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Complete Edge Smooth Finite Interpolation Method for Limit Upper Limit Analysis of Axisymmetric Structures

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Abstract: Axisymmetric structures have applications in various fields such as engineering and architecture. This type of structure exhibits complex stress distribution and failure modes when subjected to ultimate loads, and the importance of the analysis method for its upper limit in the engineering field is self-evident. Its upper limit analysis is mainly used to evaluate the stability and load-bearing capacity of axisymmetric structures under load. The intention of the fully edge smooth finite interpolation method is to improve the accuracy and efficiency of analysis. It improves the interpolation function to maintain smoothness at the boundary, and uses adaptive mesh partitioning and local refinement to make the analysis more accurate and efficient. Therefore, the purpose of this article's in-depth study on the perfect edge smooth finite interpolation method is to introduce smooth functions and finite interpolation techniques, accurately simulate structures, avoid risks, and reduce losses. This article mainly applies numerical simulation and experimental comparison to compare the axisymmetric structures of thick walled cylinders, frustums, and spheres. The ultimate load of each structure is obtained by changing the distortion coefficient and radius ratio. The experimental results show that in cylindrical testing, the performance difference between the two methods is greatest when the radius ratio is 3.5. The error between the perfectly smooth finite interpolation method and the analytical value is smaller, while the direct iteration method has a larger error.

1. Introduction

An object or structure has symmetry in a certain axis, which is called an axisymmetric structure. Pressure vessels, bearings, axles, etc. all belong to this category. Ultimate limit analysis can determine the maximum stress and deformation that a structure can achieve under ultimate loads, involving elasticity, plasticity, and fracture mechanics. It considers the nonlinear behavior of materials, geometric nonlinearity of structures, and boundary conditions to ensure the safety of the structure. With the rapid development of computer technology and numerical analysis methods, various numerical methods have been used for the analysis of such situations. Finite element method, boundary element method, finite difference method, etc. can all calculate the upper limit, and this article needs to study the advantages of fully edge smooth finite interpolation method compared to these three methods. Most limit analysis methods are based on empirical formulas and simplified models, which lead to inaccurate predictions of the ultimate bearing capacity of structures. To address this issue, this paper proposes a novel numerical analysis method - the fully edge smooth finite interpolation method, aimed at improving the accuracy and efficiency of the analysis.

The complete edge smooth finite interpolation method is used to construct an appropriate interpolation model and accurately simulate the mechanical behavior of axisymmetric structures. The smooth function smoothes the stress distribution on the structural boundary, avoiding the phenomenon of stress concentration. Arranging interpolation nodes inside the structure, interpolate the stress on the nodes, and obtain the stress distribution of the entire structure. This method not only ensures the accuracy of the analysis, but also improves the efficiency of the calculation.

This article first introduces the importance and application fields of axisymmetric structures and their limit upper limit analysis in the introduction, analyzes the purpose and advantages of using complete edge smooth finite interpolation method, and then explores the interpolation method and axisymmetric structures studied by scholars. In the upper limit analysis section of axisymmetric structures, this article elaborates on the basic concepts and solution methods of axisymmetric mechanical problems, proposes control equations and boundary conditions, studies the basic principles and advantages of the complete edge smooth finite interpolation method, and compares the direct iteration method and the complete edge smooth finite interpolation method. In the fourth part, this article uses numerical simulation to implement it, compares the numerical and theoretical results of ultimate load under different conditions, and discusses them. Finally, a summary is provided for this article.

2. Related Works

When the stress exceeds the allowable value of the material, the material enters the plastic deformation stage first. This stage is the process of the material transitioning from elastic to plastic, and the plastic region gradually expands. During this process, the internal stress of the structure is redistributed, and the load-bearing part increases, thereby improving the overall load-bearing capacity. Some scholars have conducted in-depth research on interpolation methods. For example, Karthick Sampath used finite difference methods and linear interpolation to solve coupled problems, which was an effective numerical method for dealing with such complex delay differential equations [1]. To explore the spatial distribution of atmospheric temperature, HUANG L analyzed the spatial interpolation of China's atmospheric weighted average temperature grid products and considered the vertical decline rate [2]. Natalia B D applied the Newton interpolation method to the bank interest rate of housing loans in Indonesia, providing a new method for numerical analysis of bank interest rates [3]. The spatial modeling of groundwater level can enable people to grasp the temporal and spatial changes of water level in a timely manner. Arkoc O emphasized the use of GIS and different interpolation methods to study the groundwater level in Ergene Basin, Türkiye [4]. Some scholars have also studied materials related to axisymmetric structures. Huang X simulated the water inflow problem using an axisymmetric smooth particle fluid dynamics model and a VAS scheme [5]. Tang T studied the combustion structure and flame stability in axisymmetric scramjet engines, providing valuable theories [6]. Based on temperature dependent material properties, Wang B performed stress constrained thermoelastic topology optimization on axisymmetric disks [7]. Salenko O used the finite difference method to obtain the damage behavior of multi-layer axisymmetric shells [8]. Li F had made significant contributions to the stability analysis of hypersonic flows, as he experimentally simulated the secondary instability of Görtler vortices in hypersonic boundary layers under axisymmetric configurations [9]. Shahriar A had gained a deep understanding of the vortex dynamics of axisymmetric cones at high angles of attack [10]. Establishing a global two-layer radiative transfer model for an axisymmetric and shadow covered protoplanetary disk, Okuzumi S emphasized the axisymmetric characteristics of the protoplanetary disk [11]. Based on the molecular transport effect, Mitsupoulos E P believed that estimating laminar flame velocity using axisymmetric intrinsic flames was a new computational approach [12]. Vaccaro M S studied the mechanical behavior of two-phase elastic axisymmetric nanoplates, and numerically simulated their stress and strain distributions under different conditions, as well as the mechanical properties and size effects of the material [13]. Kurenov S focused on the axisymmetric stress state of adhesive joints between thin plates with circular notches and circular patches [14]. Eipakchi H described the axisymmetric analysis of negative Poisson's ratio composite cylindrical shells with a honeycomb core layer and variable thickness [15].

Ultimate load is an important criterion for plastic failure in engineering strength design and safety assessment, and is widely used in engineering structural design specifications and regulations. At present, limit analysis mainly adopts mature grid based numerical methods such as finite element method and boundary element method. The field of computational mechanics still requires research on simple, efficient, stable, and applicable numerical algorithms for complex engineering structures. Therefore, this article studies the complete edge smooth finite interpolation method for limit analysis of axisymmetric structures.

3. Upper Limit Analysis of Axisymmetric Structures

3.1 Axisymmetric Mechanical Problems

The parts of the axisymmetric structure on both sides of the straight line are identical in shape, size, and relative position [16]. This symmetry endows objects with a sense of aesthetic balance. Many engineering structures, such as bridges, buildings, and mechanical components, adopt axisymmetric design [17-18]. Axisymmetric structures have strong stability, can withstand huge loads and wind forces, and resist the impact of natural disasters such as earthquakes [19-20]. Axisymmetric mechanics problems belong to a branch of elasticity [21]. The displacement, stress, and strain components are expressed as functions of r and z, independent of angle. Its solution involves the use of cylindrical coordinate systems, calculated using the Lefu displacement function. This article uses the Galerkin method to construct solution formats for linear elastic analysis and elastic-plastic analysis. These two formats are important steps in solving axisymmetric problems. Materials are elastic bodies that follow Hooke's law, which means there is a linear relationship between stress and strain. Axisymmetric problems are usually analyzed using cylindrical coordinates (r, a, z), where the z-axis serves as the axis of symmetry. The displacement is only radial displacement u along the r direction and axial displacement w along the z direction. The corresponding control equations and boundary conditions are:

$$di\partial + y = 0 \tag{1}$$

$$\partial \cdot \mathrm{m} = \bar{s} \tag{2}$$

Among them, $\partial = [\partial_r, \partial_z, \mu_{rz}, \partial_a]^S$, di is the differential operator, and m is the outer normal cosine. Then, the weighted residual method is used to obtain the weak form, and the displacement is obtained through the natural neighbor interpolation method. The discrete equation is expressed as:

$$LV = \mathrm{g} \tag{3}$$

V is the displacement vector of all nodes, L is the overall stiffness matrix, and g is the overall load vector. Elastoplastic analysis needs to consider the nonlinear behavior of materials. This article adopts a gradual increase in load approach to approximate the problem as a series of linear problems. Calculating the stress increment and strain increment at each load increment, and update the stress state of the material. This article uses the modified Newton Raphson method to solve incremental problems, discretizing continuous problems into finite elements or nodes for numerical calculations. This article uses finite element method and finite difference method to solve the discretized equation, and post-processes the results obtained from the solution.

3.2 Ultimate Limit Analysis

Ultimate limit analysis is a widely used mechanical analysis method in structural engineering, whose basic principle is based on the limit state of the structure. The state in which the structure is subjected to the maximum allowable load is the limit state. Applying limit limit analysis to evaluate safety and stability under maximum allowable load, as well as the lifespan and durability of the structure. Calculating the ultimate load multiplier ς as follows:

$$\varsigma = \min : \sqrt{\frac{2}{3}} \partial_t \int_W \sqrt{\nu_{ik}\nu_{ik}} cW \tag{4}$$

∂_t is the yield stress. The discrete axisymmetric plane consists of multiple smooth domains. The upper limit step based on complete edge smoothness is to solve the minimization problem of axisymmetric completely plastic states, obtain the limit load multiplier, divide the rigid and plastic zone subsets, and then solve the minimization problem. When the following equation is satisfied:

$$\left|\gamma_i - \gamma_{i-1}\right| / \gamma_i \le \nu \tag{5}$$

Introducing Lagrange multipliers into the objective function. The solution algorithm for limit analysis is based on the finite element method and boundary element method. This article believes that it is necessary to correct the defects that depend on the grid. Therefore, this article needs to use the meshless method for limit upper limit analysis.

3.3 Complete Edge Smooth Finite Interpolation Method

The fully edge smooth finite element method transforms the process of solving non smooth problems into solving problems in a smooth function space, improving the computational accuracy, convergence, and efficiency of linear elements, and reducing sensitivity to distorted meshes. This method smoothes the corresponding variable matrix and combines the signed indefinite integral and Gaussian divergence theorem to calculate the consistent mass matrix using coupled integral. It unifies the integration format of stiffness matrix and consistent mass matrix, converting domain integration into boundary integration along smooth domain boundaries, avoiding coordinate mapping and Jacobian matrix calculation.

The interpolation method accurately captures the subtle changes near stress concentration and singular points, and can also maintain the accuracy of the solution without increasing the grid density. It reduces dependence on the grid and does not require excessive refinement of the grid. This article accurately establishes the cylindrical shape of an axisymmetric cylinder in two-dimensional space, and then uses a relatively sparse but optimized distribution grid to reduce computational complexity. Introducing an additional degree of freedom on each edge allows for a better capture of local behavior. Its fixed boundary, symmetric boundary, free boundary, etc. are determined.

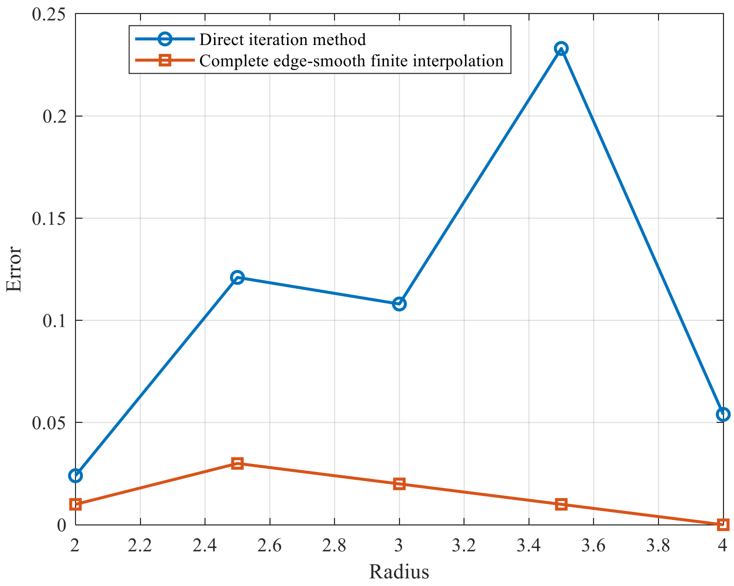

Figure 1: Comparison of numerical and theoretical solutions for ultimate load under different radii

As shown in Figure 1, this article compares the errors at radii of 2, 2.5, 3, 3.5, and 4, respectively. The comparison methods are direct iteration method and fully edge smooth finite interpolation method. The errors between the direct iteration method and the analytical values are 0.024, 0.121, 0.108, 0.233, and 0.054, respectively. The errors between the completely smooth finite interpolation method and the analytical values are 0.01, 0.03, 0.02, 0.01, and 0, respectively.

The specific situation of its ultimate load when the outer radius/inner radius=3 is shown in Table 1:

Table 1: Ultimate load at outer radius/inner radius=3

Deformation coefficient	Analytic solutions	Direct iteration method	Complete edge-smooth finite interpolation method
0.1	1.2544	1.2541	1.2543
0.2	1.2537	1.2532	1.2535
0.3	1.2532	1.2528	1.2533
0.4	1.2528	1.2523	1.2529
0.5	1.2525	1.2527	1.2525

The value of the analytical solution differs to some extent from the direct iteration method and the method proposed in this paper. We can see that when the outer radius/inner radius is 3, the ultimate load with a distortion coefficient of 0.2 has the maximum error between the direct iteration method and the fully edge smooth finite interpolation method and the analytical value. The error of the fully edge smooth finite interpolation method is smaller than that of the direct iteration method. Therefore, the fully edge smooth finite interpolation method has higher accuracy in the limit analysis of axisymmetric structures.

4. Results and Discussion

4.1 Numerical Simulation Experiments

In order to achieve effective calculation verification of correctness, this article requires a comparison of three thick walled axisymmetric structures, namely cylindrical, spherical, and frustum, to test their ultimate pressure and compare them with the analytical solution. This article uses cylinders of different materials and sizes, gradually increasing pressure, and recording their performance when reaching the ultimate pressure. A spherical object has a uniform force characteristic and undergoes a positive change when subjected to pressure. This article summarizes the deformation and failure of a sphere under different pressures, and analyzes the influencing factors of its ultimate pressure. The ultimate load of the sphere is analyzed by adjusting the distortion coefficient. A frustum is an axisymmetric structure with variable cross-section, and its ultimate pressure is closely related to the diameter, height, and wall thickness of the upper and lower bottom surfaces. This article compares the ultimate load results with distortion coefficients of 0, 0.3, and 0.5 for circular frustum bottom angles of 30, 45, 60, and 75, respectively. These three structures exhibit different behaviors when subjected to pressure.

4.2 Analysis of Ultimate Load

The fully edge smooth finite interpolation method, as a new numerical analysis method, has advantages such as high accuracy and efficiency, and can accurately predict the ultimate bearing capacity of structures. This article uses the direct iteration method and the fully edge smooth finite interpolation method to analyze axisymmetric mechanical problems. The factors that need to be considered in axisymmetric mechanical problems include displacement, stress, and strain components. The purpose of numerical simulation experiments is to verify the effectiveness and accuracy of the fully edge smooth finite interpolation method and compare it with analytical solutions. The smooth function plays a role in smoothing the stress distribution on the boundary of the structure.

The stress function method is easy to satisfy equilibrium conditions, but the boundary conditions are not easy to satisfy, and the solution process is complex. Analytical analysis method is used to determine the plastic damage limit of the structure and solve the ultimate load. The numerical analysis method can obtain more accurate ultimate load results. This article uses the complete edge smooth finite interpolation method to analyze the loads on spherical shells and frustums.

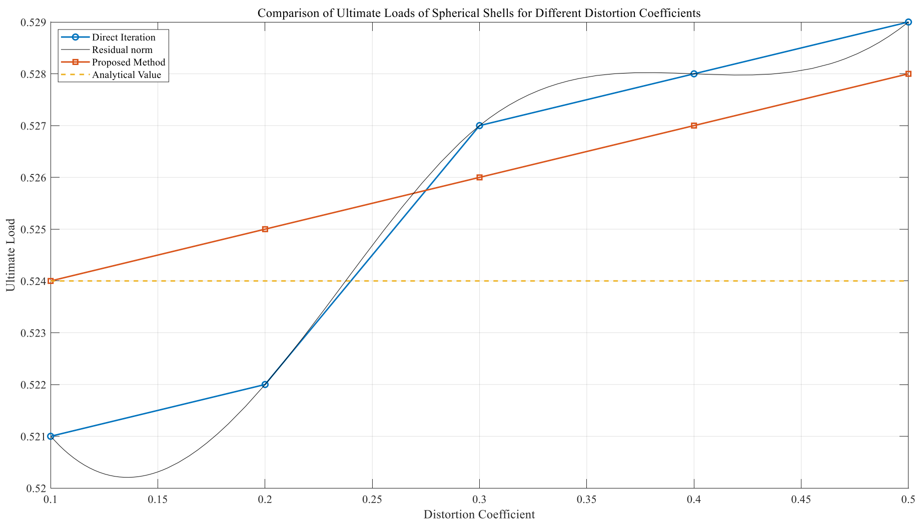

Figure 2: Ultimate load of spherical shells with different distortion coefficients

As shown in Figure 2, the distortion coefficients are 0.1, 0.2, 0.3, 0.4, 0.5, and the analytical values are 0.524. The results of the direct iteration method are 0.521, 0.522, 0.527, 0.528, and 0.529, respectively. The results of the method proposed in this paper are 0.524, 0.525, 0.526, 0.527, and 0.528, respectively. The method used in this article is significantly higher than the direct iteration method. By using Matlab to calculate the residual norm, it is found that the change in the result is tortuous, with an overall trend of first decreasing and then increasing.

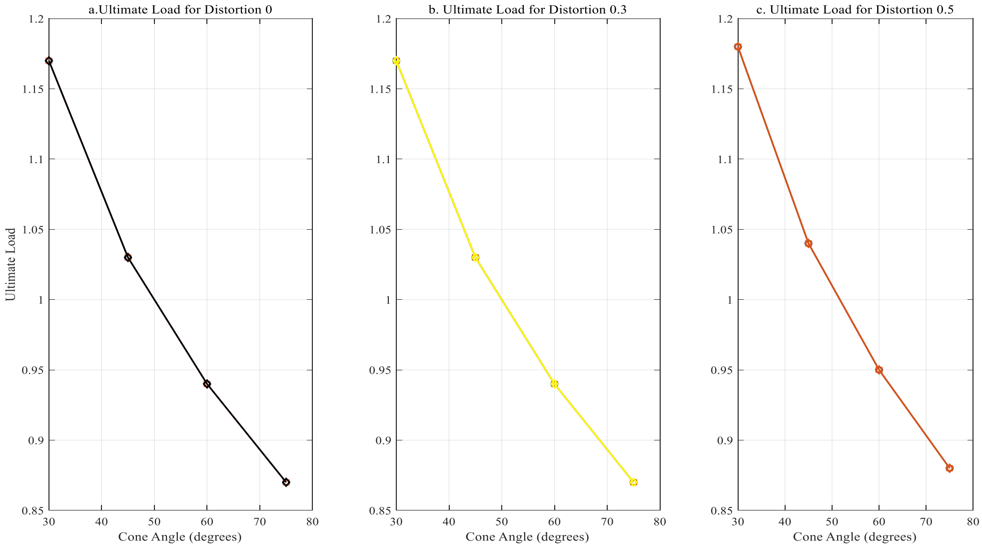

Figure 3: Ultimate load of circular frustum at different base angles

As shown in Figure 3, when the base angle is 30 °and the distortion coefficient is 0 (as shown in Figure 3a), the ultimate load shows a higher bearing capacity (1.17). When the bottom angle becomes 45 °, the ultimate load slightly decreases (1.03). When the bottom angle is 60 °, the load further decreases (0.94). When the bottom angle is 75 °, the load is 0.87. When the distortion coefficient increases to 0.3 (as shown in Figure 3b), the ultimate load remains unchanged. The ultimate load at a base angle of 30 °is 1.17, the result at 45 °is 1.03, the result at 60 °is 0.94, and the result at 75 °is 0.87. When the distortion coefficient increases to 0.5 (as shown in Figure 3c), the ultimate load at a base angle of 30 °slightly increases (1.18), the result at 45 °is 1.04, the result at 60 °is 0.95, and the result at 75 °is 0.88. This indicates that the increase in distortion coefficient has an enhancing effect on the ultimate load.

5. Conclusion

The rapid development of technology and the soaring productivity have elevated the pursuit of safety and stability in engineering structures in industries such as chemical, mechanical, and civil engineering. The call for environmental protection, energy conservation, and sustainable development is urging engineering structures to demonstrate their maximum load-bearing capacity, achieving a leap in economic benefits while also achieving careful resource management. The fully edge smooth finite interpolation method performs well in the upper limit analysis of axisymmetric structures. This method adopts smoothing technique, which is more accurate in describing stress distribution and strain field, has wide applicability, and higher computational efficiency. But its computational complexity increases, consumes more computing resources, is easily affected by the model, and has certain errors. The parameters need to be adjusted at any time. This article believes that improving algorithms can reduce computational complexity, further improve the model, and study parameter adaptive techniques, so that this method can be better applied to the analysis of large axisymmetric structures.

References

[1] Karthick Sampath, V. Subburayan: Finite difference methods with linear interpolation for solving a coupled system of hyperbolic delay differential equations. Int. J. Math. Model, 2022, 12(4): 370-389.

[2] Huang L, Li C, Xie S, et al. Spatial Interpolation of Atmospheric Weighted Mean Temperature Grid Products in China with Consideration of Vertical Lapse Rate. Geomatics and Information Science of Wuhan University, 2023, 48(2): 295-300.

[3] Natalia B D, Ditasona C, Vania R R, et al. Numerical Analysis of Home Ownership Loans Bank Interest Rates in Indonesia with Newton's Interpolation Method. EduMatSains: Jurnal Pendidikan, Matematika dan Sains, 2024, 8(2): 250-260.

[4] Arkoc O. Modeling of spatiotemporal variations of groundwater levels using different interpolation methods with the aid of GIS, case study from Ergene Basin, Turkey. Modeling Earth Systems and Environment, 2022, 8(1): 967-976.

[5] Huang X, Sun P, Lyu H, et al. Water entry problems simulated by an axisymmetric SPH model with vas scheme. Journal of Marine Science and Application, 2022, 21(2): 1-15.

[6] Tang T, Wang Z, Huang Y, et al. Investigation of combustion structure and flame stabilization in an axisymmetric scramjet. AIAA Journal, 2023, 61(2): 585-601.

[7] Wang B, Wang G, Shi Y, et al. Stress-constrained thermo-elastic topology optimization of axisymmetric disks considering temperature-dependent material properties. Mechanics of Advanced Materials and Structures, 2022, 29(28): 7459-7475.

[8] Salenko O, Drahobetskyi V, Symonova A, et al. Damage Behavior of Multilayer Axisymmetric Shells Obtained by the FDM Method. Journal of Engineering Sciences (Ukraine), 2024, 11(1): 27-35.

[9] Li F, Choudhari M, Paredes P. Secondary instability of Görtler vortices in hypersonic boundary layer over an axisymmetric configuration. Theoretical and Computational Fluid Dynamics, 2022, 36(2): 205-235.

[10] Shahriar A, Kumar R, Shoele K. Vortex dynamics of axisymmetric cones at high angles of attack. Theoretical and Computational Fluid Dynamics, 2023, 37(3): 337-356.

[11] Okuzumi S, Ueda T, Turner N J. A global two-layer radiative transfer model for axisymmetric, shadowed protoplanetary disks. Publications of the Astronomical Society of Japan, 2022, 74(4): 828-850.

[12] Mitsopoulos E P, Souflas K, Paterakis G, et al. Estimation of laminar flame speeds using axisymmetric bunsen flames: Molecular transport effects. Proceedings of the Combustion Institute, 2023, 39(2): 1861-1869.

[13] Vaccaro M S, Sedighi H M. Two-phase elastic axisymmetric nanoplates. Engineering with Computers, 2023, 39(1): 827-834.

[14] Kurennov S, Smetankina N, Barakhov K. Axisymmetric Stress State of Adhesive Joint of a Circular Patch with a Plate Weakened by a Circular Cut-out. Periodica Polytechnica Mechanical Engineering, 2023, 67(1): 12-18.

[15] Eipakchi H, Mahboubi Nasrekani F. Axisymmetric analysis of auxetic composite cylindrical shells with honeycomb core layer and variable thickness subjected to combined axial and non-uniform radial pressures. Mechanics of Advanced Materials and Structures, 2022, 29(12): 1798-1812.

[16] Makhinko A, Makhinko N, Semko V. Creation of design models of silos for grain storage under conditions of non-axisymmetric actions. Ce/papers, 2023, 6(3-4): 1935-1940.

[17] Shlyakhin D A, Savinova E V. Coupled Axisymmetric Thermoelectroelasticity Problem for a Round Rigidly Fixed Plate. Advanced Engineering Research (Rostov-on-Don), 2024, 24(1): 23-35.

[18] Qing H. Well-posedness of two-phase local/nonlocal integral polar models for consistent axisymmetric bending of circular microplates. Applied Mathematics and Mechanics, 2022, 43(5): 637-652.

[19] Panda J P, Handique J, Warrior H V. Mechanics of drag reduction of an axisymmetric body of revolution with shallow dimples. Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment, 2023, 237(1): 227-237.

[20] Farrokh M, Mousavi Fard H S. An extension of Carrera unified formulation in polar coordinates for mechanical and thermal buckling analysis of axisymmetric FG circular plate using FEM. Mechanics of Advanced Materials and Structures, 2024, 31(8): 1803-1811.

[21] Harris B L, Tailleux R, Holloway C E, et al. A moist available potential energy budget for an axisymmetric tropical cyclone. Journal of the Atmospheric Sciences, 2022, 79(10): 2493-2513.