

A preliminary study on spin and trajectory equations of table tennis

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Abstract: Table tennis is a world-popular ball sport. Table tennis has three stages attack, confrontation, and defense in the game, and its playing methods are divided into curve and fast attack. In the players who mainly play curve, table tennis rotation, curve, and landing point are more prominent. The point of table tennis mainly lies in rotation, and the strange curve and landing point caused by rotation are also the biggest points of interest. Rubber through different ways of friction will produce various types of rotation, the most basic rotation modes are top rotation, down rotation, left rotation, right rotation, cis rotation, and counterclockwise spin, which in the actual situation can produce mixed rotation. This paper will elaborate on three aspects: the introduction of rotation, the corresponding relationship between curve generation mechanism and rotation curve, and the change of trajectory velocity and landing point, combined with experimental demonstration.

1. Introduction

As China's national Table tennis team has achieved one exciting result after another in the world Series, more and more people begin to pay attention to table tennis, and the popularity of table tennis in people of all ages is also increasing. The biggest aspect of table tennis lies in the unique trajectory formed by the combination of its variable rotation and spin speed and the center of mass speed. Understanding the physical principles behind it helps us to analyze it more objectively and rationally. When we are off the field, we can analyze the way of players play; when we practice, we can better understand the ball rotation, predict the point of the ball, and make the next response. At the same time, we can extend table tennis to similar types of sports in other flow fields, in order to solve other problems.

Up, down, left, right, forward and counterclockwise spin are the most basic six rotation modes in table tennis, which can produce mixed rotation in practice, a total of 26 kinds. In this paper, we will analyze the flow field, use Bernoulli equation to analyze the influence of six basic rotation on the trajectory and landing point, and then analyze the other mixed rotation, and show some actual situations. This paper will use the simulation software to simulate the actual situation, and do some extension analysis in the actual situation. This paper is helpful to the analysis of table tennis rotation in practice.

2. Theoretical Analysis and Experimental Inquiry

2.1 Rotation profile

The basic rotation of table tennis is divided into six types: up and down rotation, forward counterclockwise spin, left and right rotation. Each rotation corresponds to the different rotation of the three-dimensional sphere around the three axis in the Cartesian coordinate system

From this on, we will build a physical model

This part will be illustrated on the (Grapher) Default coordinate system and GeoGebra Calculator Suite in the Mac operating system, using the radian system.

2.2 Up spin and down spin

top spin (figure 1) refers to the rotation of the highest point of the ball is greater than the speed of the center of mass of the ball. Improper handling of the topspin ball will be introduced. In order to hit the topspin ball, the racket should be pulled up from the back, up from the bottom, to rotate the ball forward. A compound rotation composed of topspin is the most common presence in table tennis competitions. topspin allows the ball to safely cross the net and fall on the table at high speed, reducing errors and increasing attack, and is common in the stalemate phase.[2]

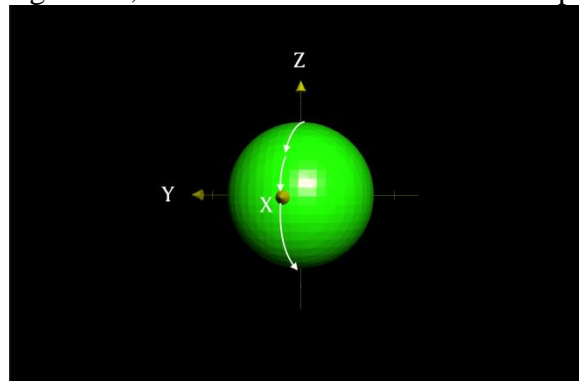


Figure 1: Epispin diagram (table tennis center velocity direction along the X axis)

Downspin (figure 2) is a rotation where the line speed of the lowest point of the ball is greater than the center speed of the ball, which is also common in games. Improper processing of the backspin ball will reduce the net. The common ways to deal with backspin ball are rub, backhand pull, forehand pull, pull backspin, slice and so on.

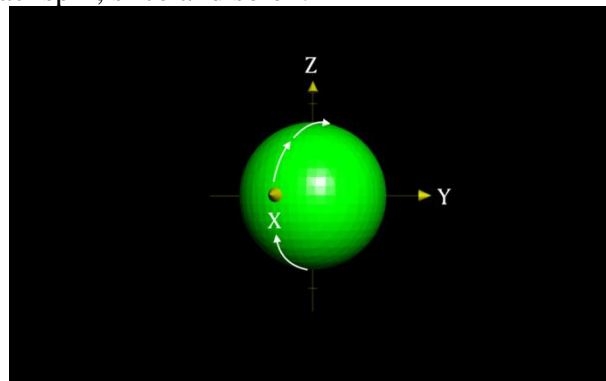


Figure 2: Schematic diagram of backspin (table tennis center velocity direction along the X axis)

2.3 Clockwise rotation and inverse rotation

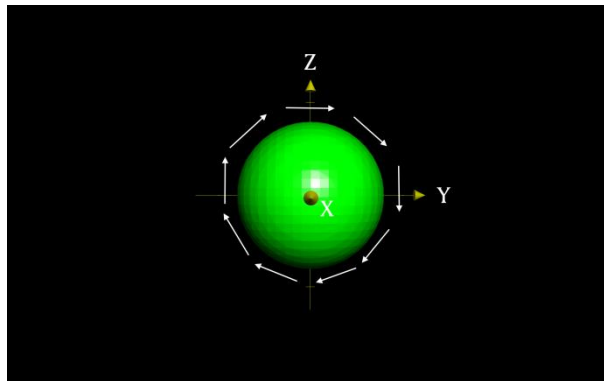


Figure 3: Schematic diagram of forward rotating ball (velocity direction of table tennis along X axis)

Clockwise spin (figure 3) refers to the clockwise rotation of the table tennis ball in the direction of the ball with the speed direction as the axis. It is usually seen in the serve, but there is no pure cisspin in practice. Most players with strong forehand prefer to serve, easy to serve to attack.

Counterclockwise spin (figure 4) refers to the counterclockwise rotation of the table tennis ball against the ball in the direction of the speed direction. It is usually seen in the serve, but there is no pure counterclockwise spin in practice. Most backhand players prefer to counterclockwise spin serve, which is easy to serve to attack.

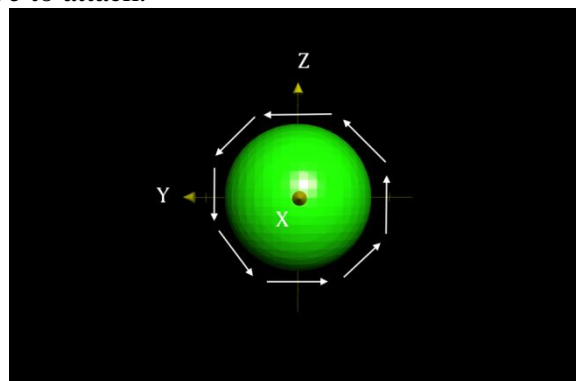


Figure 4: Diagram of reverse rotating ball (table tennis centroid velocity direction along the X axis)

2.4 Sidespin

Left rotation (figure 5) is a rotation that looks counterclockwise in the horizontal direction in the direction of the incoming ball. It can be seen on serve and other situations. Common ways to manufacture left rotation include side cutting, pull side rotation, side rotation, etc.

Right rotation (figure 6) is a rotation that looks clockwise in the horizontal direction in the direction of the opposite ball. It can be seen on serve and other situations. The common ways to manufacture the right side rotation include side cutting, pull side rotation, side rotation, etc.

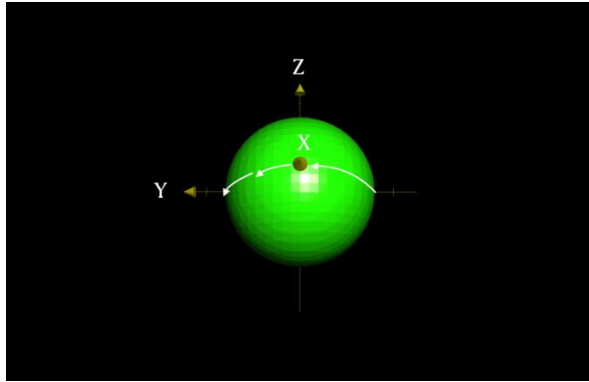


Figure 5: Schematic diagram of left rotating ball (speed direction of center of mass along X axis)

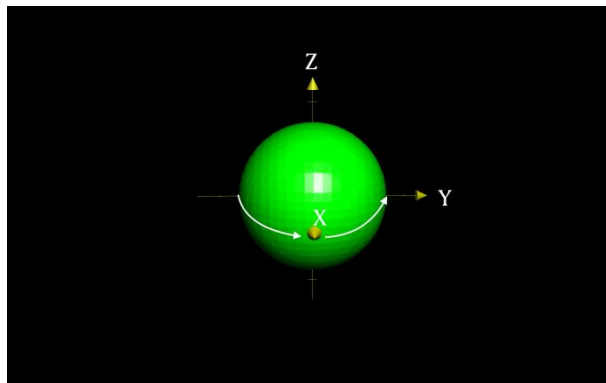


Figure 6: Schematic diagram of right rotating ball (speed direction of table tennis center of mass along X axis)

In the actual competition, there is no pure rotation, most of which are three types of compound rotation, which can be demonstrated in the (Grapher) Default coordinate system, and is not specified in this paper.

Since the angular velocity is a pseudovector, the right-hand rule of angular velocity can be more intuitive to classify six rotations, shown as Table 1:

Table 1: The angular velocity direction

The angular velocity direction $\vec{\omega}$	top spin	down rotation
	Vertical velocity direction is horizontally to the right	Vertical velocity direction is horizontally to the left
	clockwise spin	counterclockwise spin
	The speed direction is reverse	The speed direction is positive
	left spin	right spin
	Vertical speed direction is vertically downward	Vertical speed direction is oriented vertically upward

2.5 Composite rotation

There are 24 effective rotation of table tennis, 20 of which are compound rotation. In fact, most of them are three-dimensional composite rotation, and there is no basic rotation. Compound rotation occupies a very important position in real situations.

For compound rotation, the addition of the vectors is decomposed (Figure 7)

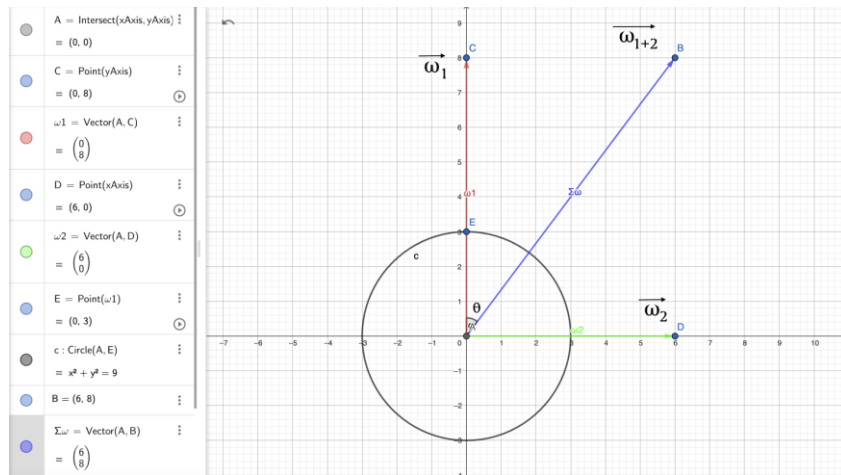


Figure 7: Schematic diagram of the angular velocity addition in two-dimensional space compound rotation (center id velocity direction vertical straight outward)

Here, taking the example of the composite rotation of right spin $\vec{\omega}_1$ and top spin $\vec{\omega}_2$ synthesis as an example, we can find that the angular speed of this composite rotation is in the plane perpendicular to the speed and in an Angle θ upward from the vertical plane.

In two-dimensional space, when the compound angular velocity is constant, according to the relation of vector modulus length, we can find that the larger the θ is, the stronger the topspin is. The smaller θ is, the stronger the right-hand spin.

3. The correspondence between the curve generation mechanism and the rotating curve

In the process of playing, we often see that the trajectory of the ball is a curve, and the curve trajectory of the ball of different rotation types is different.

There must be friction, which leads to the resultant force and the different direction of motion. At the same time, friction is also divided into two types: friction with air changes the trajectory, and friction with the table.

First, we can observe the trajectory caused by the friction with the air.

Ping-pong ball moves in the air field, according to the Bernoulli equation

$$\frac{1}{2}\rho v^2 + \rho gh + p = constant$$

Since ρgh is unchanged, the air exerts less pressure on the side with a relatively high flow velocity, and vice versa.

Then, we can explore the trajectory form of some basic rotation.

3.1 Top spin ball, backspin ball, side spin ball

In Figure 8, we select the Euler coordinate system with the table tennis ball as the origin. The particle line velocity at the top of the topspin is opposite to the flow field fluid velocity, the line velocity at the bottom particle is in the same direction as the fluid velocity of the flow field, the spin angular velocity of the table tennis is ω , the radius is r , and the center velocity of the table tennis ball is u .

Top and bottom line velocity of topspin ball in Euler coordinate system:

$$\vec{v} = \vec{\omega} \times \vec{r}$$

In practice, the air is considered to be static relative to the ground, and select the Euler coordinate system with the ping-pong ball as the origin, when the air flow flows through the ping-pong ball, the air flow velocity is u .

The lower part of the table tennis ball will accelerate the airflow, while the upper part will block the movement of the airflow and slow down.

It can be found that in the Euler coordinate system, there is a flow velocity difference between the air on the upper and lower surface, the upper velocity is small, and the lower velocity is large, and according to the Bernoulli equation, the change of ρgh in the equation is ignored, then the kinetic energy of the fluid is large, the fluid pressure on the table tennis is small, and vice versa. For table tennis peripheral force integral force, we found that the air will impose a force below, the force is also known as Magnus force, makes the topspin by the downward acceleration greater than the local gravity acceleration, regardless of the air resistance of the topspin trajectory function (parabola) quadratic term coefficient is greater than the ball trajectory function (parabola) secondary coefficient, so we can say that the topspin ball corresponding in the vertical direction of parabolic term coefficient is greater than the gravitational acceleration of the parabola.[1]

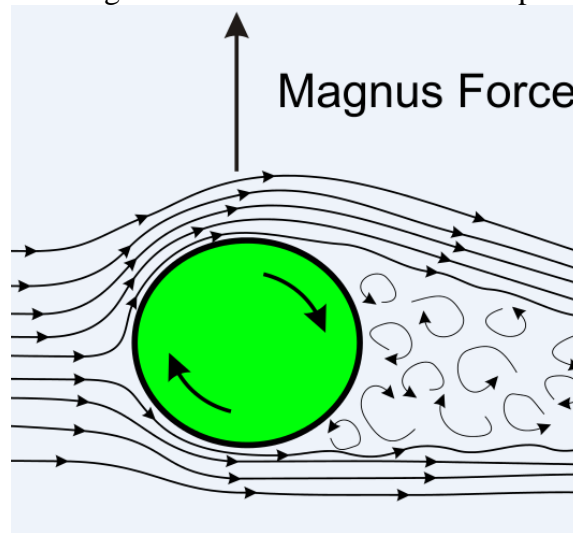


Figure 8: Schematic diagram of the side-spin table tennis ball in the flow field

Similar to the topspin sphere, we can qualitatively obtain the trajectory equation corresponding to the backspin sphere. (Figure 9)

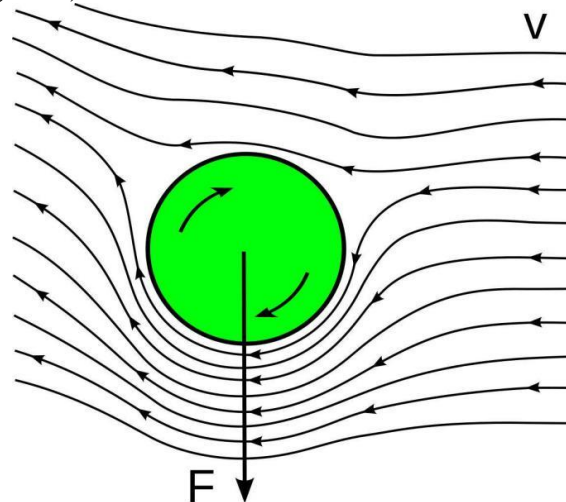


Figure 9: Schematic diagram of the backspin table tennis ball in the flow field

Similar to the topspin sphere, when the original vertical plane transposition horizontal plane, we can qualitatively obtain the trajectory equation of the side spin sphere.

Also, we choose the Euler coordinate system, for the left spin, in perspective, the surface of the airflow rate difference, the right flow is smaller, the left flow rate is larger, and according to the Bernoulli equation, the force at the same time. The table tennis, we found that the air will force (to the left, then the ball will get centripetal acceleration, namely the table tennis will turn to the left, so we can say that the left spin on the horizontal plane, with the centroid of mass speed direction for the y axis direction, the center of curvature in the curve on the right side of the function, so can be qualitative right spin trajectory equation.

Next, we will use ANSYS to demonstrate the above, taking the side spin ball as an example:

Using ANSYS simulation analysis, the flow field velocity distribution around the stationary air, shown as figure 10:

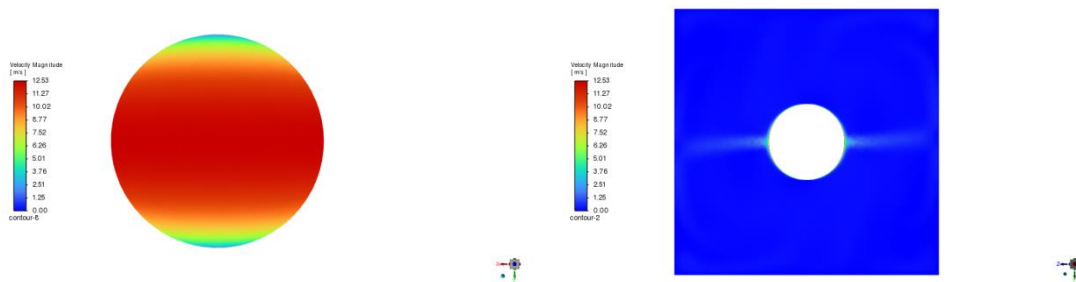


Figure 10: Flow field analysis of side spin ball (velocity)

The pressure distribution can also be obtained, shown as Figure 11:

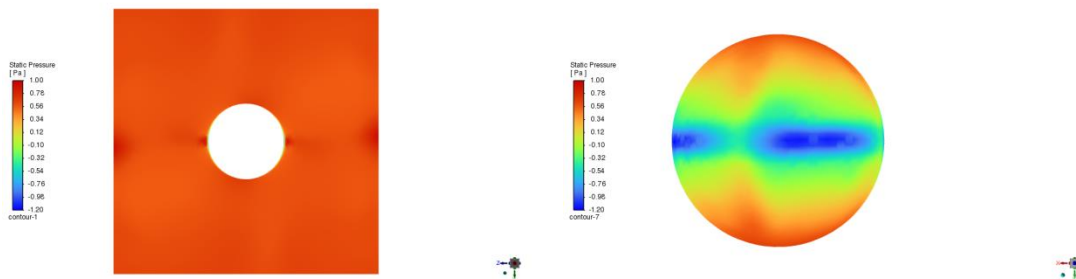


Figure 11: Flow field analysis of side spin ball (pressure)

We can clearly see that there are two opposite directions of the air for the ball pressure is small, for the blue part. In the case of a particular type of side spin, the two blue parts are different, which eventually causes the resultant force to move toward one side and changes the trajectory.

3.2 Shun-spin ball, reverse-spin ball

Next, the trajectory changes due to friction with the table.

Using ANSYS simulation analysis, the flow field velocity distribution around the stationary air, shown as Figure 12:

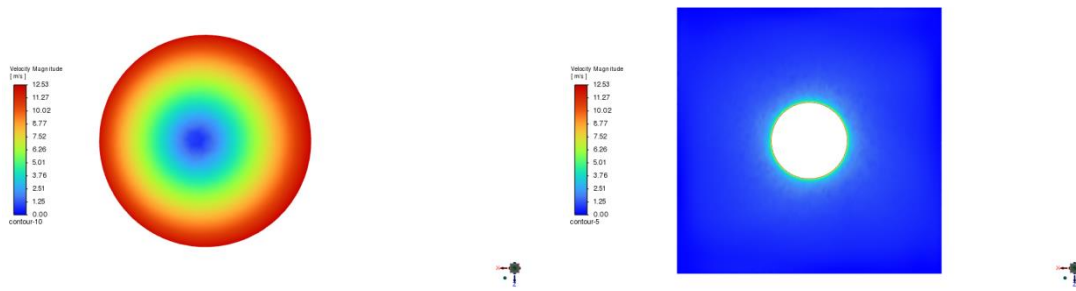


Figure 12: Flow field analysis of the inverse spin (velocity)

The pressure distribution can also be obtained, shown as Figure 13:

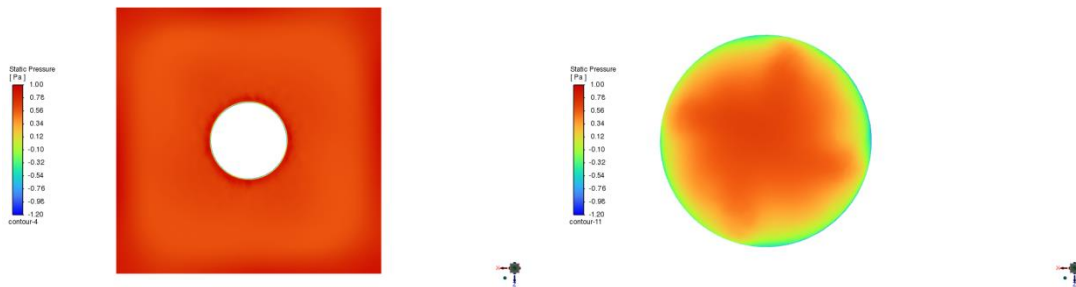


Figure 13: Flow field analysis of reverse spin (pressure)

According to the pressure around the sphere, the pressure of the sphere is approximately symmetric in the rotation plane, and the rotation symmetry is approximate at the center of the circle. It is inferred that the resultant force of the sphere is zero, and the air trajectory is almost ballistic curve.

After the landing of the ball, the situation is different. The rebound curve of the table (the second curve) will bounce to the left or right due to the friction with the table. In Figure 14, the flight curve of the reverse spin ball can be seen. The first curve of the side spin ball off flight does not significantly turn left or right, only after the left and right side turn, which is obviously different from the ordinary side spin ball.

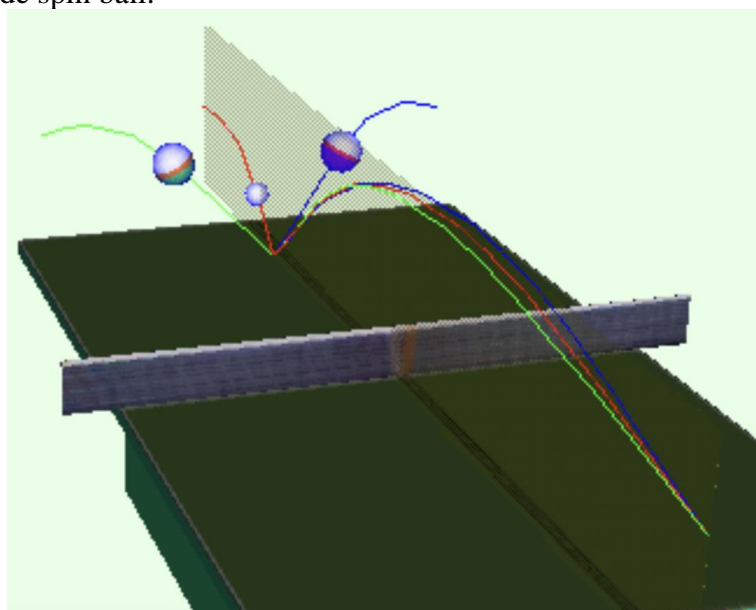


Figure 14: Schematic representation of the conand inverse rotating sphere in the flow field

Then we can summarize the inductive counterinverse rotation trajectory equation. The forward rotation corresponds to the trajectory equation in the receiver's perspective and the second curve deviates to the left relative to the original velocity direction, with the same inverse rotation. (figure 14)

4. Change of trajectory movement speed and landing point

The reason why table tennis can not be regarded as a particle in the air field is that its spin will affect the speed and direction of the movement, so the basic analysis of different kinds of rotation is analyzed.

4.1 Up spin and down spin

Placement

When the initial velocity is the same, because the downward acceleration of the topspin ball is greater than the gravity acceleration, the table tennis movement is regarded as a throwing movement by ignoring the air resistance. It can be seen that the greater the downward acceleration, the shorter the time, and the decomposed initial velocity of the topspin ball is less than the non-rotating ball. In terms of the landing point, the topspin is more advanced than the non.

When the initial speed is the same, because the downward acceleration of the backspin ball is less than the gravity acceleration, compared to the topspin ball, the smaller the downward acceleration, the longer the movement time, and the landing point of the backspin ball is lower than the rotating ball.

Speed at any time

Based on the parabolic motion, we get the following velocity equation:

$$\vec{v} = \vec{v}_x + \vec{v}_y$$

$$v = \sqrt{(v_0 \cos \theta)^2 + (v_0 \sin \theta - gt)^2}$$

Then, it can be found that in the case $a > g$, the topspin is even faster at any time during the movement from the highest point to the landing point. Similarly, the backspin ball is even faster at any time during the movement from the highest point to the landing point.[3]

4.2 Sidespin

Placement

In the receiver's perspective of the ball, there is no essential difference between the trajectory equation of the side spin in the vertical direction and the trajectory equation of the ball, while in the horizontal direction, the left spin turns to the left, so the landing point of the left spin is on the left side of the line at the initial speed of the left spin. Similarly, the landing point of the right-handed ball can be obtained by the left-handed ball analogy.

Velocity

In the receiver's perspective, the left spin has an acceleration to the left at any time, and the velocity component of the left spin can be introduced, the same with the right spin. The specific velocity equation is not discussed here.

4.3 Clockwise rotation and Counterclockwise rotation

Placement

The first curve of the ball and the ball are not different from the ball, and the trajectory parabola. Due to friction with the table, from the perspective of the receiver, the second curve of the ball deviates from the original direction, so the landing point of the ball is on the left side of the plane where the first curve is located. Inverse rotation is based on the clockwise spin rotation analogy.

Velocity

In the receiver's perspective, when the first curve changes to the second curve, the forward spin has an instantaneous acceleration to the left, pushing the velocity component to the left, the same with the reverse spin. The specific velocity equation is not discussed here.

5. Conclusion

Based on the discussion of the ball trajectory and flow field in table tennis, the following conclusions can be drawn:

1) Composite rotation affects the trajectory

Table tennis presents compound rotation in actual competition, different rotation types will lead to change the trajectory of the ball. For example, the trajectory form of topspin and downspin balls can be described by the Euler frame, and air friction can affect the trajectory. The trajectory equation of the side spin ball can be qualitatively described as the characteristic of causing the ball to make the ball turn to the left or right.

2) Flow field analysis effects

The ANSYS simulation analysis can observe the flow field velocity and pressure distribution around the lateral spin ball, the shun spin ball, and the reverse spin ball, and the effect of air on the ball. A specific type of side spin will lead to trajectory changes due to differences in air pressure distribution. The difference between the curvature of the lateral rotation and the air velocity is that the air velocity difference increases by 2 degrees when the air velocity reaches 6 m/s. With a flow velocity difference of 6 m / s, the centripetal acceleration of the side-spin ball reaches 5.9 m / s². In addition, the pressure distribution is associated with the change of the side spin ball trajectory.

3) The effect of the desktop friction on the trajectory

After the landing, due to the desktop friction, it will produce a rebound curve, turning left or right, which is different from the trajectory characteristics of the ordinary side spin ball. Due to the table friction, the curve deflection angle is 2 degrees on average. The influence of table friction on the rotation can cause the acceleration of about 3 m/s².

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