

# *Heat transfer analysis and optimization of service based on unsteady heat transfer model*

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**Abstract:** This paper discusses the problem of heat transfer in service based on unsteady heat transfer theory. Firstly, the unsteady partial differential heat transfer equations are established according to the law of conservation of energy, and the initial values and boundary conditions are determined. Two unknown convective heat transfer coefficients are involved in the heat transfer model of the work suit. The optimal heat transfer coefficient is determined by describing the numerical relationship and solving the optimization problem, which can be applied to the thickness design of the work suit. The relationship between the convective heat transfer coefficient and the measured temperature is established by the least square method. Furthermore, the unsteady heat transfer model is solved numerically by explicit difference method, and the stability and accuracy of the model are verified. Finally, according to the solution results, the temperature distribution and parameter fitting effect of the work suit are shown, which proves the effectiveness and reliability of the model in practical application.

## 1. Introduction

In the high temperature working environment, the design of high temperature working clothing should not only effectively prevent personnel from being burned, but also consider the optimization of research and development costs and cycles[1]. Key experimental data can be obtained by placing the dummy in a high temperature environment and measuring the outer skin temperature. In order to meet the design requirements[2], this paper aims to establish the unsteady heat transfer model of high temperature service[3], and use the measured temperature data to verify the model and optimize the parameters. The work suit consists of four layers, the first three of which are fabric, and the fourth layer is an air layer between fabric and skin[4]. The first layer is directly exposed to the external environment and requires effective isolation of high temperature transfer[5]. A constant temperature of 37°C was maintained inside the dummy as the initial condition in the model[6]. The unsteady heat transfer model will reflect the heat transfer characteristics of the service at different time points, and provide accurate estimation of real-time temperature distribution and heat transfer parameters for the design[7]. This paper will first analyze the background and requirements of the problem, and then describe the mathematical model and its solution in detail. Through the

establishment and solution of the model, the aim is to achieve the optimal design of high temperature working clothing, in order to meet the dual objectives of protection and cost effectiveness[8].

## 2. Unsteady heat transfer model

According to the model preparation part, the unsteady partial differential heat transfer control equation can be established based on the law of conservation of energy, and the initial value and boundary conditions can be determined. Then the unsteady heat transfer model of the service is established[9]. The two convective heat transfer coefficients in the model were unknown. The numerical relationship between the coefficient and the measured temperature was established by the heat transfer model, and the fitting optimization problem was searched and solved to obtain the optimal heat transfer coefficient, which was applied to the thickness design of the subsequent service[10].

For the unsteady heat transfer problem, the unsteady partial differential governing equation is established according to the law of conservation of energy, that is, for any microelement, the change of its thermodynamic energy (manifested as the change of temperature) is equal to the difference of the heat flow in and out of the microelement. The governing equation is:

$$\rho_j c_j \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_j \frac{\partial T}{\partial x} \right) \quad (j = 1, 2, 3, 4) \quad (1)$$

In the formula, the left term represents the change of thermal mechanical energy of the microelement. The right term represents the difference of heat flow in and out of the cell.

For the entire model, the third type of boundary conditions are applied at both ends, and the outgoing heat flows through the convection zone. It is assumed that the human body and the work clothes have reached a stable state when entering the high temperature environment, and the initial value of the work clothes temperature distribution is the dummy temperature 37°C.

$$\begin{cases} -\lambda_1 \frac{\partial T}{\partial x} |_{x=0} = h_1 (T_{en} - T(0, t)) \\ -\lambda_4 \frac{\partial T}{\partial x} |_{x=L} = h_2 (T(L, t) - T_{ren}) \\ T(x, 0) = T_{ren} \end{cases} \quad (2)$$

In the formula, h1 and h2 respectively represent the convective heat transfer coefficients at both ends, T(0,t) and T(L,t) respectively represent the interface temperature at both ends, T(x,0) is the initial condition, Ten represents the ambient temperature, and Tren represents the human body temperature. For the thermal conductivity of non-uniform materials, it has been assumed that the contact between the materials is good, the contact thermal resistance is ignored, and the interface continuity condition is met, that is, the condition of continuous temperature and heat flux on the interface is met:

$$\begin{cases} T(x_i^-, t) = T(x_i^+, t) \\ \lambda_i \frac{\partial T}{\partial x} (x_i^-, t) = \lambda_{i+1} \frac{\partial T}{\partial x} (x_i^+, t) \end{cases} \quad (i = 1, 2, 3) \quad (3)$$

Where: i represents each contact surface. As shown in Figure 1.

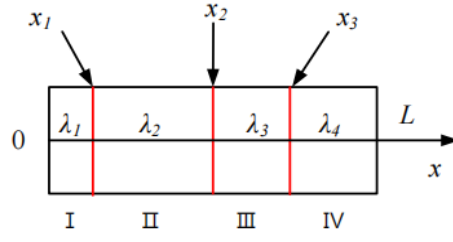


Figure 1: Material contact surface

In the unsteady one-dimensional heat transfer model, the heat transfer coefficients at both ends are unknown, and the parameter estimation model is established by least square method:

$$(\hat{h}_1, \hat{h}_2) = \operatorname{argmin}_{h_1, h_2} \sum_{i=1}^N [T(L, t_i; h_1, h_2) - T^*(t_i)]^2 \quad (4)$$

Where,  $\hat{h}_1$  and  $\hat{h}_2$  are the least squares estimates of  $h_1$  and  $h_2$ , and  $T^*(t_i)$  is the measured temperature of outer skin.

Parameter estimation:

$$(\hat{h}_1, \hat{h}_2) = \operatorname{argmin}_{h_1, h_2} \sum_{i=1}^N [T(L, t_i; h_1, h_2) - T^*(t_i)]^2. \quad (5)$$

Equation of control:

$$\rho_j c_j \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda_j \frac{\partial T}{\partial x} \right) \quad (j = 1, 2, 3, 4). \quad (6)$$

Boundary condition:

$$\begin{aligned} -\lambda_1 \frac{\partial T}{\partial x} \Big|_{x=0} &= h_1 (T_{xn} - T(0, t)), \\ -\lambda_4 \frac{\partial T}{\partial x} \Big|_{x=L} &= h_2 (T(L, t) - T_{ren}) \end{aligned} \quad (7)$$

Surface of contact:

$$\begin{aligned} T_i &= T_{i+1} \\ \lambda_j \frac{\partial T}{\partial x} &= \lambda_{j+1} \frac{\partial T}{\partial x} \end{aligned} \quad (8)$$

Initial condition:

$$T(x, 0) = T_{ren} \quad (9)$$

The basic idea of numerical solution of heat transfer problem is to discrete continuous physical quantities in time and space on each node, and use finite difference method to solve the numerical solution of physical quantities as shown in Figure 2.

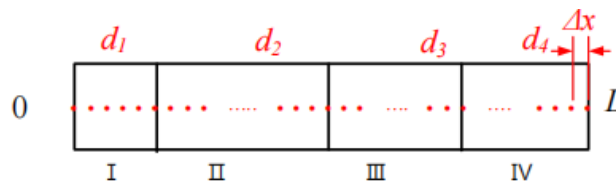


Figure 2: Discrete diagram

There are two difference schemes: explicit and implicit. For the explicit scheme, the calculation is smaller, but the accuracy and stability are not as good as the implicit scheme. Implicit difference must solve simultaneous equations, which has high stability and accuracy but large computation. Because of the large amount of data, the explicit difference scheme is used. In this paper, an explicit difference scheme is used to discretize the heat transfer model. When solving the temperature in the (n+1) time layer, the temperature information of the previous layer is relied on. There is only one unknown  $T_{i+1}$  in the discrete scheme of the governing equation:

$$\begin{aligned} \text{Governing equation: } \Delta x_j \rho_j c_j \frac{T_i^{n+1} - T_i^n}{\Delta t} &= \lambda_j \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta x_j} \\ \left\{ \begin{aligned} \frac{1}{2} \Delta x_1 \rho_1 c_1 \frac{T_1^{n+1} - T_1^n}{\Delta t} &= -h_1(T_1^n - T_{an}) - \lambda_1 \frac{T_1^n - T_2^n}{\Delta x_1} \\ \frac{1}{2} \Delta x_4 \rho_4 c_4 \frac{T_{and}^{n+1} - T_{and}^n}{\Delta t} &= -h_2(T_{and}^n - T_{and}) + \lambda_4 \frac{T_{and} - T_{-1}^n - T_{and}^n}{\Delta x_4} \\ \text{Contact point: } \frac{1}{2} (\Delta x_j \rho_j c_j + \Delta x_{j+1} \rho_{j+1} c_{j+1}) \frac{T_i^{n+1} - T_i^n}{\Delta t} &= \lambda_j \frac{T_{i-1}^n - T_i^n}{\Delta x_i} + \lambda_{j+1} \frac{T_{i+1}^n}{\Delta} \end{aligned} \right. \quad (10) \end{aligned}$$

For the explicit difference scheme, the stability of the unsteady heat transfer process needs to be considered in the discrete solution. The above explicit difference shows that the temperature at time node n+1 on space node i is affected by neighboring points on the left and right sides, and the stability constraint (Fourier grid number constraint) must be met, otherwise unreasonable oscillating solutions will appear:

$$\begin{aligned} Fo_{\Delta} &= \frac{\lambda \Delta t}{\rho c (\Delta x)^2} && \text{(Fourier grid number)} \\ Fo_{\Delta} &\leq \frac{1}{2} && \text{(Internal node restrictions)} \\ Fo_{\Delta} &\leq \frac{1}{2 \left(1 + \frac{h \Delta x}{\lambda}\right)} && \text{(Boundary constraint)} \end{aligned} \quad (11)$$

After the time-space discretization of the unsteady heat transfer model, it can be solved layer by layer on the time and space nodes according to the boundary conditions and initial value conditions. The numerical relationship between the compound heat transfer coefficient of unknown parameters and the measured temperature of the outer skin of the dummy can be established, and then the unknown coefficients h1 and h2 can be searched to solve the optimal fitting of the measured temperature data. The specific solution steps are as follows:

STEP1: The initial values of h1 and h2 were substituted, and the discrete equations of the unsteady heat transfer model were solved layer by layer to obtain the calculated values of the outer skin temperature of the dummy;

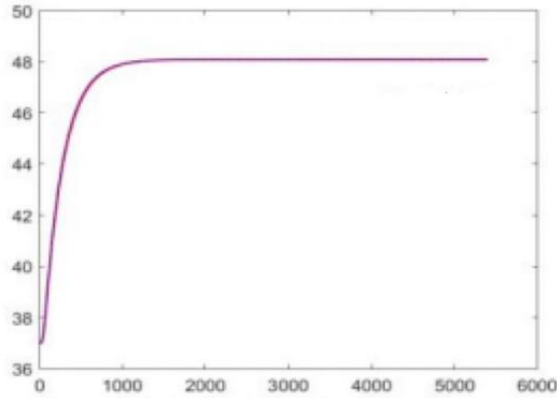
STEP2: Use the least squares method to solve the error between the calculated value and the measured value, and find the residual sum of squares;

STEP3: Update the h1 and h2 values, and bring them into the discrete equation again for solving, to get a new temperature calculation value;

STEP4: Repeat the above steps, search and optimize to find the convective heat transfer coefficient with the best fitting degree, and apply it to the subsequent service design;

STEP5: According to the best fitting convective heat transfer coefficient obtained from the search, the temperature distribution of the service is solved;

According to the above solution steps, the optimal fitted convective heat transfer coefficient was found:  $h_1 = 113 \text{ W/(m}^2 \text{ k)}$ ;  $h_2 = 8.344 \text{ W/(m}^2 \text{ k)}$ . It is found that the convective heat transfer coefficient  $h_1$  mainly affects the time to steady state;  $h_2$  mainly affects the outer skin temperature during homeostasis. At this time, the temperature outside the skin calculated from the unsteady heat transfer model is plotted against the measured temperature as shown in Figure 3 below.



(Data source: <http://www.mcm.edu.cn>)

Figure 3: Fitting diagram between simulated calculation data and measured data

Under the composite heat transfer coefficient, the residual sum of squares fitting the calculated value and the measured value is 3.6552; The error range is 0.0061, the fitting result is good. At this time, the maximum Fourier grid number of the explicit difference is:  $Fo_{\Delta \max} = 0.0472$ ; If the restriction conditions are met, there will be no unreasonable oscillation of the solution, the fitted data are shown in Table 1.

Table 1: Fitting status

$h_1/ \text{W/(m}^2 \cdot ^\circ\text{C)}$	$h_2/ \text{W/(m}^2 \cdot ^\circ\text{C)}$	Residual sum of squares	range
113	8.344	3.6552	0.0061

Cording to the heat transfer coefficients at both ends, the unsteady heat transfer model is used to calculate the temperature distribution. The three-dimensional temperature distribution diagram of skin temperature and time-space and the spatial distribution diagram of steady-state temperature are drawn as shown in Figure 4 below.

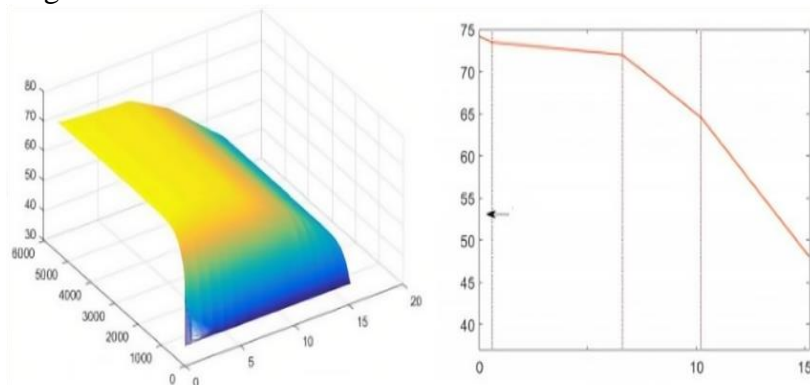


Figure 4: Time-space temperature distribution of operation service; Steady-state service temperature distribution

Part of the temperature distribution data is as Table 2 follows, where the specific temperature distribution information.

Table 2: Temperature distribution

t/min	x=0mm	x=0.6mm	x=6.6mm	x=10.2mm	x=15.2mm
5	70.18974	66.2322	59.8908	55.2576	44.3273
10	73.0906	71.5173	67.7980	62.0502	47.0590
15	73.8808	72.9568	69.9517 ...	63.9004	47.8030
80	74.1814	73.5046	72.0045	64.3007	48.0861
85	74.1814	73.5046	72.0045	64.3007	48.0861
90	74.1814	73.5046	72.0045	64.3007	48.0861

The preparation part of the model has assumed that radiative heat transfer is ignored. Here the hypothesis is tested and the unsteady heat transfer model is further extended. The model of adding radiative heat transfer term is established and analyzed. For skin, it can be approximately considered as absolute blackbody, so that  $\epsilon_{skin}=1$ ; For service clothing, emissivity  $\epsilon_g=0.02$ . According to the calculation, the radiation heat transfer is obtained and brought into the unsteady heat transfer model:

$$\begin{cases} \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) - q_{\text{Radiation}} & \text{Third layer right interface} \\ \lambda_4 \frac{\partial T}{\partial x} \Big|_{x=L} = h_2 (T_{\text{end}} - T_{\text{skin}}) + q_{\text{Radiation}} & \text{Lateral surface of skin} \end{cases} \quad (12)$$

The discrete format of the governing equation is:

$$\begin{cases} \frac{1}{2} (\Delta x_3 \rho_3 c_3 + \Delta x_4 \rho_4 c_4) \frac{T_i^{n+1} - T_i^n}{\Delta t} = -q_{\text{Radiation}} + \lambda_4 \frac{T_{i+1}^n - T_i^n}{\Delta x_4} + \lambda_3 \frac{T_{i-1}^n - T_i^n}{\Delta x_3} \\ \frac{1}{2} \Delta x_4 \rho_4 c_4 \frac{T_{\text{end}}^{n+1} - T_{\text{end}}^n}{\Delta t} = -h_2 (T_{\text{end}}^n - T_{\text{skin}}) + \lambda_4 \frac{T_{\text{end}-1}^n - T_{\text{end}}^n}{\Delta x_4} + q_{\text{Radiation}} \end{cases} \quad (13)$$

In the same step, the numerical relationship between the heat transfer coefficients  $h_1$  and  $h_2$  and the measured temperature values is established. The best fitting results are as shown in Figure 5 below.

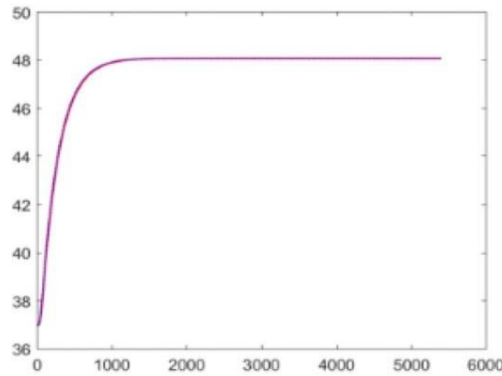


Figure 5: Fitted image considering radiation heat transfer

In this case, better fitting results can also be achieved. Compared with the absence of radiation heat transfer, the heat transfer coefficient  $h_1$  remains unchanged, and  $h_2$  changes from 8.344 W/(m<sup>2</sup> k) to 8.496 W/(m<sup>2</sup> k), with only slight changes.

Table 3: Consider the radiation fit

$h_1$ / W/(m <sup>2</sup> ·°C)	$h_2$ / W/(m <sup>2</sup> ·°C)	Residual sum of squares	range
113	8.496	3.7703	0.0021

It can be seen from Table 3 that the influence of radiation heat transfer on the whole unsteady heat

transfer process is almost negligible due to the good heat insulation effect of protective clothing and low radiation emissivity, and the assumptions in the preparation part of the model are reasonable.

### 3. Conclusion

In this study, the balance between personnel protection and R&D cost control in high temperature environment was effectively solved by establishing a design framework of high temperature work clothing based on unsteady heat transfer model. Firstly, we analyzed the hierarchical structure and material properties of the worksuit in detail, established a mathematical model for the heat transfer characteristics of each layer, and verified the accuracy and reliability of the model by using the measured temperature data outside the skin. By solving the model, we optimize the heat transfer coefficient of the working clothing to ensure the safety and comfort of human skin under high temperature working conditions. At the same time, numerical methods such as explicit difference method are used to simulate the unsteady heat transfer process effectively, which provides a scientific basis for the design optimization. In addition, we emphasized the dual consideration of protection effect and development cost in the design process, and successfully reduced the development cycle and controlled the cost through parameter optimization and model validation. These results not only provide technical support for the engineering application of high-temperature work clothing, but also lay a foundation for further research and development in related fields.

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