Research on an Even Number Which Plus 2 Can be Exactly Divided by 6 Can be Expressed as the Sum of Several Sets of Two Prime Numbers

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Abstract: The X_B number axis of a 0 to N_1 segment is shifted to the right by two number axis units and then overlapped in the same direction with the X_A number axis of a 0 to N_1 segment. The overlapped number axis points of the two primes number are a set of twin primes and the law of twin primes smaller than N_1 is found. Then, the two X_B number axes with the same length as them overlap in the opposite direction, and it is found that the overlapping number axis points of the two primes number are the P (1,1) prime pairs of N_1, which proves that N_1 can also be expressed as the sum of several groups of two primes. Comparing the two overlapping number axes, it is found that the X_B\X_B’ overlapping number axes are symmetrical, and it is found that prime numbers that make 6P_a exactly divide (N_1+2P_a) or 6P_b exactly divide (N_1-2P_b) can also increase the table method number of P(1,1) prime pairs of N_1, which proves that the number of P (1,1) primes of N_1 to the table group is about equal to or more than half of the number of twin primes smaller than N_1.

1. Introduction

By analyzing the smaller even numbers, it is found that except for 4, the other even number which plus 2 can be exactly divided by 6 can be expressed as the sum of two P_B=(6m-1) primes, and the table method group count increases with the increase of even numbers, such as 10=(5+5), 22=(5+17)=(11+11), 46=(5+41)=(17+29)=(23+23). Using the number axis overlap method, it is found that the even number which plus 2 can be exactly divided by 6 can be expressed as the sum of several groups of twin primes smaller than the even number.

2. Characteristics of Twin Primes and Their Distribution on the Number Axis [1]

Except for 2 and 3, the composite numbers that cannot be exactly divided by 2 or 3 are all in the odd numbers of A=6m+1 and B=6m-1 [2]. The prime numbers are represented by P, and the composite numbers are represented by H. If removes H_A from A, the remaining is P_A; if removes H_B from B, the remaining is P_B (the prime number in this paper does not include 2 and 3, and the composite number does not include the composite number that can be exactly divided by 2 or 3).
The even number is divided into three types according to the remainder divided by 6: \( N_1 = 6m-2, N_2 = 6m, N_3 = 6m+2 \).

The even number \( N_1 \) \((N_1 = 6m-2, m \geq 2)\) is given. Except for 4, the other even numbers which Plus 2 can be exactly divided by 6 are in \( N_1 \).

Make \( X \) number axes that show only odd numbers \( A \) and \( X \) number axes that show only odd numbers \( B \), with the origin on the Y axis. The composite number axis points with the same prime factor are connected by arcs in the form of \( y = |\sin x| \). The prime factor is the wavelength root of the frequency curve. The frequency curve \( E_p \) is above \( X_A \), \( F_p \) is below, \( G_p \) is above \( X_B \), \( S_p \) is below, the wavelength roots of \( E_p \) and \( G_p \) are \( P_A \), and the wavelength roots of \( F_p \) and \( S_p \) are \( P_B \). The axis points of the frequency-free curve are primes. The \( X_A \) is placed in the first quadrant of the planar rectangular coordinate system, and the \( X_B \) is placed in the fourth quadrants. The parallel line of the vertical axis is made through the \( N_1 \) point, and the 0 to \( N_1 \) segments are intercepted respectively. Move the \( X_B \) segment to the right by two units vertically up to overlap \( X_A \), forming a twin prime overlapping number axes from 0 to \( N_1 \) \((LXAXB)\), where the B odd numbers all overlap with the A odd number, and the two primes form a twin prime overlapping number axes point.

Features of \( LXAXB \):
1) The \( X_A \) and \( X_B \) axes overlap in the same direction.
2) The odd number A and B overlap to form \((A, B)\) overlapping axis points, and the difference between the two numbers is 2.
3) There are \( N_1/6 \) sets of \((A, B)\) overlapping number axis points (rounding, the same below).
4) Frequency curves of the same wavelength neither intersect nor overlap.
5) The frequency curves of different wavelengths of the two number axes intersect at the overlapping number axis points, forming the \( LHH \) group with overlapping composite numbers \((H_A, H_B)\) of both \( E_p \) or \((and)\) \( F_p \) and \( G_p \) or \((and)\) \( S_p \). The remaining \((#H_A-LHH)\) \( H_A \) overlaps with the \( P_B \) on \( X_B \) to form \((H_A, P_B)\) overlapping number axis points with only \( E_p \) or \((and)\) \( F_p \). The remaining \((#H_B-LHH)\) \( H_B \) overlap with the \( P_A \) on the \( X_A \) to form \((H_B, P_A)\) overlap number axis points with only \( G_p \) or \((and)\) \( S_p \). The remaining prime axis points of the two axes overlap, forming \( L_{N1} \) group \((P_A, P_B)\) with no frequency curve. The number of twin prime axis points (excluding twin primes \((3, 5)\)), and the sum of the four overlapping axis points is equal to the total number of groups \((A, B)\).

The number of groups of twin primes is obtained by using the residual relation:

\[
L_{N1} = \frac{N_1}{6} - LHH - (#H_A-LHH) - (#H_B-LHH) = \frac{N_1}{6} + LHH - (#H_A+H_B)
\]  

6) The overlapping number axis points of twin primes are gradually sparse with the extension of the number axis.

3. Characteristics of \( N_1 \) Overlapping Number Axis are Analyzed by Overlapping Two \( X_B \) Number Axes in the Opposite Direction.

3.1. Make \( X_B' \) Overlapping Number Axes

Take two \( X_B \) from 0 to \( N_1 \) and place them in the first and fourth quadrants of the planar rectangular coordinate system respectively.

The \( X_B \) of the fourth quadrant is flipped \( 180^\circ \), the \( N_1 \) point is on the Y axis, the direction is left, the \( G_p' \) is still above the number axis, and the \( S_p' \) is still below the number axis, which is called \( X_B' \) number axis. The \( X_B' \) after flipping is only opposite to the \( X_B \), and other properties remain unchanged.

\( X_B' \) vertically moves up and overlaps with \( X_B \) to form the \( X_B\backslash X_B' \) overlapping axis \((XBXB)\) of \( N_1 \).

(Fig.1)

On XBXB, the \( B' \) odd number of \( X_B' \) all overlaps with the \( B \) odd number of \( X_B \), and the two
composite numbers overlap as (H_B, H_B') overlapping axis points. The two primes overlap as (P_B, P_B') overlapping number axis points, which are a set of P (1, 1) prime pairs of N_1. Although the X_B' is in the opposite direction of X_B, the frequency curve is not directional, and the intersection rule of the frequency curve is not directional. Through the intersection rule of the frequency curve, the rule of (H_B, H_B') overlapping axis points is found, and the rule of P (1, 1) prime pairs of N_1 is found.

![Figure 1: X_B\X_B' overlapping number axis (N_1=292): The two X_B number axes are oppositely overlapped. The solid pentagram is (H_B, H_B'), the hollow pentagram is (P_B, P_B'), the rhombus with horizontal line is (H_B, P_B'), and the rhombus with vertical line is (H_B', P_B).](image)

There are two cases of XBXB:
1) If 6P_a can not exactly divide (N_1 + 2P_a) nor can 6P_b exactly divide (N_1 - 2P_b), there will be no frequency curve overlap, which is called XBXB_1.
2) If (N_1 + 2P_a) is divisible by 6P_a or (N_1 - 2P_b) is divisible by 6P_b, then there is a corresponding frequency curve overlap, called XBXB_2.

3.2. Characteristics of the Overlapping Number Axis Points and the Group Count of XBXB_1

3.2.1. Characteristics of XBXB_1
1) Two identical X_B overlap in the opposite directions.
2) Two B odd numbers overlap to form (B, B') overlapping number axis points, and the two numbers are complementary, and their sum is equal to N_1.
3) There are N_1/6 sets of (B, B') overlapping number axis points.
4) Frequency curves with the same wavelength neither intersect nor overlap.
5) The (P_B, P_B') overlapping axis points are evenly distributed on XBXB_1.

3.2.2. The number of (H_B, H_B') overlapping composite group count of frequency curve intersection

He frequency curves of different wavelengths of the two axes will intersect on XBXB_1 to form (H_B, H_B') overlapping number axis points with both Gp or Sp and Gp' or Sp'. The group count of (H_B, H_B') is equal to the quotient obtained by dividing the length of the overlapping number axis by 6 times the product of the wavelength roots of the relevant frequency curve.

The Ep\Gp intersection ^1 of LXAXB is the same as the Gp\Gp' intersection of XBXB_1. Gp and
Gp' with the same wavelength neither intersect nor overlap, but Gp and other Gp' do intersect. One or more Gp will also intersect with one or more Gp' at the same overlapping number axes point, with $\sum_{gg'}$ comprehensively representing the Gp\Gp' intersection (H, H') group count.

The Fp\Sp intersection $[1]$ of LXAXB is the same as the Gp\Sp' intersection of XBXB1. Both Gp and Sp' intersect, and one or more Gp will also intersect with one or more Sp at the same overlapping number axes point, and the group count of (H, H') where Gp\Sp' intersects is represented by $\sum_{gs'}$.

The Fp\Gp intersection $[1]$ of LXAXB is the same as the Sp\Gp' intersection of XBXB1. Sp and Gp' intersect with each other, and one or more Sp will also intersect with one or more Gp' at the same overlapping number axis point. The group count of (H, H') where Sp\Gp' intersects is represented by $\sum_{gs'}$.

The intersection of multiple frequency curves on XBXB1 and LXAXB is the same. The two frequency curves on XBXB1 can also intersect at the same point with the third frequency curve, and the three frequency curves can also intersect at the same point with the fourth frequency curve. The intersection of multiple frequency curves takes up more frequency curve axis points, which reduces the group count of (H, H') counted by the intersection of the two frequency curves, and such reduced group count is represented by $\sum_{gs'\times'}$.

$N_{1HH}$ is used to represent the group count of all (H, H') overlapping number axis points:

$$N_{1HH} = \sum_{gg} + \sum_{gs} + \sum_{ss} + \sum_{gs'\times'}$$

It is the same as the LHH calculation formula $[1]$ of the group count of overlapping composite of LXAXB, and the number is basically equal.

### 3.2.3. (H, P) group count of overlapping number Axis Points

X_B has #H_B H_B. Except for the N_{1HH} groups of (H_B, H_B') composed of H_B and H_B', the remaining H_B overlaps with P_B'. There are only (H_B, P_B') overlapping number axis points of Gp or Sp in the (#H_B-N_{1HH}) group, which are basically equal to the number of (H_B, P_A) overlapping number axis points of H_B and P_A on LXAXB.

X_B' has #H_B H_B'. Except for the N_{1HH} groups of (H_B, H_B') composed of H_B' and H_B, the remaining H_B overlaps with P_A. There are only (H_B', P_A) overlapping number axis points of Gp' or Sp' in the (#H_B'-N_{1HH}) group, which is basically the same as the number of (H_B, P_A) overlapping number axis points of H_B and P_A on LXAXB.

### 3.2.4. The group count of (P_B, P_B') overlapping axis points without frequency curve

The remaining prime axis points overlap to form (P_B, P_B') overlapping axis points without frequency curve. The sum of the four overlapping axis points is equal to the total group count of (B, B') overlapping axis points.

The group count of (P_B,P_B') overlapping number axes points is obtained by using the residual relation:

$$d = N_i/6 - N_iHH - (#H_B-N_iHH) - (#H_B'-N_iHH) = N_i/6 + N_iHH - 2#H_B$$  \hspace{1cm} (2)

It is known that the number of composite axis points of X_A and X_B with equal length is basically equal, and the number of 2#H_B and (#H_A+#H_B) is basically equal, so the number of d and L_{N1} groups is basically equal.
3.2.5. Symmetry of XBXB

If the axis points B₁ of Xᵦ and (N₁-B₁)' of Xᵦ' overlap, then (N₁-B₁) of Xᵦ must overlap with the complementary B₁' of Xᵦ'. Although there are two different sets of overlapping axis points on XBXB₁, their odd pairs are the same. The distance between one set of overlapping axis points and the origin of Xᵦ is equal to the distance between the other set of overlapping axis points and the origin of Xᵦ', that is, they are symmetrical for the midpoint of XBXB₁.

If N₁/6 is an even number, the midpoint of XBXB₁ is not on the (B, B') overlapping axis points, and the axis points on both sides of the midpoint are symmetrical. There are N₁/12 sets of (B, B') odd pairs. If N₁/6 is an odd number, the midpoint of XBXB₁ is a set of the same B odd numbers, then there are (N₁/6+1)/2 groups of (B, B') odd pairs, and this difference is ignored when N₁ is larger.

Therefore, the group count of the composite pairs corresponding to (Hᵦᵦ, Hᵦᵦ') on XBXB₁ is equal to half of the group count of (Hᵦᵦ, Hᵦᵦ') overlapping axis points N₁HH, and the group count of (P₁₁, P₁₁) prime pairs corresponding to (P₁₁, P₁₁) overlapping axis points D_N₁ is equal to half of the group count of (P₁₁, P₁₁') overlapping axis points d.

3.2.6. The similarities and differences between XBXB₁ and LXAXB

3.2.6.1. The similarities

1) There are N₁/6 groups of odd pairs of overlapping number axis points.
2) The rule of frequency curve intersection is consistent.
3) Frequency curves with the same wavelength neither intersect nor overlap.
4) The group count of N₁HH and LHH is basically equal.
5) The group count of (P₁₁, P₁₁') overlapping axis points d is basically equal to the group count of (P₁₁, P₁₁) overlapping axis points Lₜ₁.

3.2.6.2. The differences

1) XBXB₁ is composed of two Xᵦ with opposite directions. LXAXB is composed of one Xₐ and one Xᵦ, and the directions of the two axes are the same.
2) XBXB₁ overlapping axis points are two B odd numbers, two numbers are complementary, and their sum is equal to N₁; the overlapping axis points of LXAXB are one A and one B, and the difference between the two numbers is 2.
3) XBXB₁ is symmetrical for the midpoint; LXAXB has no symmetry.
4) The (P₁₁, P₁₁') overlapping axis points are evenly distributed on XBXB₁. The (P₁₁, P₁₁) overlapping axis points are dense first and then sparse on LXAXB.
5) The P (1, 1) prime pair group count D_N₁ of N₁ is equal to half of the group count d of the (P₁₁, P₁₁') overlapping axis points of XBXB₁. The group count of twin primes Lₜ₁ which is smaller than N₁ is equal to the group count of (P₁₁, P₁₁) overlapping axis points of LXAXB. Therefore, D_N₁ is approximately equal to half of Lₜ₁.

3.2.7. Variance analysis

According to the comparison between randomly selected XBXB₁ and LXAXB of equal length, the group count of P (1, 1) prime pairs of XBXB₁ is slightly less than half of the group count of twin prime pairs of LXAXB of equal length.

From the analysis of the residual relationship, #Hᵦ is only related to the size of N₁. Only the group count of (Hᵦᵦ, Hᵦᵦ') decreases, then the group count of (P₁₁, P₁₁') will decrease.
3.2.7.1. The intersection of non-composite number dash lines of a frequency curve

Sp extends to the left side of the Y axis with a dash line, and has a dash line intersection with Xb, which is the non-composite number dash line intersection of the frequency curve. It is the prime number of the wavelength root itself, and the non-composite dash line intersection of Sp is at the left end. Gp has no dash line intersection.

Xb flips 180° to become Xb', and the non-composite dash line intersection of Sp' moves to the right.

3.2.7.2. The influence of the intersection point of non-composite dash line on (HB, HB')

The number axis segment from 0 to the intersection of non-composite dash lines is the non-composite segment (NOH) of the frequency curve. Remove NOH, the remaining number axis segment is the segment where the frequency curve intersects with other frequency curves. The exact group count of (HB, HB') is obtained by dividing the length of the intersection area by 6 times the product of the wavelength roots of the correlated frequency curve.

Both Ep and Sp have NOH, and the NOH of Ep and Sp on LXAXB is at the left end. The intersection of Ep\Sp: The two NOH overlap and the intersection area only removes the larger wavelength NOH.

The NOH of Sp and Sp' on XBXB are separated at both ends. The intersection of Sp\Sp': remove the NOH of the left end Sp, and then remove the NOH of the right end Sp'. Compared with the Ep\Sp intersection area with both NOH of LXAXB, the smaller wavelength NOH is removed, which shortens the intersection area and may reduce one group (HB, HB'). The larger the even number, the more the frequency curve; the more the intersection of Sp\Sp', and the more the chance of reducing one group (HB, HB'), but the proportion of reduced groups in the whole NiHH is very small. According to the group count of all Sp\Sp' intersection, the area of the curved trapezoid is counted. The curved trapezoid is composed of (#Sp-1)*#Sp vertical lines representing Sp\Sp' intersection. Each vertical line is composed of N1/(6Pb*Pb') points representing the group count of (HB, HB'). The denominator of the reduced group count in the proportion of N1HH is the area of the curved trapezoid, and the numerator is half the length of the curved edge (calculated as equal or less than half of the group count of each vertical line (HB, HB')) , so the proportion of (HB, HB') reduction is very small.

3.2.7.3. Influence of intersection starting point on (HB, HB')

Gp\Gp' has no NOH, Sp\Gp', Gp\Sp' has only one NOH, and these three kinds of intersections have not shortened the intersection area because of Xb flipping 180°, but the overlapping composite number with LXAXB intersection also has group count differences due to different starting points, resulting in differences between the group count of (HB, HB') and (HA, HB).

The intersection starting point of two frequency curves on LXAXB is fixed, such as 301, the intersection starting point of 7-Ep\13-Gp (abscissa); 247, the intersection starting point of 13-Ep\7-Gp. The intersection starting point of XBXB is related to N1. When N1 is equal to 1000, 10000 and 100000, the intersection starting point of 7-Gp\13-Gp' is 77, 497 and 329, and the intersection starting point of 13-Gp\7-Gp' is 377, 767 and 299. The intersection area is slightly longer when the starting point of the intersection is small, and the intersection area is slightly shorter when the starting point of the intersection is large, which may make (HB, HB') less than 1 group or more than 1 group of (HA, HB) of LXAXB. The smaller even number will affect the ratio of the table method group count to the group count of twin prime, but the differences caused by different starting points are partially offset by the intersection of many frequency curves and the intersection of multiple frequency curves. Although the number of differences is not small, it has an impact on the group
count of overlapping composite number, but it has little effect on the ratio of the group count of twin prime, especially the large even number.

Due to the combined effect of these factors, the overlapping composite number of XBXB1 is slightly less than that of LXAXB, so (Pｂ, Pｂ′) is reduced, but the proportion of reduction is very small. According to the comparison between randomly selected XBXB1 and isometric LXAXB, the average group count of P (1, 1) prime pairs of XBXB1 is about 1.2% less than half of the group count of twin prime pairs of LXAXB, but not up to 5%. According to statistical regulations, there is no significant difference.

Therefore, the group count of P (1, 1) prime pairs corresponding to (Pｂ, Pｂ′) of XBXB1 is about half of the group count of twin prime pairs smaller than N1: D[N1≥0.5LN1]

3.3. Characteristics of the Overlapping Number Axis Points and the Group Count of XBXB2

On XBXB1, the frequency curves with the same wavelength do not intersect or overlap, but intersect with other frequency curves. The unintersected number axis points overlap with the prime number axis points of another number axis, forming (Hｂ′, Pｂ) and (Hｂ, Pｂ′) overlapping number axis points.

On XBXB2, if (N1+2Pｂ) can be exactly divided by 6Pｂ, then the Gp with Pｂ as the wavelength root completely overlaps with the Gp' with the same wavelength, and the total number of composite number axis points(Hｂ, Hｂ′), then (Hｂ, Hｂ′) increases. At the same time, the overlap with the other number axis primes is reduced, so that (Pｂ, Pｂ′) increases. The increased group count is approximately equal to half of the group count of twin prime, and every (Pｂ-1) N1, there is a N1 that can be exactly divided by the 6Pｂ, so that (Pｂ, Pｂ′) increases.

If (N1-2Pｂ) can be exactly divided by 6Pｂ, then the Sp of the Pｂ for the wavelength root completely overlaps with the Sp' with the same wavelength root, which reduces the overlap with the prime number of the other number axis, and increases the (Pｂ, Pｂ′). The increased group count is approximately equal to half of the group count of twin prime, and every (Pｂ-1) N1, there is a N1 that can be exactly divided by the 6Pｂ, so that (Pｂ, Pｂ′) increases. These primes that can have (N1+2Pｂ) exactly divided by 6Pｂ or (N1-2Pｂ) exactly divided by 6Pｂ and that increase (Pｂ, Pｂ′) are called prime factors of the even number.

If multiple Pｂ can make 6Pｂ exactly divide (N1+2Pｂ) or multiple Pｂ can make 6P𝑏 exactly divide (N1-2Pｂ), or both Pｂ and Pｂ can make 6Pｂ exactly divide (N1+2Pｂ), and 6Pｂ exactly divide (N1-2Pｂ), then (Pｂ, Pｂ′) is affected by these prime factors. (Fig. 2)

The increase of (Pｂ, Pｂ′) is inversely proportional to the size of the prime factor. The smaller the prime factor, the more (Pｂ, Pｂ′) increases. It is also related to the number of prime factors. The more prime factors, the more (Pｂ, Pｂ′) increase. However, due to symmetry, the increased P (1, 1) prime pairs are only half of the increased (Pｂ, Pｂ′) overlapping axis points.

Therefore, the group count of (Pｂ, Pｂ′) overlapping axis points of XBXB is approximately equal to or more than the group count of (Pｂ, Pｂ) overlapping axis points of LXAXB, and the group count of P (1, 1) prime pairs corresponding to (Pｂ, Pｂ′) is approximately equal to or more than half of the group count of twin prime pairs: D[N1≥0.5LN1].

3.4. The Group Count of P (1, 1) Prime Pair of Infinity N1

Pｂ is infinite [2]. When N1 increases to (2N1+2), there will be no Pｂ from 0 to N1 number axis and no P_b from N1 to (2N1+2) number axis, and the intersection rule of frequency curve remains unchanged. Therefore, N1 doubling still follows the rule of D[N1≥0.5LN1]. N1 infinitely doubles, and these properties will not change. At present, although it is not certain that the group count of twin
prime is infinite, the known number of twin primes will not disappear. The group count of P (1, 1) prime pairs of infinite N₁ must be more than half of the known group count of twin prime.

3.5. Group Count of P (1, 1) Prime Pairs Adjacent to N₁

In the planar rectangular coordinate system, a parabola (L_{N₁}/2 curve) is formed by connecting the points with N₁ as the abscissa and 0.5L_{N₁} as the vertical coordinate, which is half of the group count of twin prime smaller than N₁. Then connect the points with N₁ as the abscissa and the P (1, 1) prime pair of N₁ as the vertical coordinate to the group count D_{N₁} (D_{N₁} connection). The influence of the prime factor makes the D_{N₁} of the adjacent N₁ vary greatly, and the standard curve cannot be formed. The above of the connection line is uneven and irregular, like a crocodile tail. With the increase of the even number, the shape is like a hand-painted porcupine back, and the many lowest points below the connection line (the (P₁, P₁′) group count of XBXB₁ is the point of the vertical coordinate) form a smooth curve, which is basically consistent with the L_{N₁}/2 curve. The connection line does not break through the L_{N₁}/2 curve downward (basically) no matter how it fluctuates. The overall trend of the connection increases with the increase of N₁. (Fig. 2)

If (N₁-3) is a prime, the N₁ can also be expressed as (Pₐ+3) prime pairs (w=1 in Figure 2 indicates that the even number has the N₁=(Pₐ+3) table method, but the group count is negligible).

![Figure 2: The influence of prime factors on D_{N₁}](image)

4. Conclusion

The even number 4 can be expressed as the sum of (2+2), and the other even number which plus 2 can be exactly divided by 6 can be expressed as the sum of two Pb. The table method group count is approximately equal to or more than half of the group count of twin prime smaller than the even number.

References