

Model-Based Compensation for Sliding Mode Trajectory Tracking Control of Remote Operated Vehicle

Juncheng Zhao^{1,a,*}, Xiaoming Xia^{1,b}, Luoluo Li^{2,c}

¹*School of Ocean Engineering, Jiangsu Ocean University, Lianyungang, Jiangsu, China*

²*Makarov College of Marine Engineering, Jiangsu Ocean University, Lianyungang, Jiangsu, China*

^a*Zhao_Jun_Cheng@126.com*, ^b*xiaxiaoming@jou.edu.cn*, ^c*546427124@qq.com*

**Corresponding author*

Keywords: Remote Operated Vehicle, Model Compensation, Sliding Mode Control, Trajectory Tracking

Abstract: To address the issues of accuracy degradation and insufficient disturbance rejection capabilities faced by underwater robots during trajectory tracking, this paper proposes a sliding mode control method based on model compensation. The method constructs a sliding surface and introduces state error and its dynamic characteristics to design a controller that includes reference velocity and acceleration compensation terms. Using the Lyapunov stability theory, the global asymptotic stability of the closed-loop system was proven. Simulation results show that the proposed method ensures rapid convergence of the trajectory tracking error while demonstrating good robustness and interference resistance, providing an effective means for precise control of underwater robots in complex environments.

1. Introduction

With the increasing depletion of land resources, the development and utilization of marine resources has become an important direction for global development. Against this backdrop, Underwater Remotely Operated Vehicles (ROVs), as key equipment for deep-sea resource exploration and operations, are gradually playing an increasingly important role. It is widely used in underwater environment surveying, underwater target search and identification, and other tasks, and has important application value in marine scientific research, engineering construction, national defense security, and other fields. In actual operation, underwater robots need to run precisely along predetermined trajectories to ensure the stability and reliability of the task[1]-[2]. However, due to the significant dynamic changes in the marine environment, such as complex interference factors like underwater currents and swells, the operational stability and positioning accuracy of underwater robots are often significantly affected. In addition, underwater robots have complex structures and are susceptible to factors such as fluid dynamic disturbances, structural parameter changes, and sensor errors, resulting in highly uncertain and nonlinear motion models. Therefore, how to improve the trajectory tracking performance of underwater robots in complex environments and enhance their robustness to external disturbances and internal model uncertainties has become one of the key issues in current underwater robot control research [3]. Based on the above background, this paper focuses on anti-disturbance control strategies for underwater robot trajectory tracking.

Methods for addressing the issues of parameter variability and external disturbances in underwater robot models can be categorized into active and passive approaches. Active methods introduce observers to estimate unknown disturbances and compensate for them in advance. Passive methods utilize error feedback to design controllers that enhance system robustness. Reference [4] proposed a double-loop sliding mode controller using the inverse tangent function as a new switching term. Reference [5] designed an adaptive fault-tolerant control scheme based on a hyperbolic tangent sliding mode observer, combining the hyperbolic tangent sliding mode observer with projection relaxation technology to solve the problem of deep coupling between external uncertainty and actuator failure encountered by underwater salvage robots during operation. In reference [6], an adaptive sliding mode control scheme combining the superhelix algorithm, low-pass filter, and leakage-type adaptive law is proposed, the LTA law was introduced to estimate the upper bound of unknown disturbances in real time, eliminating the need for prior disturbance information and alleviating the problem of overestimation. In Reference [7], an adaptive fast nonsingular integral terminal sliding mode control (AFNITSMC) method is proposed. Reference [8] proposed a novel adaptive dynamical sliding mode control based methodology to design control algorithms for the trajectory tracking of underactuated unmanned underwater vehicles (UUVs). In Reference [9], The proposed control scheme in this work is based on the conventional SMC approach, however it incorporates higher-order sliding modes in control law that results in improved transient performance and reduces chattering completely while preserving its robustness characteristics. Reference [10] focuses on the design of an adaptive second-order fast nonsingular terminal sliding mode control scheme for the trajectory tracking of fully actuated autonomous underwater vehicles (AUVs) in the presence of dynamic uncertainties and time-varying external disturbances.

Based on the above analysis, this paper proposes a sliding mode controller based on model compensation. First, the reference speed is constructed using the position error and desired speed, and the reference acceleration is constructed using the speed error and desired acceleration; Then, based on position error and velocity error, a sliding mode surface was selected, and a sliding mode controller (MCSMC) with reference velocity and reference acceleration compensation models was designed; Finally, the stability of the system is proven using the Lyapunov direct method, and the effectiveness of the proposed strategy is verified using simulation results. The proposed MCSMC method was compared with simulation tests, verifying that the proposed control method can effectively suppress the jitter of the sliding mode controller and accelerate the convergence speed of the tracking error.

2. Model Establishment and Problem Description

2.1. ROV kinematics model and dynamics model

To better describe the position, velocity, and attitude information of the underwater robot, a body-fixed reference frame ($O\text{-}xyz$) and an inertial reference frame ($E\text{-}XYZ$) are established, as shown in Fig. 1, to characterize the motion of the vehicle. The origin of the body-fixed reference frame is located at the geometric center of the underwater robot's body. The positive direction of the x -axis is the forward direction of the underwater robot, the positive direction of the y -axis points to its right side, and the positive direction of the z -axis points directly below it. The origin of the inertial coordinate system can be selected at any geographical location on Earth as the starting point of motion, with the X -axis pointing north, the Y -axis pointing east, and the Z -axis pointing vertically downward toward the ground [11].

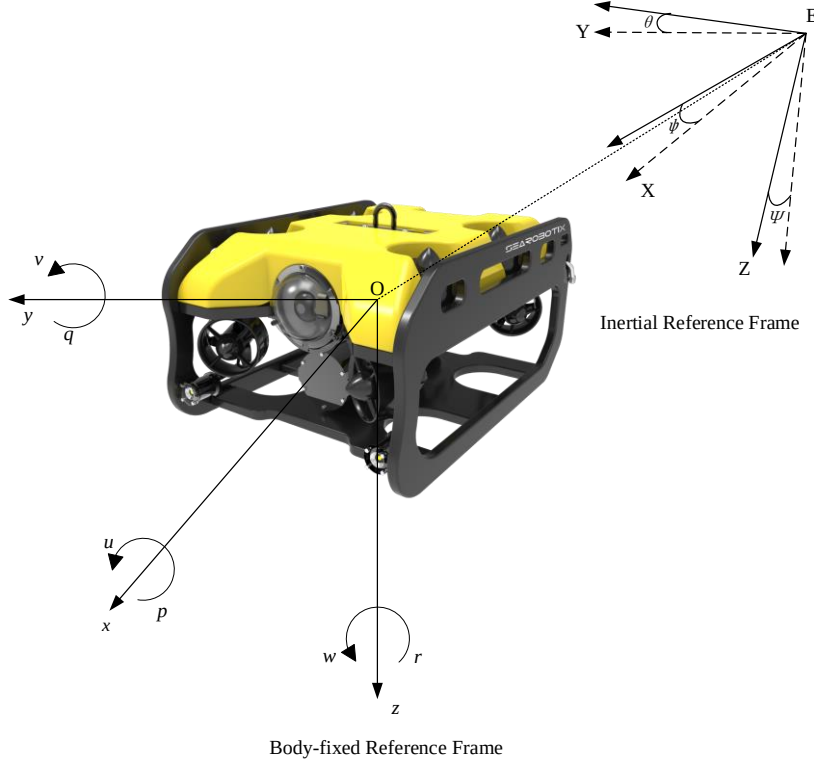


Figure 1: Inertial reference frame and body-fixed reference frame for the ROV
The symbols used for ROV are shown in Table 1:

Table 1 Definition of Parameters in the Coordinate System.

reference system	Position and orientation inertial reference frame	velocity coordinate system	Force and torque body system
surge	x	u	F_x
sway	y	v	F_y
heave	z	w	F_z
roll	φ	p	M_K
pitch	θ	q	M_M
yaw	ψ	r	M_N

For the convenience of research, the influence of water on the carrier is disregarded in the study of rigid-body dynamics, and the analysis is performed in the body-fixed reference frame based on Newton's laws and Euler's equations of motion. Therefore, the underwater robot is analyzed mathematically under the following two assumptions: 1) Underwater robots are considered ideal rigid bodies, and all mechanical forces acting on them can be equated to net external forces (moments); 2) The forces generated by the Earth's rotation do not affect the inertial coordinate system defined on Earth [12]. Therefore, the six degrees of freedom dynamics and kinematics equations of the underwater robot system can be described as:

$$\begin{cases} \dot{\eta} = J(\eta)v \\ M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau_T + \tau_d \end{cases} \quad (1)$$

In the equation: $\eta = [x, y, z, \varphi, \theta, \psi]^T$ represents the position and attitude angle matrix of the

underwater robot in the inertial coordinate system, where and represent the longitudinal, transverse, and vertical displacements, as well as the roll angle, pitch angle, and yaw angle in the inertial coordinate system, respectively. $\eta = [x, y, z, \varphi, \theta, \psi]^T$ is the velocity and angular velocity matrix of the underwater robot in the body coordinate system, where u, v, w, p, q, r are the longitudinal, lateral, and vertical velocities, as well as the roll, pitch, and yaw angular velocities in the body coordinate system. $J(\eta)$ is the pose transformation matrix between the attached coordinate system and the inertial coordinate system; where $M = M_{RB} + M_A$, the inertial tensor M is composed of the sum of the mass inertia matrix M_{RB} and the added mass matrix M_A . $C(v)$ is the coriolis and centripetal matrix; $D(v)$ is the hydrodynamic damping matrix; $g(\eta)$ are the Gravity and buoyancy (hydrostatic) forces; τ_T and τ_d are the external forces and Vector of disturbances, respectively.

2.2. Problem description

The control objective of this paper is to design a model-compensated double-loop sliding mode controller for ROV trajectory tracking under environmental disturbances and model uncertainties, enabling the underwater robot to track a given trajectory at the desired distance and angle, The error between the expected trajectory η_d and the actual value η of the six degrees of freedom is defined as: $\eta_e = \eta - \eta_d$, where $\eta_d = [x_d, y_d, z_d, \varphi_d, \theta_d, \psi_d]^T$ is the expected trajectory.

To verify the controller's ability to ensure satisfactory tracking performance under external disturbances and model uncertainty, this paper compares the conventional sliding mode controller with the model compensation sliding mode controller. Both methods are proven to be asymptotically stable and satisfy Lyapunov's theorem. Based on the kinetics expressed in Equation (1), the following assumptions are made to facilitate controller design and closed-loop system stability analysis.

Assumption 1: The effects of all model uncertainties and external disturbances τ_{di} in the underwater robot system are bounded, i.e., there exists a normal number ξ such that $|\tau_{di}| \leq \xi$, $i = 1, 2, 3, 4, 5, 6$.

Assumption 2: The expected trajectory is a continuous and smooth curve with $\dot{\eta}_d$ and $\ddot{\eta}_d$ being bounded.

Lemma 1: Let V be function mapping the time interval to the real number set $\mathbb{R}$. There exists an inequality equation $\dot{V} \leq -\alpha V + f$, for $\forall t \geq t_0$, where t_0 is a time constant and $t_0 \geq 0$, then

$$V(t) \leq e^{-\alpha(t-t_0)}V(t_0) + \int_{t_0}^t e^{-\alpha(t-\tau)} f(\tau) d\tau \quad (2)$$

In this case, α is a normal number.

If $f = 0$, then the solution for $\dot{V} \leq -\alpha V$ is

$$V(t) \leq e^{-\alpha(t-t_0)}V(t_0) \quad (3)$$

3. Controller Design

3.1. Conventional Sliding Mode Design (LSM)

The conventional sliding mode surface is defined as

$$S = \dot{\eta}_e + \Lambda \eta_e \quad (4)$$

In the formula, $\Lambda > 0$ is the gain coefficient.

Deriving equation (1) yields:

$$\ddot{\eta} = \dot{J}(\eta)v + J(\eta)\dot{v} \quad (5)$$

$$\dot{v} = M^{-1}[\tau + \tau_d - (C(v)v + D(v)v + g(\eta))] \quad (6)$$

Differentiate equation (2) and substitute it into equation (3) and equation (4) to obtain:

$$\begin{aligned} \dot{S} &= \dot{J}(\eta)v + J(\eta)\dot{v} - \ddot{\eta}_d + \Lambda(\dot{\eta} - \dot{\eta}_d) \\ &= \dot{J}(\eta)v + J(\eta)M^{-1}[\tau_T + \tau_d - (C(v)v + D(v)v + g(\eta))] + \Lambda(\dot{\eta} - \dot{\eta}_d) \end{aligned} \quad (7)$$

For equation (5), an exponential approximation law is used, and the switching term is replaced by a saturation function instead of the traditional sign function. The saturation function is defined as follows:

$$\text{sat}(S) = \begin{cases} 1 & S > \Delta \\ tS & |S| \leq \Delta \\ -1 & S < -\Delta \end{cases} \quad t = 1/\Delta \quad (8)$$

In the equation, $1/\Delta$ is the thickness of the boundary layer, and $1/\Delta < 1$ is $\Delta > 1$.

Let $\dot{S} = -\varepsilon \text{sat}(S) - kS$, where $\varepsilon > 0, k > 0$, substitute equation (5) into the exponential approximation law to obtain

$$\dot{J}(\eta)v + J(\eta)M^{-1}[\tau_T + \tau_d - (C(v)v + D(v)v + g(\eta))] + \Lambda(\dot{\eta} - \dot{\eta}_d) + \varepsilon \text{sat}(S) + kS = 0 \quad (9)$$

After organizing the above equation, we obtain the control law τ_T :

$$\tau_T = MJ^{-1}[\ddot{\eta}_d - \Lambda(\dot{\eta} - \dot{\eta}_d) - \dot{J}(\eta)v - \varepsilon \text{sat}(S) - kS] + C(v)v + D(v)v + g(\eta) - \tau_d \quad (10)$$

Let a Lyapunov function candidate V be defined as:

$$V = \frac{1}{2}S^2 \quad (11)$$

Using the exponential approximation law, we can obtain:

$$\dot{V} = \begin{cases} -\varepsilon|S| - kS^2 & |S| > \Delta \\ -\varepsilon tS^2 - kS^2 & |S| \leq \Delta \end{cases} = \begin{cases} -2kV - \varepsilon|S| & |S| > \Delta \\ -2kV - \varepsilon tS^2 & |S| \leq \Delta \end{cases} \quad (12)$$

Clearly, $\dot{V} \leq -2kV$, using Lemma 1, for the inequality equation $\dot{V} \leq -2kV$, there is $\alpha = 2k, f = 0$, whose solution is

$$V(t) \leq e^{-2k(t-t_0)}V(t_0) \quad (13)$$

It can be seen that the $V(t)$ index converges to zero, so that the S index converges to zero, and the convergence speed depends on k . The constant-speed convergence term $\dot{S} = -\varepsilon \text{sat}(s)$ ensures that when S approaches zero, the convergence speed is ε rather than zero, thereby guaranteeing arrival within a finite time.

In index convergence, the convergence speed gradually decreases from a large value to zero, which not only shortens the convergence time but also reduces the speed of the moving point when it reaches the switching surface. Therefore, in order to ensure rapid convergence while weakening oscillation, appropriate parameters k 、 ε are selected, such that k is increased while ε is decreased.

3.2. Model Compensation Sliding Mode Controller Design(MCSMC)

The error between the expected trajectory η_d and the actual value η is defined as $\tilde{\eta} = \eta - \eta_d$, and the error between the expected velocity and angular velocity v_d and the actual values v is defined as $\tilde{v} = v - v_d$.

The reference velocity is defined as $v_r = v_d - \sigma \tilde{\eta}$, the reference acceleration is defined as $\dot{v}_r = \dot{v}_d - \sigma \dot{\tilde{\eta}}$, and the sliding mode function is expressed as follows:

$$S = \tilde{v} + \sigma \tilde{\eta} \quad (14)$$

Among them, σ is a constant array, $\sigma > 0$

The sliding mode control law based on model compensation is designed as follows:

$$\tau = M \dot{v}_r + C v_r + D v_r + g - K_D S \quad (15)$$

Among them, K_D is a positive definite matrix.

Differentiating equation (14) yields:

$$\begin{aligned} \dot{S} &= \dot{\tilde{v}} + \sigma \dot{\tilde{\eta}} \\ &= \dot{v} - \dot{v}_d + \dot{v}_d - \dot{v}_r \\ &= \dot{v} - \dot{v}_r \end{aligned} \quad (16)$$

To analyze the stability of the system, the following two dynamic characteristics of the underwater robot system are given:

Property 1: The inertia matrix M in the underwater robot motion model is a positive definite diagonal matrix, i.e., $M = M^T$, and $M > 0$.

Property 2: $M - 2C$ is a skew-symmetric matrix, i.e., for all $\forall x$, $x^T (M - 2C)x = 0$ holds.

$$V(t) = \frac{1}{2} S^T M S \quad (17)$$

So

$$\begin{aligned} \dot{V}(t) &= S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S = S^T M (\dot{v} - \dot{v}_r) + \frac{1}{2} S^T \dot{M} S \\ &= S^T (M \dot{v} - M \dot{v}_r) + \frac{1}{2} S^T \dot{M} S = S^T (\tau - C v - D v - g - M \dot{v}_r) + \frac{1}{2} S^T \dot{M} S \\ &= S^T (M \dot{v}_r + C v_r + D v_r + g - K_D S - C v - D v - g - M \dot{v}_r) + \frac{1}{2} S^T \dot{M} S \\ &= S^T (C + D)(v_r - v) - S^T K_D S + \frac{1}{2} S^T \dot{M} S = -S^T D S - K_D S^T S + \frac{1}{2} S^T (\dot{M} - 2C) S \\ &= -S^T D S - K_D S^T S \leq 0 \end{aligned} \quad (18)$$

Since $\dot{V}$ is semi-definite and K_D is positive definite, when $\dot{V} \equiv 0$, $S \equiv 0$, according to LaSalle's invariance principle, the closed-loop system is asymptotically stable, and when $t \rightarrow \infty$, $S \rightarrow 0$.

4. Simulation Results

To validate the effectiveness of the model-based compensation sliding mode control method proposed in this paper, a simulation verification method was used to compare the designed controller with a sliding mode controller based on a saturation switching function. Numerical simulation experiments were conducted using simulation software to analyze the positional motion and trajectory motion of underwater robots, with control parameters $\Lambda = \text{diag}\{10, 10, 10, 10, 10, 10\}$, $\varepsilon = \text{diag}\{2.5, 2.5, 2.5, 2.5, 2.5, 2.5\}$, $k = \text{diag}\{3, 3, 3, 3, 3, 3\}$, $\sigma = \text{diag}\{10, 10, 10, 10, 10, 10\}$, $K_d = \text{diag}\{1000, 1000, 1000, 500, 1000, 500\}$.

Based on the standard range defined by the heading, the bow angle of underwater robots is usually limited to a finite range $[-180^\circ, 180^\circ]$ as a physical constraint. This paper focuses on the trajectory movement of underwater robots in the x,y,z directions, and the effect of the bow angle direction on the overall trajectory movement is ignored. In order to verify the underwater trajectory tracking effect, Set the initial position vector $\eta = [-2\text{m}, 8\text{m}, -2\text{m}, 0^\circ, 0^\circ, 0^\circ]^T$ and initial velocity vector $v = [0\text{m/s}, 0\text{m/s}, 0\text{m/s}, 0^\circ/\text{s}, 0^\circ/\text{s}, 0^\circ/\text{s}]^T$ of the ROV. The sampling period is 0.1s, the simulation time is 20s, and the expected trajectory is set as:

$$\begin{aligned} x_d(t) &= \begin{cases} 0.2*t, 0 \leq t < 90 \\ 2*\sin(\pi/20*(t-90)-\pi/2)+20, 90 \leq t \leq 200 \end{cases} \\ y_d(t) &= \begin{cases} 0.2*t, 0 \leq t < 90 \\ 2*\cos(\pi/20*(t-90)-\pi/2)+20, 90 \leq t \leq 200 \end{cases} \\ z_d(t) &= \begin{cases} -0.5*t, 0 \leq t < 90 \\ -0.5*t, 90 \leq t \leq 200 \end{cases} \end{aligned} \quad (19)$$

In order to verify that the controller has good robustness and stability under external disturbances, a simulation comparison was conducted between the model-compensated sliding mode controller and the conventional sliding mode controller. A sine function is used to simulate periodic interference in a real underwater environment, describing a multi-frequency disturbance environment and approximating overall random interference. Random interference is introduced as follows:

$$\tau_d = \begin{cases} \tau_{dx} = 5*\sin(\pi*t/3+7*\pi/15)+4 \\ \tau_{dy} = 2*\cos(\pi*t/2-3*\pi/5) \\ \tau_{dz} = 3*\cos(\pi*t/5-9*\pi/5)-5 \\ \tau_{du} = 5*\sin(\pi*t+7*\pi/15)+7 \\ \tau_{dv} = 2*\cos(\pi*t/4-3*\pi/5) \\ \tau_{dw} = \cos(\pi*t/2-11*\pi/4)-5 \end{cases} \quad (20)$$

The simulation results are illustrated in Figures 2 to 5. Results indicate that while LSM can suppress external disturbances to some extent and achieve basic trajectory tracking, it exhibits significant chattering during the system's transient phase. This leads to large fluctuations in control inputs, resulting in insufficient tracking accuracy for position and attitude, as well as slow

convergence of steady-state error. In contrast, the MCSMC incorporates a nonlinear convergence mechanism and a combined sliding surface design into its control law. This enables the system to exhibit enhanced robustness and faster error convergence characteristics during dynamic processes.

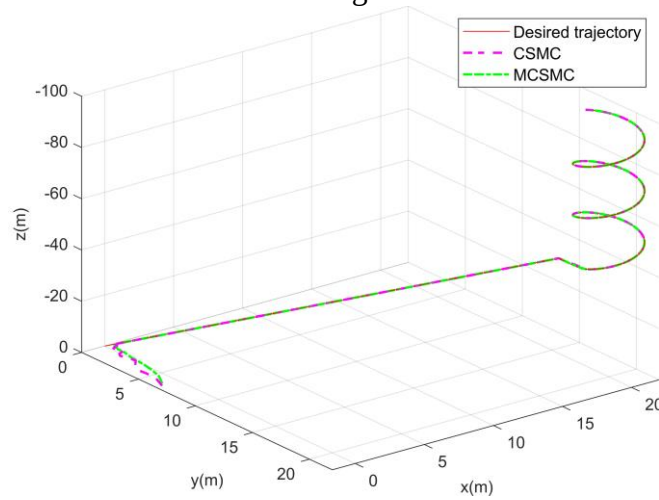


Figure 2: 3D trajectory tracking curve of underwater robot

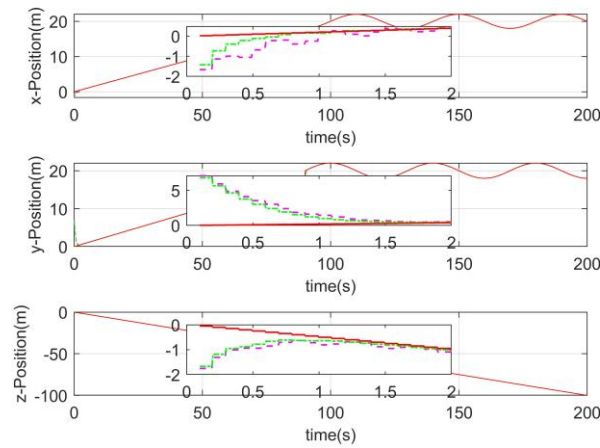


Figure 3: Pose tracking curves

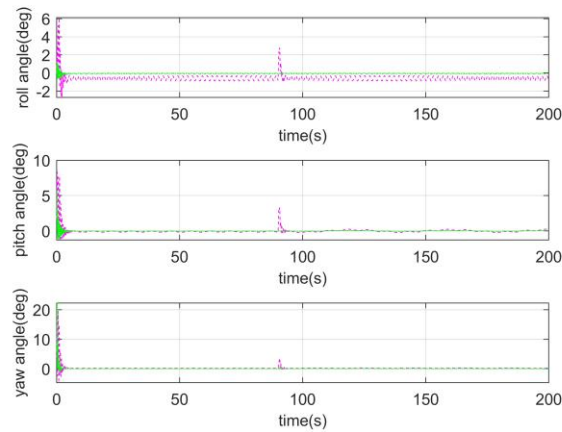


Figure 4: Posture tracking curves

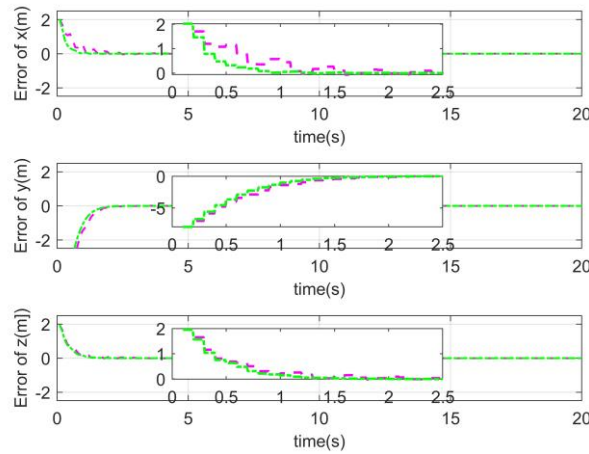


Figure 5: Position tracking error curve

Specifically, as shown in Figure 2-4, the LSM exhibits noticeable chattering and overshoot during the system transient phase, with residual error persisting in the steady-state phase. In contrast, the MCSMC suppresses high-frequency chattering caused by sliding mode switching by introducing reference velocity and acceleration compensation terms. This enables the attitude and position curves to converge rapidly and smoothly toward the desired values, with significantly reduced overshoot. As shown in Figure 5, the comparison of positional errors further validates the aforementioned conclusions. The error curve of LSM exhibits significant fluctuations and converges slowly, whereas MCSMC converges the error to near zero within a finite time and maintains steady-state error within ± 0.05 . Overall convergence speed improves by approximately 40% compared to LSM while maintaining higher control accuracy.

5. Conclusion

Given the characteristics of external interference and system uncertainty in the motion control of underwater robots, a model-compensated sliding mode controller was designed to achieve spatial trajectory tracking of underwater robots. Using the Lyapunov method, the global stability of the sliding surface was theoretically proven, and the tracking error ultimately converged to zero within a finite time. A simulation comparison was conducted between the designed MCSMC controller and the LSM controller. The results showed that the MCSMC control strategy significantly improved the accuracy and convergence speed of trajectory tracking, suppressed overshoot and oscillation during the control process, and was more suitable for actual control scenarios requiring high precision and reliability. Subsequent steps will involve optimizing and improving parameter tuning, combined with actual testing for verification, linking theory with practice, and incorporating adaptive control algorithms to address more complex operating conditions.

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