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Exploration and Reflection on the Development of an Ideological and Political Case Repository in Complex Variable Function Courses

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Abstract: This paper examines the development of an ideological and political case base for the Complex Function course, focusing on three key dimensions: mathematical humanities, mathematical aesthetics, and mathematical philosophy. Building upon these aspects, this study integrates ideological and political education throughout the entire teaching process of the course. This approach aims to enhance students' ideological and political awareness while fostering a correct worldview, outlook on life, and set of values alongside the instruction of professional knowledge.

1. Introduction

Complex Function Theory is a mandatory and core course for mathematics majors in colleges and universities. It builds upon and extends the principles of Mathematical Analysis, serving as a foundational pillar for Real Function Theory, Functional Analysis, Differential Equations, Probability Theory, and other advanced disciplines. Consequently, this course holds significant importance for mathematics students. Following the 18th National Congress of the Communist Party of China, the Party and government have placed considerable emphasis on ideological and political education in higher education institutions. In this new era, ideological and political education should prioritize moral development and talent cultivation, aiming to nurture a cadre of high-quality individuals who are both virtuous and competent. Compared with Mathematical Analysis, Complex Function Theory has fewer class hours but presents greater challenges, particularly for students with weaker foundations in Mathematical Analysis, who may develop anxiety towards the subject. To address this issue, the teaching team discussed strategies to enhance teaching methods by integrating ideological and political elements into the curriculum. This approach aims to stimulate students' enthusiasm for learning, foster their spirit of exploration, and cultivate their scientific literacy.

This paper selects the National Excellent Textbook for Higher Education, "Complex Function Theory" authored by Zhong Yuquan, as a case study to explore course-based ideological and political education (IPE) practices. This exploration aims to provide material support for the IPE reform of this course. The main contents of "Complex Function Theory" encompass complex numbers and functions, analytic functions, integrals of complex functions, power series and Laurent series of analytic functions, residue theory, and conformal mapping [1]. Presently, research on IPE in complex function courses is limited. For example, Niu Yingchun examined the integration of IPE elements in the blended teaching reform of "Complex Function Theory" [2], Pang Hongbo conducted preliminary explorations into IPE within the complex function course [3], and Ruan Shihua explored and practiced IPE teaching methods for the complex function course [4]. Consequently, ongoing research focuses on deeply exploring IPE cases and organically integrating them with course knowledge for effective incorporation. As one of the pioneering "demonstration courses of ideological and political education" at our university, it has served as a benchmark and guide in teaching practice, achieving certain results and construction experience. This paper will introduce the basic situation of the IPE case library construction of this course from three perspectives: mathematical humanism, mathematical aesthetics, and mathematical philosophy.

2. The development status of the ideological and political case repository for the complex variable function course

2.1. Case Study in Mathematical Humanities

The development of a discipline is the culmination of the relentless efforts of several generations of scientists. Similarly, the field of complex variable functions has evolved through the meticulous research of mathematicians both domestically and internationally. Numerous humanistic and historical narratives, along with the achievements of these mathematicians, are integrated into the curriculum as ideological and political elements.

Case One: The history of imaginary numbers. In the introduction, we provide a concise overview of the historical development of complex functions, with particular emphasis on the emergence of imaginary numbers. The concept of imaginary numbers first appeared in the 16th century when mathematicians encountered solutions to certain equations that could not be represented using real numbers. In 1545, the Italian mathematician Gerolamo Cardano referred to these newly discovered quantities as "fictitious" and "sophistical", grouping them with negative numbers under the term "false numbers", while positive numbers were termed "true numbers". Their existence sparked considerable debate within the mathematical community. For example, the French mathematician François Viète and his student Thomas Harriot challenged Cardano's views, arguing that although imaginary numbers were difficult to comprehend, they should not be dismissed outright. It was not until the 18th century, through the pioneering work of mathematicians like Leonhard Euler and Carl Friedrich Gauss, that imaginary numbers gained broader acceptance and became an integral part of modern mathematics.

The history of imaginary numbers underscores the importance of exploring uncharted territories in learning and research, daring to question established norms, and persistently seeking solutions. Just as Cardano demonstrated courage and perseverance in confronting imaginary numbers, students should similarly challenge traditional concepts and pursue truth. This spirit fosters academic advancement and cultivates innovative thinking and problem-solving skills.

Moreover, the development of imaginary numbers exemplifies a key feature of scientific progress: many significant mathematical concepts may initially face skepticism or misunderstanding, but over time, through continuous exploration and validation, they become indispensable components of the knowledge system. Therefore, we should encourage students to maintain an open mindset, embrace challenges, and continually seek new knowledge is a crucial educational objective.

Case Two: Introducing the mathematician Euler. When discussing the three representations of complex numbers, we introduced Euler's formula to transform complex numbers from their trigonometric form into exponential form. This transformation provides a solid foundation for further study and deepens students' understanding of complex number theory. Additionally, it introduces Leonhard Euler, an influential figure in the history of mathematics, offering students a historical perspective on these mathematical concepts.

Euler was one of the most prolific mathematicians in history, with contributions spanning analysis, number theory, algebra, mechanics, and physics. He published a total of 886 works and papers during his lifetime, with 40% focused on analysis, number theory, and algebra, and 28% on mechanics and physics. The St. Petersburg Academy of Sciences spent 47 years compiling and publishing his complete works, underscoring the breadth and depth of his contributions.

Euler's life was marked by remarkable resilience. Between 1765 and 1771, he gradually lost his eyesight but continued his research undeterred. Incredibly, during the seven years after becoming completely blind, Euler produced more than half of his remaining works at an astonishing rate. Historical records show that he often worked while holding a child on his lap, demonstrating an indomitable spirit that commands admiration.

In 1748, Euler introduced the famous "Euler's Formula": $e^{i\theta} = \cos\theta + i\sin\theta$. This formula is celebrated as "the most beautiful formula in the world" due to its elegant connection between the trigonometric and exponential forms of complex numbers, revealing profound mathematical relationships. Euler's formula holds significant theoretical importance and has wide-ranging applications in engineering, physics, and other fields.

By studying Euler's life, students can cultivate qualities such as perseverance and courage in the face of difficulties. Euler's story illustrates that with unwavering belief and persistent effort, one can overcome challenges and achieve success. This spirit fosters academic progress and helps develop innovative thinking and problem-solving skills.

Case Three: Understanding the mathematical history of the Cauchy-Riemann equations and the life of Cauchy. When studying the criteria for analytic functions, we introduce and derive the Cauchy-Riemann equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$, which serve as the primary condition for determining analyticity. In teaching, it is beneficial to explain why this equation bears the name "Cauchy-Riemann." In 1746, d'Alembert first mentioned a set of relationships: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ in his work on fluid mechanics. These relationships were later revisited by Riemann in his 1851 paper " The Basic Principles of the Theory of Complex Functions," where he conducted extensive research based on these relations and used them to define analytic functions. The same relationships also appeared frequently in Cauchy's works. For many years, Cauchy struggled to determine the conditions under which the functions he studied could be classified as analytic. It was not until later in his career that he successfully distinguished analytic functions. Consequently, these relationships became known as the "Cauchy-Riemann equations."

Through this historical context, we can delve deeper into the life of Augustin-Louis Cauchy, one of the most influential French mathematicians. Despite achieving early success and fame in his youth, Cauchy's life was marked by significant challenges. Political upheavals forced him into exile, and changes in government limited his professional opportunities. Nevertheless, Cauchy made profound contributions to both pure and applied mathematics, becoming one of the founders of complex function theory. In terms of productivity, Cauchy ranks second only to Euler, having authored an impressive number of papers. Through the story of Cauchy's perseverance in the face of adversity, students can be inspired to set lofty goals and aspirations, while cultivating qualities such as adherence to truth, proactive thinking, rigorous analysis, and courageous exploration.

Case Four: Understanding the mathematician Abel. When discussing the convergence and divergence of power series, students will encounter Abel's Theorem, which plays a crucial role in understanding the radius of convergence of power series. To pique students' interest, it is beneficial to introduce them to Abel's life and contributions. Niels Henrik Abel was one of the greatest mathematicians in history. Despite his tragically short 27-year life, his research made significant contributions to the advancement of science and technology. Unfortunately, his talents and achievements were not fully recognized during his lifetime and were largely overlooked.

Abel's work has had a profound impact on various fields of science. For instance, the famous Yang-Mills theory, developed by Yang Zhenning and American physicist Robert Mills in the late 1950s, relies on concepts derived from Abel's mathematical theories. This theory forms the basis of quantum chromodynamics (QCD). Many Nobel laureates in Physics have utilized Abel's mathematical insights in their groundbreaking research. Even today, Abel's theories continue to be applied in cutting-edge scientific endeavors. A notable example is the discovery of the Higgs boson, often referred to as the "God particle," in 2009 at the world's largest particle collider. Physicist Peter Higgs used Abel's mathematical framework to predict the existence of this new boson.

Through studying Abel's life and contributions, students can reflect on the purpose of human efforts to explore and advance science. Abel's life, marked by adversity and lack of recognition, exemplifies the pursuit of knowledge for its own sake. His dedication to mathematics was driven by a passion for solving complex problems rather than seeking wealth or honor. By exploring Abel's legacy, students can gain insight into the true meaning of scientific endeavor and the importance of perseverance in the face of challenges.

In addition to the mathematicians introduced above, there are numerous other mathematicians whose stories can be explored as ideological and political elements. For instance, the French mathematician Édouard Goursat made significant contributions to function theory and differential equations, profoundly influencing the French mathematical community. The British mathematician Brook Taylor was a pioneer in the development of the theory of finite differences. In China, Hua Luogeng's groundbreaking work on multivariate complex functions, along with Zhang Guanghou and Yang Le's research on the value distribution of complex functions, have all played pivotal roles in shaping modern and contemporary Chinese mathematics. These mathematicians exemplify the dedication and perseverance required to advance scientific knowledge, providing valuable lessons for students.

2.2. Cases of Mathematical Aesthetics

In the process of studying mathematics, learners gradually come to appreciate that mathematics, much like music and poetry, possesses significant aesthetic value.

Case Five: Euler's formula. The Euler's formula mentioned in Case Three, $e^{\pi i}+1=0$, is often regarded as one of the most elegant and perfect formulas in mathematics. This equation elegantly links five fundamental constants—e (the base of the natural logarithm), i (the imaginary unit), π (pi), 1 (the multiplicative identity), and 0 (the additive identity)—in a single, concise expression. The combination of these seemingly unrelated mathematical symbols into one formula is not only aesthetically pleasing but also profound, evoking admiration from mathematicians who view it with the same appreciation as a beautiful poem.

Euler's formula is not only a masterpiece of mathematical beauty but also has significant practical applications. It plays an indispensable role in various fields such as the study of alternating current, signal analysis, quantum mechanics, polar coordinate transformations, the evaluation of improper integrals, and the analysis of circular motion. Through this formula, we can observe that the beauty of mathematics is both abstract and intuitive, rigorous and harmonious. It can inspire students to recognize the charm and elegance of mathematics, thereby fostering enthusiasm and interest in learning the subject.

By appreciating the beauty of mathematics, students are more likely to find the subject engaging rather than dull. Therefore, teachers can leverage students' cognitive experiences of mathematical aesthetics to enhance their aesthetic education, encouraging them to cultivate a deeper interest in mathematics within a pleasant learning environment.

In complex analysis, there are numerous other beautiful formulas and theorems that exemplify the elegance and power of mathematics. For instance, the Cauchy integral theorem reveals that the integral of an analytic function along any closed curve in a simply connected domain is zero—a property that is unimaginable for real functions. This unique characteristic allows complex analysis to solve problems that are difficult or impossible to address within the realm of real numbers. Another remarkable concept is analytic continuation, which extends the domain of an analytic function from a smaller region to a larger one. A notable example is the Riemann zeta function, whose properties derived through analytic continuation have far-reaching implications in modern mathematics and physics.

These stunning and powerful theorems in complex analysis require students to think critically, deeply understand the underlying principles, and appreciate the beauty of the results. By doing so, they can develop a genuine passion for mathematics and experience the joy of exploration even in challenging areas of study.

2.3. Cases in the Philosophy of Mathematics

In the teaching of complex variable functions, there are abundant elements of materialist dialectics embedded within the subject matter. These include the law of unity of opposites, the dialectical relationship between content and form, and the principle of transformation from quantitative change to qualitative change. By exploring these concepts, students can gain a deeper understanding of the inherent philosophical underpinnings that govern mathematical principles.

Case Six: The ideological and political elements contained in the "complex number field".The complex number field, as a natural extension of the real number field, not only infuses new vitality into mathematical theories but also broadens the scope for practical applications.

In the real number domain, certain operations are either impossible or lack meaning. For example, the square root of a negative number is undefined in the real number domain. However, in the complex number domain, not only can the square root be computed, but four distinct fourth roots can also be found. This extension allows many previously unsolvable problems to find solutions within the complex number domain.

The exponential function in the complex field exhibits periodic characteristics. According to Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$, the exponential function forms periodic waveforms on the complex plane, meaning that the value of $e^{i\theta}$ repeats every 2π. This periodicity is a phenomenon unique to the complex domain and does not occur in the real number field.

In the complex domain, sine and cosine functions are no longer confined to bounded ranges. In the real domain, these functions are restricted to values between $[-1,1]$. However, in the complex domain, their values can span the entire complex plane.

The change in the number field can be seen as a transformation of the environment in which functions exist. Extending functions from the real number field to the complex number field provides them with a broader stage, allowing them to exhibit more diverse characteristics and behaviors. Personal development and professional growth often require finding breakthroughs by adjusting one's environment. Just as the transition from the real number domain to the complex number domain brings new opportunities and challenges, changing one's environment can foster self-improvement, entrepreneurship, and employment. By adapting to new environments, individuals can gain more resources and support, thereby better achieving their goals.

Case Seven: The convergence and divergence of complex series reveal the law of quantitative change leading to qualitative change. When studying the convergence or divergence of series with complex terms, we often encounter a phenomenon where, despite each term of the series approaching zero, the entire series may still diverge. This can be aptly described by an ancient saying: "A journey of a thousand miles begins with a single step." This adage not only reveals profound mathematical truths but also embodies rich philosophical insights.

From a mathematical perspective, even if each individual term becomes arbitrarily small, if the number of these terms is sufficiently large, their cumulative effect can still cause the series to diverge. For instance, in the harmonic series $\sum_{n=1}^{\infty} \frac{i}{n}$, although each term tends to zero, the series as a whole diverges. This occurs because, despite the smallness of each term, the cumulative effect of an infinite number of terms leads to an unbounded sum.

This phenomenon exemplifies the principle of transformation from quantitative change to qualitative change in dialectical materialism. According to dialectical materialism, the development of things involves a process where quantitative changes accumulate to a critical point, triggering qualitative changes. Just as water dripping on a stone eventually wears it away, or a rope sawing through wood over time, every small action and incremental progress, after long-term accumulation, can lead to a qualitative leap.

This principle applies not only to mathematics and philosophy but also to various aspects of real life. Whether in personal growth, teamwork, or national development, the importance of accumulating quantity cannot be overstated. As the ancients said: "A tree that fills your arms grows from a tiny sprout; a nine-story platform is built from a heap of earth." Every small effort and every persistent action lays a solid foundation for future success.

For students, this means starting from small things and gradually accumulating efforts. Daily academic progress, adherence to rules, and the cultivation of good habits are all cornerstones of future success. Through continuous accumulation, students can enhance their overall quality and contribute to societal progress and development.

The issue of convergence or divergence in complex series is not just a mathematical concept but also embodies profound philosophical implications. By understanding this phenomenon, we can better grasp the principle of transformation from quantitative to qualitative change and apply it to various aspects of life, motivating ourselves to strive continuously for excellence.

Case Eight: The "uniqueness theorem" implies life philosophy. In complex analysis, if two analytic functions $f_1(z)$ and $f_2(z)$ are equal on a certain sub-region (or a small arc) within a domain D, they must be identical throughout the entire domain D [1]. This is known as the uniqueness theorem. The uniqueness theorem in complex analysis reveals a profound and distinctive property: the local characteristics of an analytic function can reflect its overall behavior. This phenomenon also applies to many aspects of life, where certain key choices or events can have a profound impact on one's overall trajectory. Here are some specific examples:

In terms of academic advancement, a critical decision can significantly influence one's future career. For instance, choosing to pursue a master's or doctoral degree not only affects future employment opportunities, work locations, and income levels but also impacts promotion prospects. Even seemingly minor decisions, such as selecting a particular supervisor or participating in a specific research project, can become pivotal moments in a career. Over time, these small choices accumulate and ultimately shape an individual's professional path.

In academia, the quality of learning in foundational courses during the first year of university plays a crucial role. For example, the mastery of Mathematical Analysis directly influences the success in subsequent advanced courses like Complex Function Theory, Probability Theory, and Ordinary Differential Equations. A solid foundation in Mathematical Analysis facilitates smoother learning and deeper understanding in later studies, while a weak foundation can lead to greater challenges.

Life is replete with similar examples. A significant interview performance, a crucial exam score, or an unexpected social event—these seemingly minor incidents often have far-reaching consequences. A positive first impression can open doors to new opportunities; an outstanding exam result can secure scholarships or recommendation letters; a meaningful social interaction can introduce important connections. The accumulation of these "points" gradually forms the "surface" of one's life, shaping different developmental paths.

The uniqueness theorem in complex function theory not only uncovers profound mathematical truths but also provides valuable insights into life. Just as local information in mathematics can determine global behavior, every critical choice and effort in life can have lasting effects on the future. Therefore, it is essential to value each present decision and effort, as they may be key factors in achieving future success.

Case Nine: Conformal mapping theory reflects the relationship between humans and environment. In the theory of conformal mapping, analytic transformations exhibit several key properties: domain preservation, angle preservation, and scale preservation. Domain preservation ensures that the topological structure of a region remains unchanged before and after the transformation; angle preservation guarantees that angles are conserved during the transformation process; and scale preservation maintains the local scale ratio. These characteristics make analytic transformations highly valuable in complex analysis and geometry. These "preservation" properties facilitate the analysis of transformed figures through invariants, thereby simplifying problems and providing a more intuitive understanding.

This leads us to reflect on the deep-seated relationship between individuals and their environment: how can we better adapt in a constantly changing world? In fact, if we consistently maintain diligence, confidence, and inner kindness, we can navigate any environment, resist temptations, and overcome challenges without losing our way.

Diligence forms the foundation for adapting to changes. Regardless of environmental shifts, continuous effort and relentless pursuit help us find direction amidst uncertainty. For example, in the rapidly evolving technology sector, staying updated with new knowledge and mastering emerging technologies is crucial for maintaining competitiveness. A diligent individual actively seeks opportunities for self-improvement rather than passively waiting for change.

Self-confidence serves as a psychological pillar when facing challenges. Confidence allows us to handle various situations calmly in a dynamic environment. For instance, encountering difficulties at work, self-confidence enables rational thinking and finding optimal solutions. A confident person is not easily swayed by external negative factors but trusts their own abilities and judgment.

Kindness is the cornerstone of personal character. No matter how complex and unpredictable the external environment becomes, inner kindness remains invaluable. Kindness manifests not only in caring for and helping others but also in one's attitude toward oneself. A kind person remains optimistic in adversity, actively seeking solutions rather than complaining or giving up.

Many real-life examples illustrate this point. Scientist Tu Youyou faced numerous failures and setbacks during her research on artemisinin but remained diligent and confident, ultimately discovering a life-saving drug. Her story demonstrates that persistence and a positive mindset can lead to success even in the most challenging circumstances.

The "preservation" properties of analytic transformations and fractional linear transformations in conformal mapping theory reveal profound mathematical truths and offer valuable life lessons. Just as these transformations preserve key attributes in complex geometric transformations, we should cultivate diligence, confidence, and kindness to navigate changing environments. This approach ensures that we can handle any situation calmly and achieve success in all endeavors.

3. Conclusions

Complex function theory, as a core course in higher education institutions, presents an important opportunity to integrate ideological and political education. Appropriately incorporating ideological and political education cases into complex function theory can inject new vitality into the curriculum. This approach not only enhances students' self-worth, national pride, and moral cultivation but also fosters their interest in learning and perseverance in overcoming difficulties. Consequently, it achieves a systematic integration of value guidance, knowledge acquisition, and skill development[5].

The content of complex function theory offers ample opportunities for exploring ideological and political elements. While this article highlights only a few selected cases, there remains significant potential for further exploration. These cases effectively combine scientific content with humanistic spirit, aesthetics, and philosophy, thereby enhancing students' professional learning abilities and ideological and political qualities. They also assist students in establishing correct worldviews, values, and outlooks on life.

In future research and teaching practices of complex function theory, we will continue to explore and refine ideological and political elements. By studying better implementation strategies, we aim to seamlessly integrate these elements into the curriculum, leading to tangible improvements in teaching outcomes and cultivating well-rounded, high-quality talents.

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