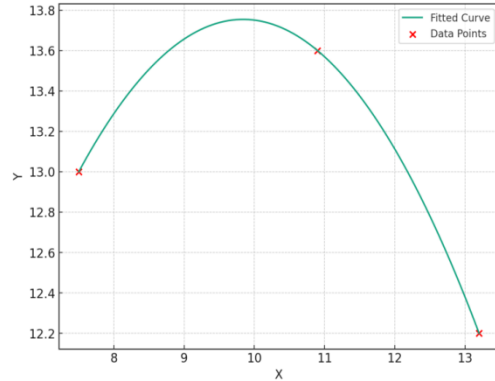


Figure 5: The curve of on the eye

The range of values of x in the lower ear is known to be (16, 18.5) and the range of values of y is known to be (-6.8, -8), which leads to the equation of the corresponding curve:

$$y = 0.08x^2 - 2.28x + 8.00, x \in (16, 18.5) \quad (7)$$

It is known that the range of values of x on the lateral side of the ear is (18.5, 23), and the range of values of y is (-6.8, 14), so the corresponding curve equation is:

$$y = -0.249x^4 + 2.88x^3 - 6.63x - 4.08, x \in (18.5, 23) \quad (8)$$

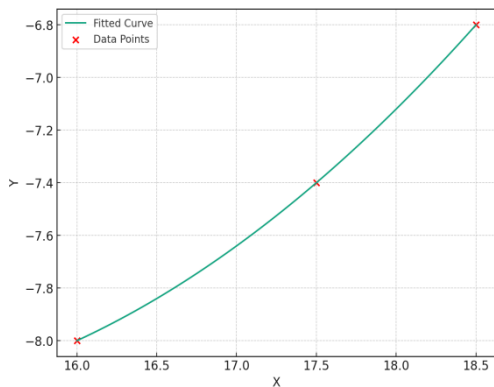


Figure 6: The curve of next ear

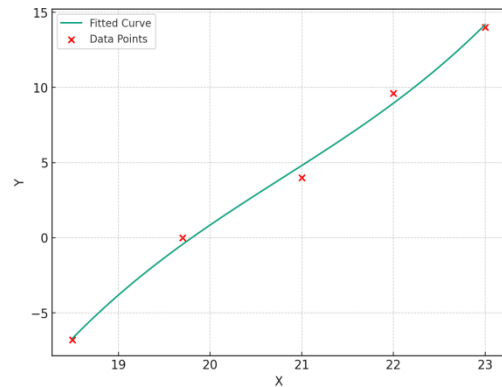


Figure 7: The curve of ear side

It is known that on the upper ear side, x ranges (17.5, 23), y ranges (14, 16.4), so the corresponding curve equation is:

$$y = 0.19x^3 + 11.49x^2 - 2.87x + 15.97, x \in (17.5, 23) \quad (9)$$

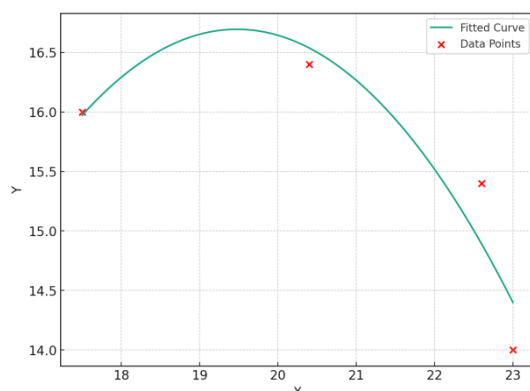


Figure 8: The curve of on the ear

The ears are wide, the eyes are hollow, and the surface area of the golden mask needs to subtract the surface area of the eye plus the surface area of the ear.

Using the surface area of the eye and ear:

$$S_{\text{eye}} = 120.21 \text{ cm}^2$$

$$S_{\text{ear}} = 148.98 \text{ cm}^2$$

The surface area of the semi-gold mask is

$$S = 721.54 \text{ cm}^2.$$

the volume:

$$V = S \cdot H$$

According to the thickness (0.2 mm is 0.02 cm, the thickest part is 0.4 mm is 0.04 cm, 0.03 cm, the average is 0.03 cm),

$$721.54 \times 0.03 = 21.65 \text{ cm}^3$$

Finally, according to its gold content is about 85%, the content of silver is about 13% to 14%, and the density (the density of gold is 19.32 grams per cubic centimeter, the density of silver is 10.53 g):

$$\begin{aligned} M &= \rho \cdot V \\ &= 524.13 \text{ g} \end{aligned}$$

4. Model inspection and evaluation

"Lei Yu, the leader of the archaeological excavation in the sacrificial area of Sanxingdui Site, said: "the weight of the half gold mask found is about 280 g, and the total weight is expected to be more than 500 g"^[5]. In this model, the body of the semi-gold mask is a cuboid shape, which is actually an irregular shape. It is inevitable that there are errors in the measurement, and the data used is only an approximation of the actual data. This paper adopts the mathematical method of horizontal coordinate system and curve equation to establish the model, and solves the practical problems through curve fitting and integration, which has certain practicality.

5. Improvement and extension of the model

The modeling idea of combining polar coordinate system and curve equation can clearly describe

complex geometric figures, which are applied in a wide range of fields, such as navigation, engineering, 3D animation, physical experiment and so on. Curve fitting can be used in teaching data analysis and statistics of students' performance, and integral is also used in medicine to calculate body mass, volume and surface area^[8].

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