

Optimization of Bench Dancing Dragon Team Motion Based on Collision Detection Model and Numerical Simulation

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Abstract: As an important folk cultural treasure, “Bench Dragon” attracts many audiences with its unique dance form [1]. Based on mathematical modeling and optimization algorithm, this paper discusses the dynamic marching process of the dragon dance team on the spiral line, focusing on the path planning and speed optimization of the coil in and out, aiming to improve the smoothness and safety of the dragon dance performance, so as to enhance the visual impact of its cultural heritage. Firstly, the position of the dragon head over time is deduced through the conversion of polar coordinates and Cartesian coordinates, and the positions of the dragon body and tail parts in each second are calculated step by step by using the geometrical characteristics of the dragon [2]. Meanwhile, the central difference method was applied to solve the velocity of the key nodes, and the model results were visually examined to verify the validity of the model [3]. Then, the position model and the collision detection model of the multi-bench system were constructed, the minimum distance between benches was calculated, and the collision was detected by the differential method, which finally led to the conclusion that the dragon's head collided with the 8th board of the dragon's body at about 412.6 seconds [4]. Finally, based on the previous research results, the collision detection model was constructed and the critical point was measured, and the range of the pitch to satisfy the condition was determined to be 0.450~0.451 meters using the global traversal method. The research in this paper provides theoretical support for the safety and fluidity of non-heritage dragon dance activities.

1. Introduction

Intangible cultural heritage is an important part of national and ethnic culture, carrying rich historical and artistic values. As national cultural self-confidence grows, the awareness of protecting and inheriting intangible cultural heritage continues to rise. As a representative of China's traditional folk culture, the “bench dragon” has attracted many audiences with its unique dance form and

profound cultural connotation, and has become an important carrier of the national spirit. However, with the popularization of dragon dance activities, it faces challenges in the process of inheritance, such as safety and fluency.

In order to better protect and inherit the “bench dragon”, scientific research and optimization are crucial. In this paper, through mathematical modeling and optimization algorithms, we analyze the movement laws of the dragon dance team in dynamic performances, and explore reasonable path planning and speed optimization in order to improve the safety and viewability of the performance. It is hoped that through the accurate calculation of the movement states of the dragon head, body and tail, it can provide theoretical support and practical guidance for the inheritance of “Bench Dragon” and promote the sustainable development of this traditional culture.

2. Position and velocity of a dynamic object per second

2.1 Parametric equations for isometric solenoids

In order to better study the path optimization problem of the complex motion of objects, this paper takes the example of “bench dragon”, a traditional folk activity, which needs to be able to coil in and out freely in a limited space, in line with the requirements of the complex trajectory of objects. Assuming that the dragon dance team needs to be coiled clockwise along the isometric thread with a pitch of 55cm. The traveling speed of the front handle of the dragon is constant at 1m/s, starting from point A of the 16th turn of the thread. It is necessary to calculate and record the position and speed of the whole dragon dance team every second from the initial moment to 300 seconds, and pay special attention to the data of several key time points.

In this paper, the parametric equations of the isometric solenoid are chosen to describe the traveling path of the dragon dance team [5]. Since the speed of the front handle of the dragon head is constant, we can derive the position of the dragon head over time by combining the parametric equation of the spiral line with the transformation relationship between polar coordinates and Cartesian coordinates.

$$\begin{cases} x_{(\theta)} = r_{(\theta)} \cos \theta = k \cdot \theta \cos \theta \\ y_{(\theta)} = r_{(\theta)} \sin \theta = k \cdot \theta \sin \theta \end{cases} \quad (1)$$

That is, the mathematical expression for the helix is:

$$r_{(\theta)} = k\theta \quad (2)$$

A linear relationship between r and θ in an isometric helix gives:

$$k = d/2\pi \quad (3)$$

Using the geometric features of the bench dragon (e.g. bench length, width, position of holes, etc.) and the position of the dragon's head, the position of each part of the dragon's body and tail at each second was gradually deduced. This process reflects the idea of moving from the local to the whole, i.e., starting from the position of the dragon's head, gradually derive the position of the whole dragon dance team.

The traveling speed from the dragon's head to the handle is always 1m/s, considering the dragon's head entering direction with polar coordinates, know:

$$|ds / dt| = v = 1 \quad (4)$$

$$ds = \sqrt{r^2 + (dr / d\theta)^2} d\theta \quad (5)$$

The above equation shows that the rate of change of angular velocity of the faucet as it moves along the helix is:

$$\frac{d\theta}{dt} = \frac{-1}{k\sqrt{1+\theta^2}} \quad (6)$$

Let the positions between neighboring holes be (x, y) and (x_1, y_1)

$$M = \sqrt{(x - x_1)^2 + (y - y_1)^2} \quad (7)$$

That is, given the coordinates (x_1, y_1) and distance M of the previous hole, determine the angle θ of the next hole

$$M^2 = (k \cdot \theta \cos \theta - x_{\theta 1})^2 + (k \cdot \theta \sin \theta - y_{\theta 1})^2 \quad (8)$$

At this time, the constraint equation of the distance is more than one value, here we need to add constraint judgment conditions, that is, there are a number of can be made to meet the distance M at any moment can be found θ , by the center of the difference method of the derivative discretization of the angular velocity at the moment of the solution i .

$$\left. \frac{d\theta}{dt} \right|_{t_i} = v(t_i) = \frac{\theta(t_i + 1) - \theta(t_i - 1)}{2\Delta t} \quad (9)$$

The velocities of the front handle of the dragon's head as well as the nodes of the rear handle of the dragon's tail are solved using forward and backward differencing:

$$\begin{cases} v(t_1) = \frac{\theta(t_2) - \theta(t_1)}{\Delta t} \\ v(t_n) = \frac{\theta(t_n) - \theta(t_{n-1})}{\Delta t} \end{cases} \quad (10)$$

Velocity change of each bench wrench (hole) at different points in time:

$$v = -k \cdot \sqrt{1 + \theta^2} \frac{\Delta \theta}{\Delta t} \quad (11)$$

For the arc length L between two neighboring points, it must be ensured that the new hole is in the same solenoid as the previous hole and not jumping to another solenoid. As shown in Figure 1.

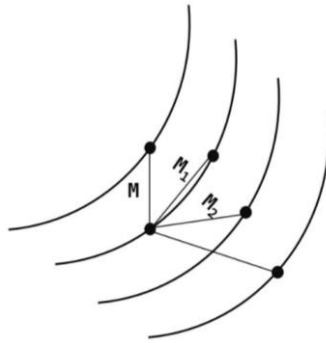


Figure 1: Spacing between the two handle points of the screw

$$L = k \cdot \Delta\theta \quad \left(|k\theta_2 - k\theta_1| < \frac{d}{2} \right) \quad (12)$$

$$\left\{ \begin{array}{l} x_{(\theta)} = k \cdot \theta \cos \theta \quad y_{(\theta)} = k \cdot \theta \sin \theta \\ r_{(\theta)} = k\theta \\ k = \frac{d}{2\pi} \\ \frac{d\theta}{dt} = \frac{-1}{k\sqrt{1+\theta^2}} \\ M = \sqrt{(k \cdot \theta \cos \theta - x_{\theta 1})^2 + (k \cdot \theta \sin \theta - y_{\theta 1})^2} \\ v = -k \cdot \sqrt{1+\theta^2} \frac{\Delta\theta}{\Delta t} \end{array} \right. \quad (13)$$

2.2 Visualization of model results

Combined with the above model, we first simulate the trajectory of the center hole of the front handle of the “Bench Dragon”, and then simulate the trajectory of the whole “Bench Dragon” coiled inward for 300s in an isometric spiral trajectory, as shown in Figure 2:

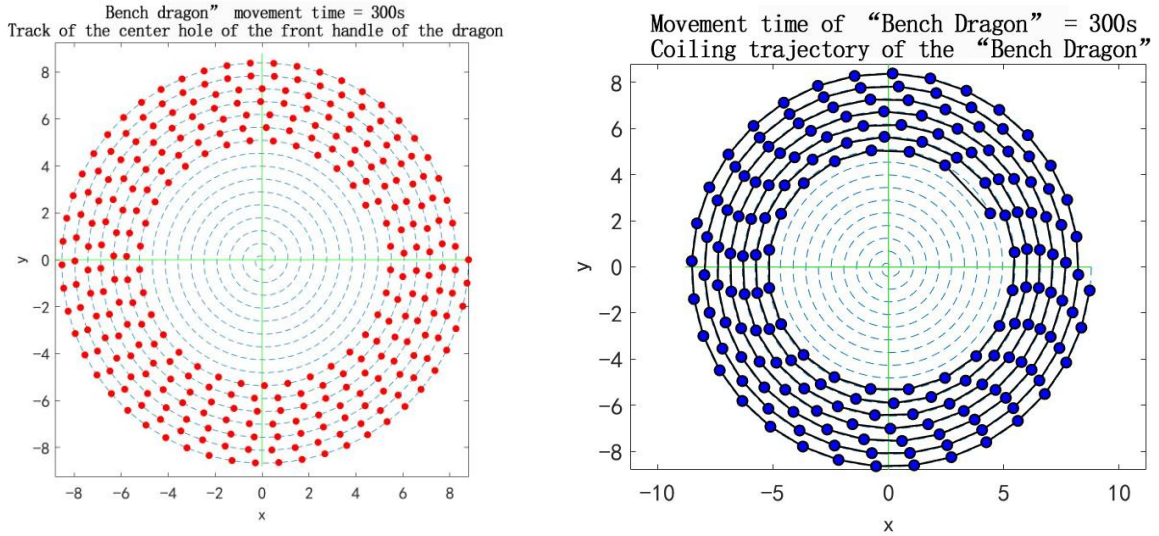


Figure 2: Trajectory of the center hole of the front handle of the dragon head and the trajectory of the “bench dragon” (t=300s)

As shown in the figure, the trajectory of the center hole of the front handle of the “Bench Dragon” drawn after 300s does not reach 224 points within 16 circles of the isometric screw, and then analyze the simulation results of the whole “Bench Dragon”, which shows that the “Bench Dragon” cannot enter all the points within 16 circles of the isometric screw within 300s. It can be analyzed that the whole “bench dragon” can't enter into the isometric thread within 16 circles of point A within 300s.

3. Position and velocity of an object at the moment of termination of its motion

3.1 Collision Detection Model

In this section, it is necessary to determine the moment of termination of the dragon dance team's disking in along the solenoid, i.e., the point in time when it is ensured that there is no collision between the benches, and to give the position and velocity of the team at this point. This requires an in-depth analysis of the geometric properties of the dragon dance team, the geometry of the screw thread, and the principles of kinematics. Let the length of the dragon's head bench be L_1 , and the lengths of the benches at the dragon's body and tail be L_2 :

$$\begin{cases} L_1 = 341\text{cm} = 3.41\text{m} \\ L_2 = 220\text{cm} = 2.2\text{m} \end{cases} \quad (14)$$

Since the center of each hole is 27.5 cm from the head of the nearest plate, which is 0.275 cm, the distance between the two handle holes on each bench is respectively.

$$\begin{cases} M_1 = L_1 - 2 \times 0.275 = 2.86\text{m} \\ M_2 = L_2 - 2 \times 0.275 = 1.65\text{m} \end{cases} \quad (15)$$

The holes on each bench are located on the centerline of the bench, and the distance between the two handle holes (M1 or M2) determines the connection of adjacent benches, due to the constant updating of the angle and coordinates during the dragon dance, in order to calculate the position of the holes of the individual benches during this process. Assuming that the initial angle of the dragon is θ_0 at the initial time $t=300\text{s}$, the motion state at each time point is gradually updated by the step Δt (set to 0.01s), i.e., the dragon's head is being coiled in along the solenoid from the outside to the inside, and the time zone is $[300, 300 + \Delta t]$, i.e., from $[0, \Delta t]$, i.e., the polar angle at the next moment of each bench is:

$$\frac{d\theta}{dt} = \frac{-1}{k \cdot \sqrt{1 + \theta^2}} \quad (16)$$

A multi-bench system modeling of the dragon dance activity was conducted so as to establish the coordinates of each perspective:

$$\begin{cases} x_i(\theta_i) = k \cdot \theta_i \cdot \cos(\theta_i) \\ y_i(\theta_i) = k \cdot \theta_i \cdot \sin(\theta_i) \end{cases} \quad (17)$$

At each time Δt , instead of considering the possibility of a collision within the same circle, it is necessary to consider a collision between the circle in which the selected bench is located and the outer circle (i.e., θ outside the collision point $\theta + 2\pi$), i.e., whether the benches (rectangles) overlap.

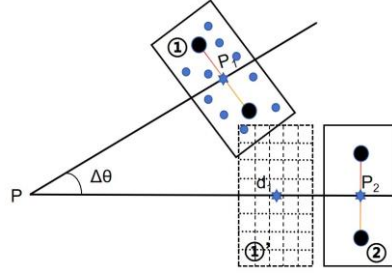


Figure 3: Two-plate collision analysis diagram

As shown in Figure 3. Based on the collision detection model, the board ① is infinitely subdivided, and the differential idea is used to discretize the board ① into countless points, i.e., whether the points discretized from the board ① exist in the board ② is judged to realize the judgement of whether the two boards collide or not. Known at any moment in the handle hole position and angle, assuming that the board has been fixed, by calculating the first board in all the points whether to appear in the second board can be obtained whether the collision, that is, the calculation of the direction of the angle of the center line of each bench and the relative angle of the clamp $\Delta\theta$,

$$\begin{cases} |x - x_{p_2}| \leq \frac{d}{2} \\ |y - 0| \leq \frac{L_2}{2} \end{cases}$$

PP2 for the x-axis, it can be seen that ① the board area is of the tilt angle, ① board is in the state of the tilt angle, assuming that the board rotates to the center of the ① board in the state of the tilt angle, assuming that ① board rotation $\Delta\theta$ to ① center point P, at this time and the center point distance is d, that is, ① center point is d. ① board meets

$$\begin{cases} -\frac{30}{2} \leq x - d \leq \frac{30}{2} \\ -\frac{L_1}{2} \leq y \leq \frac{L_1}{2} \end{cases}, \text{ the rotation matrix } T \text{ to: } T = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix}, \text{ discrete point grid}$$

generated in the first rectangle of the point (x,y) and rotated to the new matrix ① ' in

$$XY_{new} = T \begin{bmatrix} X \\ Y \end{bmatrix}, \text{ to determine whether the rotated point falls in the ② rectangle region, that is,}$$

$$\begin{cases} |x - x_{p_2}| \leq \frac{d}{2} \\ |y - 0| \leq \frac{L_2}{2} \end{cases}, \text{ if the intersection of the satisfaction of the collision, then the collision occurs.}$$

The average speed of each bench handle is:

$$v_i = -k\sqrt{1 + \theta_i^2} \frac{d\theta_i}{dt} \quad (18)$$

Calculated by the post-differential method:

$$v_i = -k\sqrt{1 + \theta_i^2} \cdot \frac{\theta_i(t) - \theta_i(t - \Delta t)}{\Delta t} \quad (19)$$

$$\begin{cases} x_i(\theta_i) = k\theta_i \cdot \cos(\theta_i) \\ y_i(\theta_i) = k\theta_i \cdot \sin(\theta_i) \\ v_i = -k\sqrt{1 + \theta_i^2} \cdot \frac{\theta_i(t) - \theta_i(t - \Delta t)}{\Delta t} \end{cases} \quad (20)$$

3.2 Visualization analysis

On the basis of the model in the previous section, it is first assumed that no collision occurs in its motion in the first 300 s. Combined with the established collision model, the remaining trajectory is simulated and the motion time is recorded to obtain the trajectory as shown in the left panel of Figure 4:

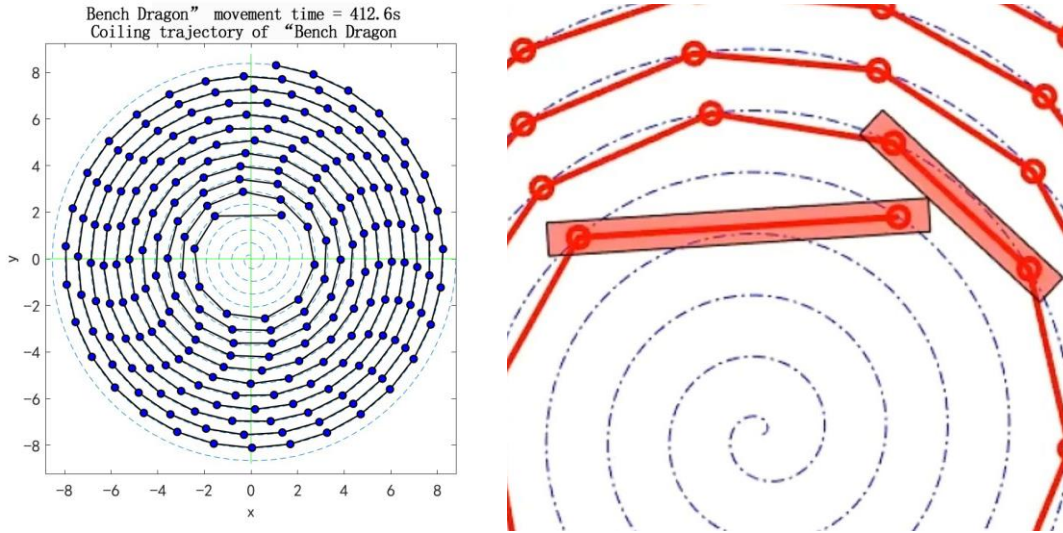


Figure 4: Localized enlargement of the disk-in trajectory of the “Bench Dragon” (t=412.6s) and the moment of collision

The left figure in Figure 4 shows the process diagram of the “bench dragon” traveling inward until the collision, which is the visualization of the collision detection model, and the right figure is the local enlargement of the left figure at the moment of the collision, which is able to more intuitively reflect the collision of the two benches, and the final result from the model is that the dragon's head and the dragon's body are collided with the 8th board at the time point, when the model is set up.

4. Turnaround space model

4.1 Minimum Pitch of Spatial Boundary Based on Collision Detection Modeling

Obtained based on isometric spirals and the relationship between r and θ in isometric spirals:

$$\begin{cases} r = k\theta \\ k = \frac{d}{2\pi} \end{cases} \implies r = \frac{d}{2\pi} \cdot \theta \text{ i.e. } \theta = \frac{2\pi r}{d} \quad (21)$$

Based on the collision detection model to determine whether the critical point of collision, assuming that the turnaround space is from the center of the thread as the center of a circle, the diameter of the circular area of 9m, from which the turnaround space ($2r = 9m$) and the thread has an intersection point E, said point E for the critical point. Dancing dragon forward in the process of θ is constantly getting smaller, when the screw line d increases, the bench can move forward more, at this time θ decreases. When the thread d decreases, the bench is able to move forward less, at which time θ increases. As d decreases, the collision occurs earlier, and the stay position θ is greater than the θ angle when d increases. The optimal pitch d is the minimum pitch that allows the faucet handle to be adjusted without collision even if it is obtained: $\theta_{\min} \geq \theta_E$ (angle of the junction of the circumference and the threads of the threads) The minimum angle θ_{\min} is the smallest angle that can be achieved by the faucet without collision. In order to avoid collisions, the faucet front handle must not collide with any benches on the body or tail of the faucet within the header area. At the outermost level of the screw threads, the smaller the pitch, the greater the density of the threads, which means that the minimum angle θ_{\min} is smaller. The relationship between the minimum angle θ_{\min} and the pitch d can be approximated as:

$$\theta_{\min} = \frac{2\pi r_{\max}}{d} \Rightarrow \theta_{\min} = \frac{9\pi}{d} \quad (22)$$

Where r_{\max} allows the largest helix radius, it is known that there is an inverse relationship between pitch d and minimum angle θ_{\min} , i.e., the smaller the pitch, the larger the minimum angle.

$$\theta_{\min} = \frac{2\pi r_{\max}}{d} \quad (23)$$

It can be seen that with the gradual decrease of pitch, the front handle of the faucet is able to get closer to the boundary of the turnaround space. At the same time, the collision detection mechanism ensures that there is no collision between the parts of the bench faucet during the whole coiling process. There is an inverse relationship between the pitch and the minimum angle that the faucet can reach, i.e. the smaller the pitch, the larger the minimum angle that the faucet can reach.

4.2 Hierarchical global traversal

Using the idea of hierarchical global traversal, the step size of the first global traversal is set to 0.005m, and the optimal angle versus the solenoidal polar angle is derived after about 34 min (one traversal), as shown in the following Figure 5

The search is performed globally, and it can be seen by looking at the image that after one traversal it is obtained that the interval of the minimum pitch at this point is m .

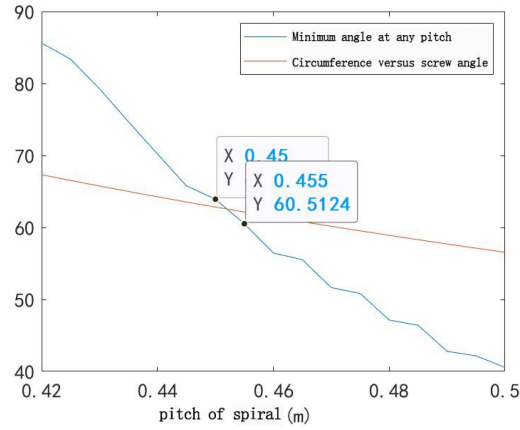


Figure 5: Optimal angle versus solenoidal polar angle (one traversal)

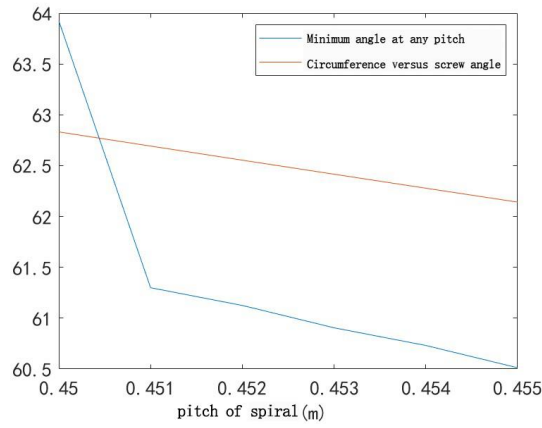


Figure 6: Optimal angle versus solenoidal polar angle (quadratic traversal)

From the results obtained from the traversal (as shown in Figure 6), a quadratic interval global traversal is performed on the interval (0.45, 0.455), and the final interval of the minimum pitch value under the optimal angle is within (0.45, 0.451) m. The global traversal under the larger interval of the model reduces a large amount of computation and thus reduces the program running time.

5. Conclusions

This paper presents an in-depth analysis of the dynamic motion in the performance of “Bench Dragon” through mathematical modeling and optimization algorithms. Firstly, the trajectories of the dragon head, body and tail during the performance are accurately depicted using polar and Cartesian coordinate transformations. Second, the velocity of the key nodes was calculated using the central difference method, and the validity of the model was verified by visualization means.

In terms of collision detection, the position model of the multi-bench system was constructed, the minimum distance between benches was calculated, and the differential method was applied to detect the potential collision, and the collision between the dragon's head and the 8th plate of the dragon's body at about 412.6 seconds was finally determined. Based on this, a collision detection model was established and the pitch range of 0.450 to 0.451 m was determined by the global traversal method.

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