

Study on design and optimization of Marine mooring system based on statics model

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Abstract: In this paper, the static characteristics and optimization of mooring system design are studied through mechanical and mathematical analysis. Firstly, based on the principles of static balance and moment balance, the stability of the mooring system and the stress of each component are analyzed when the sea surface wind speed is 12m/s and 24m/s. The relationship between the vertical projection length and the total length of the anchor chain is deduced by using the calculus method, and the critical wind speed is calculated to be 21.92m/s. Under specific conditions, when the wind speed is 12m/s, the tilt Angle of the steel drum is 2.2 degrees, the tilt Angle of each section of the steel pipe gradually increases, the anchor chain part lies flat on the seabed, the remaining part is in a curve shape, the draft depth of the buoy is 0.6816m, and the swimming area is a circle with a radius of 14.676m centered on the anchor. When the wind speed is 24m/s, the tilt Angle of the steel drum increases to 4.584 degrees, the tilt Angle of the steel pipe increases step by step, the Angle between the tangential line at the bottom of the anchor chain and the seabed is 4.4 degrees, the draft depth of the buoy increases to 0.6957m, and the swimming area is a circle with a radius of 17.7918m. The research results of this paper provide theoretical basis and specific parameter reference for rational design and optimization of mooring system

1. Introduction

The stability and reliability of the Marine mooring system, which is the key element connecting the Marine engineering and the ground structure, is very important to ensure the safe operation of the offshore facilities[1]. With the development and deepening of ocean engineering, the requirements for mooring system design are getting higher and higher[2], especially in the face of complex and changeable Marine environment. In this paper, statics and mathematical modeling methods are used to study the stability of mooring system and the optimal design of its components under different conditions of sea surface wind speed. In practical application[3], the mooring system is affected by many mechanical factors, such as wind speed, ocean current, surge, etc., these factors pose challenges to the force and stability of the components of the mooring system. Based on the statics theory, this paper establishes the mechanical model of the mooring system[4], analyzes and optimizes the layout and parameter Settings of components such as anchor chain, buoy, steel drum and steel pipe in the

system, aiming to improve the stability and safety of the system. In the process of modeling, we treat the mooring system as a statically determinate problem, and derive the force equations of each component in the system by using the principles of static balance and moment balance[5]. Through mathematical method and calculation simulation, the inclination Angle of each component, the shape of anchor chain, the draft depth of buoy and the swimming area of buoy are determined under different conditions of sea surface wind speed, and the system design is optimized according to this, so as to improve its stability and adaptability[6]. In summary, this paper aims to discuss the optimization design of Marine mooring system based on statics model[7], and provide theoretical support and practical guidance for the design and construction of future Marine engineering mooring system by analyzing the mechanical characteristics under different sea environments[8].

2. Model construction

In the equilibrium state, the static analysis of each part of the mooring system is carried out by the isolation method, the tension at both ends of the anchor chain is calculated by the element method, and the force analysis is carried out by isolating the steel pipe[9], the heavy ball and the buoy. The equations are listed, the conditions of one of the equations are set up as the objective, the depth of the buoy is selected as the variable, and the traversal method is used to solve it[10].

The structure of the buoy is a 2m long cylinder with a base diameter of 2m. Without considering the wind and waves, the buoy is subjected to gravity, buoyancy, and the pulling force of the steel pipe (it is considered that the three forces form a plane confluence force system), and the direction is vertical. In the first question, the wind speed is considered, and the near sea breeze load (hereinafter referred to as wind) is added on the basis of the above, and the buoy is subjected to four forces: gravity, buoyancy, tension of steel pipe, and wind. In static equilibrium, the four forces form a plane confluence system. As shown in Figure 1.

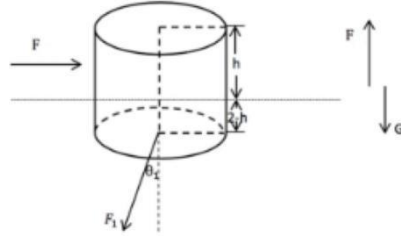


Figure 1: Force diagram of buoy system

According to the relevant knowledge of the plane confluence force system, the force points are equivalent to the center of gravity of the buoy system, and the plane rectangular coordinate system is established with the center of gravity of the buoy system as the origin. The static equilibrium equations are listed in the x and y directions respectively:

$$\begin{aligned}\sum F_x = 0: F_{[a]} - F_1 \sin \theta_1 &= 0 \\ \sum F_y = 0: F_{\text{floating}} - G - F_1 \cos \theta_1 &= 0\end{aligned}\quad (1)$$

Results:

$$\begin{aligned}0.625v^2 \times R_f h &= F_1 \sin \theta_1 \\ \rho g \pi (R_f - h) &= G_{\text{mark}} + F_1 \cos \theta_1\end{aligned}\quad (2)$$

In fact, the effect of stroke on the buoy also includes making it tilt, that is, there is a tilt Angle under the wind tilt moment. From the basic definition of torque:

$$M = F_0 \times h_0^4 \quad (3)$$

Where, F_0 is the wind force on the windward side, and h_0 is the vertical distance from the center of the windward side to the water surface. If the dynamic wind inclination is φ , the work done by the wind inclination moment can be obtained, as shown in Figure 2.

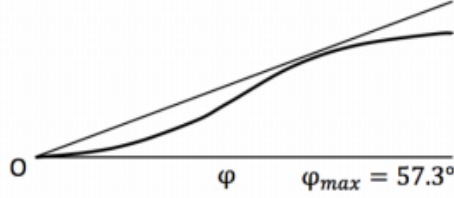


Figure 2: Dynamic stability curve

The recovery moment affected by the displacement of the buoy in the water, the height of the center of gravity, the distance of the center of buoyancy and other factors is related to the wind Angle. In the dynamic stability curve shown in the figure, the slope of the tangent line at each point is the wind inclination moment. When the maximum value of φ is 57.3° , $M = W$ in value. Let T be the weight of the structure and GM be the high initial stability, and the expression of wind inclination θ is obtained: Because wind speed, draft depth and other conditions within a certain range of wind tilt torque generated by the dip Angle is small, the buoy's wind resistance to meet the requirements, the wind tilt on the axis of the buoy cannot be considered.

The steel pipe part is composed of four identical columns with a length of 1m, a diameter of 50mm and a mass of 10kg. Without considering the influence factors such as resistance in the water, each steel pipe receives buoyancy, gravity, and the tension exerted by the adjacent steel pipe or the adjacent buoy or anchor chain at the center of the two ends of its geometric shape. The four forces form a confluence force system in the vertical plane, and the steel pipe is in static equilibrium. The stress of Section i steel pipe ($i=1,2,3,4$) is shown in the figure3 below:

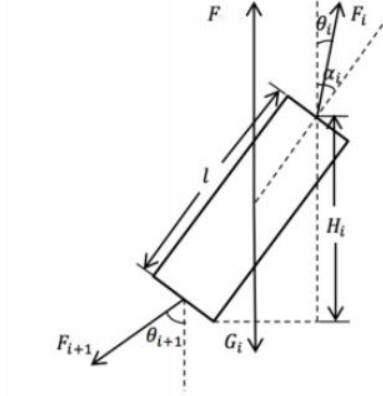


Figure 3: Force diagram of single steel pipe

Static equilibrium equations are listed in the x - and Y-axis directions respectively:

$$\begin{aligned} \sum F_x = 0: & \quad F_i \sin \theta_i - F_{i+1} \sin \theta_{i+1} = 0 \\ \sum F_y = 0: & \quad F_{i+1} \cos \theta_{i+1} - F_i \cos \theta_i - F_{\text{float } i} - G_i = 0 \end{aligned} \quad (4)$$

Balanced by torque:

$$F_i \times \frac{1}{2} l \times \sin(\alpha_i - \theta_i) - F_{i+1} \times \frac{1}{2} l \times \sin(\theta_{i+1} - \alpha_i) = 0 \quad (5)$$

Derived from geometric relations:

$$H_i = l \cos \alpha_i \quad (6)$$

The above four equations represent four different cases of four steel pipes. The fourth steel pipe is attached to the steel drum containing the underwater acoustic communication equipment. The length is 1m, the outer diameter is 30cm, and the total mass is 100kg. The steel drum is connected to the electric welded anchor chain to suspend the heavy ball, so that the tilt Angle of the steel drum (the Angle between the steel drum and the vertical line) is as small as possible. The following discusses the effect of the buoyancy force on the actual situation of the heavy ball in the sea water, and the solution is:

$$F_{\text{floating}} = \frac{\rho_{\text{seawater}}}{\rho} \cdot G \quad (7)$$

For the selection of the material of the heavy ball, the quality relationship between the heavy ball and the steel drum, the density of the heavy ball and its pressure and corrosion resistance in seawater are considered comprehensively. We first assume gold (stable and dense, $\rho = 19.32 \text{g} \cdot \text{cm}^{-3}$) and plug in the data:

$$F_{\text{floating } 1} = \frac{1.025}{1932} \cdot G \approx 0.053G \quad (8)$$

Multiplies are more related. Combined with the actual production and living conditions, the material of the heavy ball should be 10CrMoAl ($\rho = 7.65 \text{g} \cdot \text{cm}^{-3}$), then:

$$R = \sqrt[3]{\frac{m}{\rho \pi} \cdot \frac{3}{4}} \approx 0.335 \text{m} \approx 2.233 R_g \quad (9)$$

Meet the requirements, and $F_{\text{float } 1} < F_{\text{float } 2}$. Therefore, the buoyancy force of the heavy ball cannot be ignored. The small Angle deflection degree α barrel of steel drum is less than 5° , which is not easy to be reflected in the force analysis diagram, but can not be ignored in the equation analysis. Therefore, the static equilibrium state of the whole composed of the steel bucket and the heavy ball is shown as follows:

Static equilibrium equations are listed in the x - and Y-axis directions respectively:

$$\begin{aligned} \sum F_x = 0: F_5 \sin \theta_5 - F_{\text{anchor}} \sin \theta_{\text{anchor}} &= 0 \\ \sum F_y = 0: F_5 \cos \theta_5 + F_{\text{Drum float}} + F_{\text{Float ball}} - G_{\text{barrel}} - G_{\text{ball}} - F_{\text{anchor}} \cos \theta_{\text{anchor}} &= 0 \end{aligned} \quad (10)$$

Taking the center of mass of the steel drum as the reference point, the balance of torque is obtained:

$$F_5 \times \frac{1}{2} \times \sin(\alpha_{\text{barrel}} - \theta_4) - F_{\text{anchor}} \times \frac{1}{2} \times \sin(\theta_{\text{anchor}} - \alpha_{\text{barrel}}) = 0 \quad (11)$$

Derived from geometric relations:

$$H_{\text{barrel}} = \cos \alpha_{\text{barrel}} \quad (12)$$

The length of each section of the anchor chain is not large, and according to the catenary theory, the anchor chain can be regarded as a uniform linear cable. Taking a small section of anchor chain (ds), the vertical projection component (dy) of anchor chain overhang length can be represented by ds, and the expression y and s can be obtained by integrating from the bottom end of the anchor chain

to the top end, as shown in Figure 4.

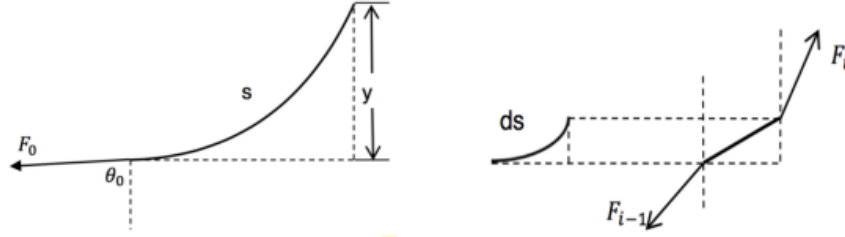


Figure 4: Analysis diagram of anchor chain

$$\sin\varphi = \frac{F_{\text{Straightening up}}}{\sqrt{F_{\text{Straightening up}}^2 + F_{\text{level}}^2}} \quad (13)$$

Then it is concluded that:

$$dy = ds \cdot \sin\varphi = ds \cdot \frac{F_{\text{Straightening up}}}{\sqrt{F_{\text{Straight up}}^2 + F_{\text{level}}^2}} \quad (14)$$

Where, F vertically represents the component force of the tension of the chain element along the horizontal direction, and F horizontally represents the component force of the tension of the chain element along the vertical direction. The static analysis of the anchor chain shows that the component force of the element tension of the anchor chain along the horizontal direction should be equal to the tension F_0 of the anchor on the anchor chain. The element of the anchor chain is subject to gravity and the buoyancy of sea water, so the component force of the tension of the element of the anchor chain along the vertical direction should be equal to the resultant force of the gravity and buoyancy, that is:

$$\begin{aligned} F_{\text{level}} &= F_{\text{anchor}} \sin\theta_{\text{anchor}} = F_0 \sin\theta_0 \\ F_{\text{Straight up}} &= F_{\text{anchor}} \cos\theta_{\text{anchor}} = mgs - \rho gAs + F_0 \cos\theta_0 \end{aligned} \quad (15)$$

The integral of the expression for dy has

$$y = \int_0^s dy = \int_0^s ds \cdot \sin\varphi = \int_0^s ds \cdot \frac{F_{\text{Straight up}}}{\sqrt{F_{\text{Straight up}}^2 + F_{\text{level}}^2}} \quad (16)$$

The above analysis is based on the premise that the entire length of the anchor chain is suspended in the sea water (no part of it lies flat on the seabed). However, when the wind speed is less than the critical value, the chain can be divided into two parts lying flat on the seabed and suspended in the sea water, and the Angle between the former tangent direction and the seabed normal is θ_0 . When θ_0 is just not 90° , the anchor chain is no longer towed, and the length of the anchor chain suspended in the sea water is the actual total length of the anchor chain, as shown in Figure 5.

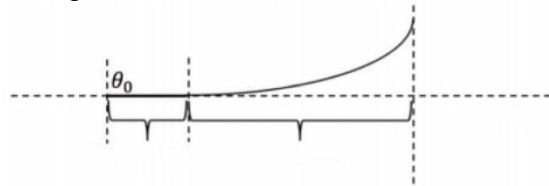


Figure 5: Schematic diagram of mopping the anchor chain

Total length = buoy draft depth + total vertical length of steel pipe + vertical length of steel drum + total vertical length of anchor chain:

$$(2 - h) + \sum_{i=1}^4 H_i + H_{\text{barrel}} + y_{\text{Anchor chain}} = 18 \quad (17)$$

The realv value of the critical wind speed is estimated to be in the range of 10-30m·s⁻¹. Since θ_0 and s are both known quantities, by combining the above equations (1) ~ (26) and narrowing down the two ranges, realv = 21.9200m·S⁻¹ is obtained. The vertical projection length y of each component of the mooring system is equal to the depth of the sea water as the goal, and the elevation h of the buoy system is the independent variable, and the control independent variable is between 0 and 2. By adjusting the independent variables, MATLAB software is used to find the solution that the vertical projection length of each component of the mooring system is closest to the depth of the sea water of 18m by using the cyclic traversal method. The cases of wind speed of 12m/s (anchor chain dragging) and 24m/s (anchor chain not dragging) are discussed respectively. Detailed results are shown in the following in table 1:

Table 1: Structural state of part of the system at 12m/s and 24m/s wind speed

Wind speed		12m/s	24m/s
Steel bucket tilt Angle α bucket		1.209 °	4.584 °
Each steel pipe is tilted	$\alpha 1$	1.163 °	4.429 °
	$\alpha 2$	1.175 °	4.458 °
	$\alpha 3$	1.180 °	4.486 °
Angle alpha i i is equal to 1,2,3,4	$\alpha 4$	1.186 °	4.521 °
Cable shape		6.253m long section lying flat on the seabed; Parametric equation expression of the remaining part: $y = 3.9763 \left(\sqrt{0.2515s^2 + 1} - 1 \right)$	In the initial state, the Angle between the anchor chain and the seabed is 4.561 °; Parametric equation expression: $y = 15.7338 \left(\sqrt{(0.0636s + 0.0798)^2 - 1.0032} \right)$ $x = 15.7338 \left[\ln \left(\sqrt{(0.0636s + 0.0798)^2 + 0.0636s + 0.0798} \right) - 0.4703 \right]$
Draft depth h		0.6816m	0.6958m
Travel area half price R		14.6591m	17.7716m

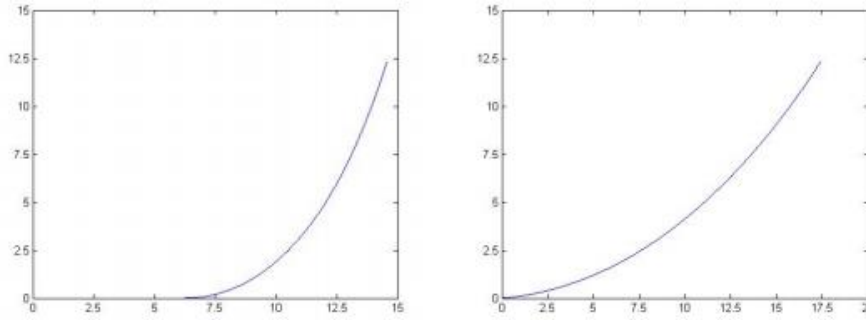


Figure 6: Cable shape diagram at 12m/s and 24m/s wind speed

As shown in Figure 6. The anchor chain is actually composed of 210 chains. In the establishment of the model, we regard the anchor chain as a catenane and use the calculus method to solve it.

$$y = a \left(\cosh \left(\frac{x}{a} \right) - 1 \right) \quad (18)$$

Among them:

$$a = \frac{T_0}{P} \quad (19)$$

T_0 represents the horizontal component of the tension at the top of the anchor chain. Since the stress in the horizontal direction of the anchor chain is balanced, F_0 can be used to represent T_0 . Since θ_0 is small, T_0 is approximately considered equal to F_0 here. When the wind speed is 12m/s, part of the anchor chain is on the seabed, and when the wind speed is 24m/s, all the anchor chain floats, so the comparison is based on the data when the wind speed is 24m/s. The value of T_0 is equal to 941.87 N and the value of P is equal to 68.6 N/m. Draw the images of y and x represented by Model 1 and the catenary equation in the same coordinate system. As shown in Figure 7.

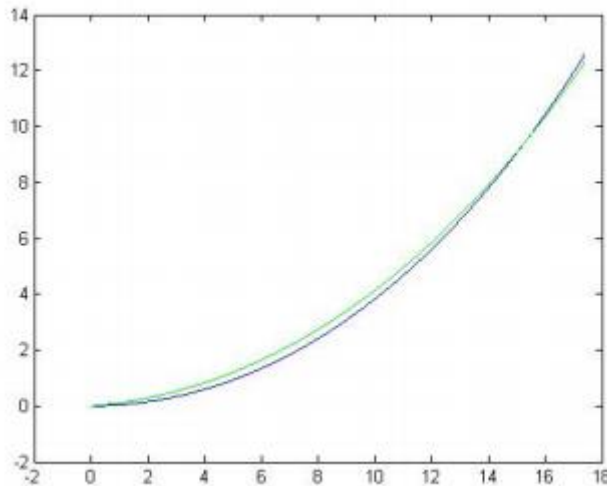


Figure 7: Fitting diagram

It can be seen that the shape of the two curves (the lower curve is the anchor chain shape curve obtained by Model 1) is relatively close, and there is little difference between the y values of 12.3012m (Model 1) and 12.5824m (catenary equation) respectively. After calculation, the parameter values of the mooring system calculated by the catenary equation method are not much different from those calculated by Model 1. The rationality and accuracy of Model 1 are verified from the side.

3. Conclusion

Based on the static analysis and mathematical modeling methods, the stability of ocean mooring system under different wind speed conditions and the optimization design of key components are deeply studied in this study. Mooring system plays a crucial role in ocean engineering, which not only supports the safe operation of various Marine facilities such as buoys and floating wind power generation equipment, but also directly affects the sustainability of Marine environmental protection and resource development and utilization.

In this paper, the stress characteristics of the mooring system under the static equilibrium condition are firstly analyzed. By establishing the static balance and moment balance equations, the key parameters of the tilt Angle of the steel barrel and steel pipe, the shape of the anchor chain, the draft depth of the buoy and its swimming area are accurately calculated when the wind speed is 12m/s and 24m/s respectively. These results not only provide a theoretical basis for optimizing the structural design of the mooring system, but also provide practical guidance for improving its stability and reliability.

By deeply exploring the mechanical properties of the components of the mooring system and their interactions, we have verified the validity and applicability of the proposed mathematical model. At the same time, for the complex nonlinear equation system solving problem, we use a combination of loop traversal method and catenation theory, effectively solve the key parameters, and carry out multiple verification of the results, to ensure the reliability and accuracy of the study.

In summary, this study not only deepens the theoretical understanding of the design optimization of Marine mooring systems, but also provides an important theoretical basis and practical application value for the safety and sustainable development of mooring systems in future offshore engineering.

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