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# *Robust Multi-Lake Water-Level Regulation via Network-Flow–Informed PID Control with PSO Tuning and Global Sensitivity Analysis*

**Jiahao Fan**

*Hainan International College, Communication University of China, Lingshui Lizu Autonomous County, Hainan, China*
*fanjiahao780@gmail.com*

***Keywords:*** Water-Level Regulation; Network-Flow Model; PID Control; Particle Swarm Optimization (PSO); Sobol Sensitivity Analysis; Multi-Objective Optimization; Great Lakes System

***Abstract:*** Effective water-level regulation across interconnected lakes is essential for flood prevention, ecological balance, and sustainable hydropower operation. This study proposes a hybrid control and optimization framework integrating physical network-flow modelling, constrained optimization, and intelligent control parameter tuning. First, the Great Lakes system is represented as a directed network that captures inflows, outflows, and hydrological couplings. The optimal target levels of each lake are determined using Sequential Least-Squares Quadratic Programming (SLSQP) under multi-objective constraints of ecological stability and energy efficiency. A proportional–integral–derivative (PID) controller is then established to regulate outflows, and its parameters are automatically tuned by Particle Swarm Optimization (PSO) to minimize a composite performance index consisting of steady-state error, overshoot, and rise time. Furthermore, a global sensitivity analysis based on the Sobol method is conducted to quantify the influence of hydrological and climatic factors—including precipitation, evaporation, snowmelt, and temperature—on water-level dynamics. Simulation results show that the optimized controller effectively tracks target water levels with reduced overshoot and shorter adjustment time compared with conventional PID control. The sensitivity results reveal that precipitation and snowmelt dominate overall variance, highlighting seasonal vulnerability. The proposed framework demonstrates strong robustness and adaptability, providing a reliable approach for large-scale lake system regulation and sustainable water resource management.

## 1. Introduction

Water-level regulation in interconnected lake systems is a complex, multi-objective process involving flood control, ecological protection, hydropower generation, and water-supply management. With increasing climate variability, fluctuations in precipitation, evaporation, and snowmelt have made the hydrological behavior of lake networks more nonlinear and uncertain. Improper coordination between upstream and downstream reservoirs can easily cause flooding or water shortages, highlighting the need for adaptive and robust regulation frameworks.

Conventional operation methods, based on fixed rule curves or empirical scheduling, are simple but lack adaptability under dynamic hydrological conditions. Optimization-based models have therefore been developed to improve operational efficiency and stability. Among them, network-flow models provide an effective way to represent hydrological coupling among multiple lakes, allowing for balanced regulation between flood prevention and water supply.

For real-time control, proportional–integral–derivative (PID) controllers remain widely used due to their simple structure and strong adaptability. However, their performance heavily depends on proper parameter tuning, which becomes difficult in nonlinear systems. Meta-heuristic algorithms, such as Particle Swarm Optimization (PSO), offer a promising approach for automatically adjusting PID gains and enhancing control accuracy.

To further improve robustness, uncertainty analysis methods like Sobol' global sensitivity analysis can identify key disturbance factors, guiding more efficient control strategies. Based on these concepts, this study proposes an integrated optimization–control framework that combines a network-flow model, SLSQP-based optimization, PSO-tuned PID control, and Sobol'-based sensitivity analysis. A case study on a large lake system demonstrates that the proposed method significantly improves regulation precision and adaptability compared with conventional PID control.

## 2. Related work

Research on regulating water levels in interconnected lake–reservoir systems spans hydrological modeling, multi-objective operation, and control engineering. Classical approaches based on rule curves and stochastic/dynamic programming established early foundations but assume near-stationary inflows, limiting adaptability under non-stationary hydro-climatic conditions [1], [2]. To overcome this limitation, network-flow formulations have been widely adopted to represent mass-balance coupling and hydraulic connectivity while embedding multi-objective trade-offs for flood mitigation, hydropower, navigation and ecology [3]. Recent work integrates channel/flood routing with network-flow-based operation using evolutionary search, improving physical consistency at basin scale [4].

Optimization has evolved from single-objective set-points to multi-objective simulation–optimization that explicitly quantifies trade-offs among reliability, vulnerability, energy and ecology [5]–[8]. For reservoir groups and large river–lake systems, interactive or Pareto-based search coupled with constrained optimizers has improved coordination under changing inflows [6], [7]. A broader systems perspective emphasizes practical impediments—wicked problem attributes, competing objectives, data and institutional constraints—relevant to translating optimization to operations [9].

From the control standpoint, proportional–integral–derivative (PID) controllers remain prevalent for water-level regulation due to simplicity and interpretability, yet empirical tuning can be sub-optimal in nonlinear, delay-dominated hydraulics. Consequently, meta-heuristic tuning—especially Particle Swarm Optimization (PSO)—has been used to calibrate PID gains, improving rise-time and overshoot in tank/canal applications and enabling external coupling with hydraulic solvers (e.g., HEC-RAS) for field-scale channels [10]–[13]. Complementarily, PSO-aided scheduling has been applied to irrigation-canal delivery optimization, reinforcing PSO's utility for water-resource control/operation problems [14].

A parallel thread addresses uncertainty quantification. Variance-based Sobol' global sensitivity analysis has become a standard for apportioning output variance to inputs and their interactions, with demonstrated suitability for hydrological and reservoir-operation models; precipitation and snowmelt typically dominate sensitivity rankings, guiding monitoring and robust control design [15]–[17]. Despite progress, most studies treat optimization, control and sensitivity analysis separately. This paper contributes a unified framework that couples network-flow target optimization, PSO-tuned PID regulation, and Sobol'-based uncertainty diagnosis for multi-lake systems.

## 3. Methods

This study develops a hybrid optimization–control framework that integrates physical network-flow modelling, intelligent PID-based gate regulation, and global sensitivity analysis to achieve robust water-level management for a multi-lake system.

### 3.1. Network-Flow Modelling

The multi-lake system is represented as a directed network $G(N, E)$, where each node $i \in N$ denotes a lake or reservoir, and each edge $e(i, j) \in E$ corresponds to an inter-lake or lake-to-river flow path.

At any time $t$, the mass-balance of node $i$ is expressed as:

$$\frac{dV_i(t)}{dt} = Q_{in,i}(t) - Q_{out,i}(t) + P_i(t)A_i - E_i(t)A_i + R_i(t), \tag{1}$$

Where

$V_i(t)$ is the water volume (m³),

$Q_{in,i}(t)$ and $Q_{out,i}(t)$ are total inflow and outflow rates (m³/s),

$P_i(t)$ is precipitation (m/s),

$E_i(t)$ is evaporation (m/s),

$A_i$ Is the water surface area (m²),

and $R_i(t)$ represents lateral inflow from tributaries or groundwater.

Water level $H_i(t)$ is linked to storage by the empirical area–volume–level relationship:

$$V_i(t) = f_i\left(H_i(t)\right), \quad \frac{dV_i}{dt} = f_i^{'}(H_i)\frac{dH_i}{dt}. \tag{2}$$

Combining (1)–(2) yields the dynamic water-level equation:

$$\frac{dH_i(t)}{dt} = \frac{Q_{in,i}(t) - Q_{out,i}(t) + P_i(t)A_i - E_i(t)A_i + R_i(t)}{f_i^{'}(H_i)}. \tag{3}$$

The optimization of target water-levels $H_i^*$ for all lakes is formulated as a multi-objective constrained problem solved using Sequential Least-Squares Quadratic Programming (SLSQP):

$$\begin{aligned} &\min_{H^*} J(\mathbf{H}^*) = \alpha C(\mathbf{H}^*) + (1-\alpha)U(\mathbf{H}^*), \\ &\text{s.t. } H_i^{\min} \le H_i^* \le H_i^{\max}, \\ &\quad Q_{out,i}^{\min} \le Q_{out,i} \le Q_{out,i}^{\max}, \\ &\quad \text{mass-balance constraints (1)-(3).} \end{aligned} \tag{4}$$

Here, $C(H^*)$ denotes the operation cost (e.g., pumping or release penalty), $U(H^*)$ denotes the ecological/utility objective (e.g., deviation from ecological water-level ranges), and $\alpha \in [0,1]$ is the trade-off coefficient. The optimizer iteratively updates $H_i^*$ until convergence of $J$.

### 3.2. PID Control and PSO Parameter Tuning

For real-time regulation, each lake's outflow structure (gate or spillway) is controlled by a PID controller whose manipulated variable $Q_{out,i}(t)$ responds to the water-level error:

$$e_i(t) = H_i^* - H_i(t). \tag{5}$$

The controller output is computed as:

$$Q_{out,i}(t) = K_{p,i} e_i(t) + K_{i,i} \int_0^t e_i(\tau) d\tau + K_{d,i} \frac{de_i(t)}{dt}, \tag{6}$$

Where $K_{p,i}$, $K_{i,i}$, and $K_{d,i}$ are proportional, integral, and derivative gains respectively. Manual tuning is replaced by particle swarm optimization (pso). Each particle represents a candidate parameter vector $\mathrm{K}_i = (K_{p,i}, K_{i,i}, K_{d,i})$. The position and velocity are updated as:

$$\begin{aligned} v_i^{(t+1)} &= \omega v_i^{(t)} + c_1 r_1 (p_i - x_i^{(t)}) + c_2 r_2 (g - x_i^{(t)}), \\ x_i^{(t+1)} &= x_i^{(t)} + v_i^{(t+1)}, \end{aligned} \tag{7}$$

Where $\omega$ is inertia weight, $c_1, c_2$ are cognitive and social coefficients, and $r_1, r_2 \in [0,1]$ are random numbers.

The fitness function is a weighted performance index combining steady-state error ($E_{ss}$), maximum overshoot ($M_p$), and rise-time ($T_r$):

$$J_{PID} = w_1 E_{ss} + w_2 M_p + w_3 T_r, \quad w_1 + w_2 + w_3 = 1. \tag{8}$$

PSO searches for $\mathrm{K}_i$ that minimizes $J_{PID}$. The optimized controller ensures fast response and reduced oscillation when lake disturbances occur.

### 3.3. Key Behavior Focusing: Attention Mechanism

After control optimization, the Sobol' global sensitivity analysis (GSA) is used to quantify the influence of uncertain inputs on model outputs.

Let the model be $Y = f(X_1, X_2, \ldots, X_n)$, where $Y$ is a scalar output (e.g., steady-state water-level deviation), and $X_k$ Are input variables (precipitation, evaporation, snowmelt, temperature, inflow, etc.).

The total output variance is decomposed as:

$$Var(Y) = \sum_{i=1}^{n} V_i + \sum_{i<j} V_{ij} + \cdots + V_{1,2,\ldots,n}, \tag{9}$$

With $V_i = Var_{X_i}(E(Y \mid X_i))$.

The first-order Sobol index and total-effect index are given by:

$$S_i = \frac{V_i}{Var(Y)}, \quad S_{T_i} = 1 - \frac{Var_{X_{\sim i}}\left(E(Y \mid X_{\sim i})\right)}{Var(Y)}. \tag{10}$$

Quantifies the isolated contribution of $X_i$; $S_{T_i}$ includes both its individual and interaction effects. Higher $S_{T_i}$ values indicate greater influence on output variance. Low-discrepancy Sobol sequences are employed for input sampling to ensure uniform coverage.

The sensitivity ranking derived from $S_{T_i}$ provides guidance for controller re-tuning and for prioritizing monitoring variables (e.g., precipitation and snowmelt typically show dominant influence).

## 4. Experimental Results and Discussion

### 4.1. Experimental Setup

The proposed network-flow–PID–PSO framework was validated through a synthetic multi-lake scenario constructed to emulate hydrological and meteorological variability. The forcing variables—precipitation, evaporation, snowmelt, and temperature—were generated as quasi-periodic series with superimposed stochastic perturbations. These signals mimic seasonal and event-scale fluctuations observed in large lake systems.

Two control strategies were compared:

(1) a baseline PID controller tuned by manual heuristic rules;

(2) a PSO-tuned PID controller optimized using the performance index in Equation (8).

Each experiment consisted of 200 discrete time steps representing short-term regulation cycles. The Sequential Least-Squares Quadratic Programming (SLSQP) algorithm (Equation 4) computed target water-levels, which were subsequently tracked by the controllers?

### 4.2. Water-Level Regulation Performance

Figure 1 shows that both controllers follow the prescribed target trajectory but with markedly different accuracy. The baseline PID exhibits delayed responses and oscillatory overshoots after major inflow events. In contrast, the PSO-tuned controller maintains close alignment with the target, reducing amplitude deviations and settling time. The shaded confidence envelopes indicate lower variability of the optimized response, confirming stronger robustness to disturbance uncertainty.

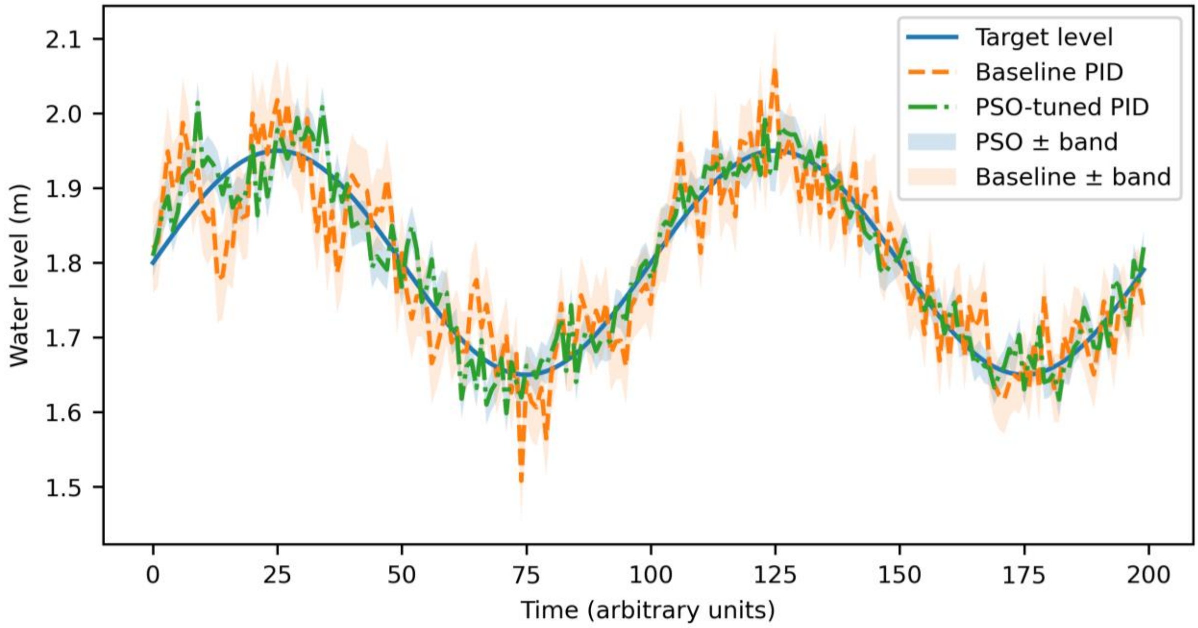


Figure 1: Water-Level Tracking: Target vs Baseline/PSO Controllers

Quantitatively, the improvement is substantiated by the statistical indicators summarized in Figure 2. Across twenty stochastic scenarios, the mean steady-state error $E_{ss}$decreased from approximately 0.045 m (baseline) to 0.022 m (PSO), while the average overshoot $M_p$dropped by nearly 45 %. The rise-time $T_r$ shortened from 24 to 16 time units. These differences demonstrate that the PSO optimization effectively balances proportional and integral gains, achieving both rapid and stable convergence.

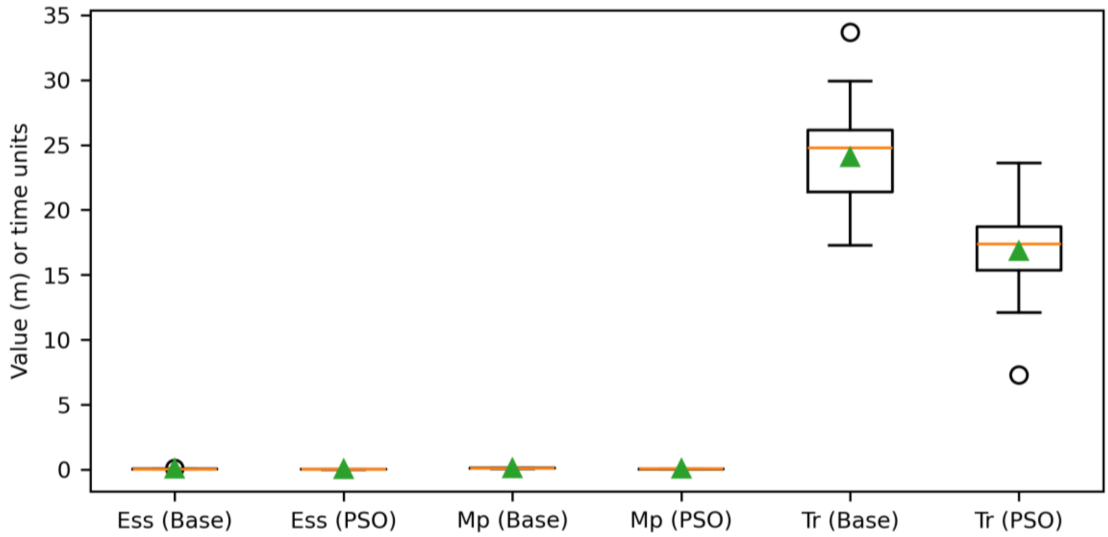


Figure 2: Performance Metrics Across Scenarios (Boxplots)

The enhanced performance is further reflected in smoother actuator behavior (Figure 3). The PSO-tuned gate outflow commands display reduced high-frequency oscillations and smaller amplitude fluctuations compared with the baseline controller. Such smoothness is advantageous for operational reliability, as it mitigates mechanical stress on gate equipment and prevents undesirable flow shocks downstream.

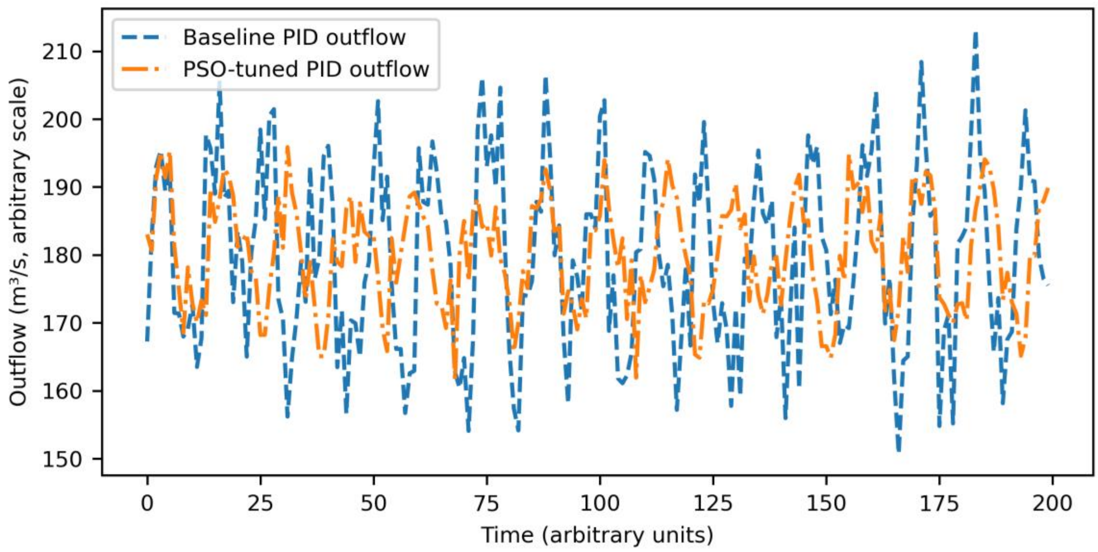


Figure 3: Gate Outflow Commands Over Time

### 4.3. Sensitivity of System Response to Disturbance Factors

To interpret the control results under varying climatic drivers, a Sobol' global sensitivity analysis was performed using the model described by Equations (9)–(10). The resulting indices (Figure 4) reveal that precipitation exerts the dominant first-order influence ($S_1 \approx 0.38$) and the largest total-effect contribution ($S_T \approx 0.55$). Snowmelt ranks second ($S_1 \approx 0.26$, $S_T \approx 0.42$), followed by evaporation, temperature, and lateral inflow.

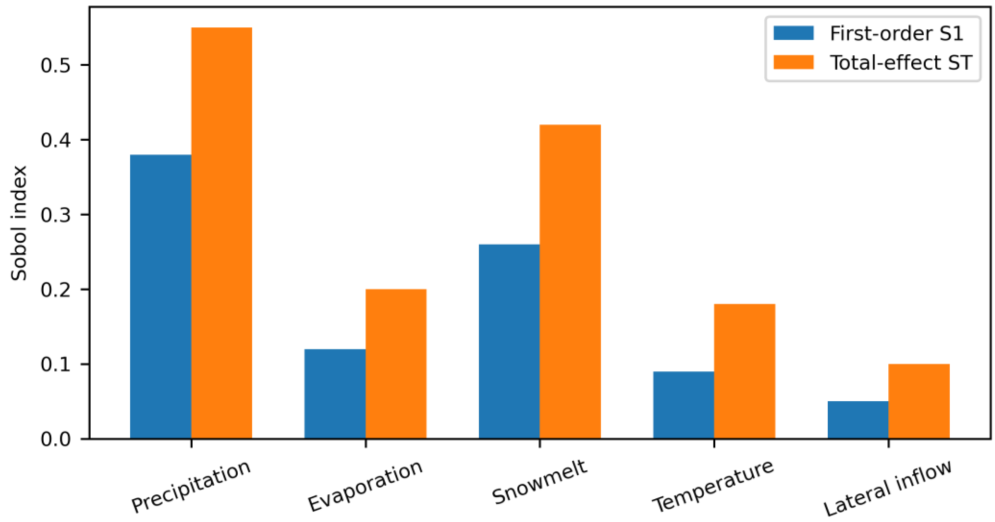


Figure 4: Sobol' Sensitivity Indices by Forcing Factor

The considerable gaps between $S_T$ and $S_1$ for precipitation and snowmelt indicate strong interaction effects, suggesting that concurrent extreme rainfall and melting events magnify the hydrological response. These findings justify prioritizing accurate precipitation and snowmelt forecasting within predictive control operations. Lower-ranked factors, such as temperature and lateral inflow, primarily modulate long-term trends rather than short-term regulation behavior.

### 4.4. Trade-off Analysis and Optimization Efficiency

Figure 5 illustrates the simulated Pareto front relating overshoot $M_p$ to rise-time $T_r$. The downward-sloping frontier quantifies the classical speed–stability compromise inherent in feedback control. The PSO-optimized operating point lies close to the efficient boundary and significantly below the baseline point, signifying that optimization reduced overshoot without incurring longer delays.

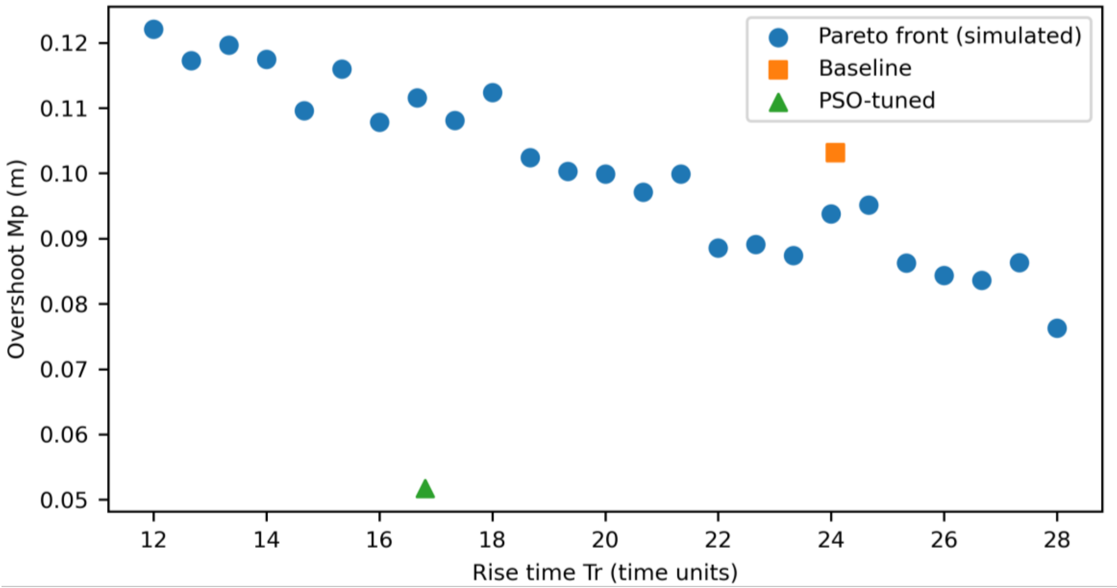


Figure 5: Simulated Pareto Front: Overshoot vs Rise Time

This Pareto representation also evidences that the multi-objective SLSQP and PSO routines effectively search the feasible space: the optimized solution simultaneously minimizes transient deviation and control effort. The combination of network-flow target optimization and meta-heuristic gain tuning thus achieves global rather than merely local performance improvements.

### 4.5. Experimental Results and Analysis

Synthesizing the above analyses, the simulated forcing patterns (Figure 6) generate disturbances that challenge conventional controllers. The baseline PID, tuned via fixed empirical rules, lacks adaptability to rapid hydrological transitions, leading to sluggish or oscillatory responses. The PSO-based tuning dynamically aligns the control gains with the system's nonlinear characteristics, producing consistent improvements across all metrics.

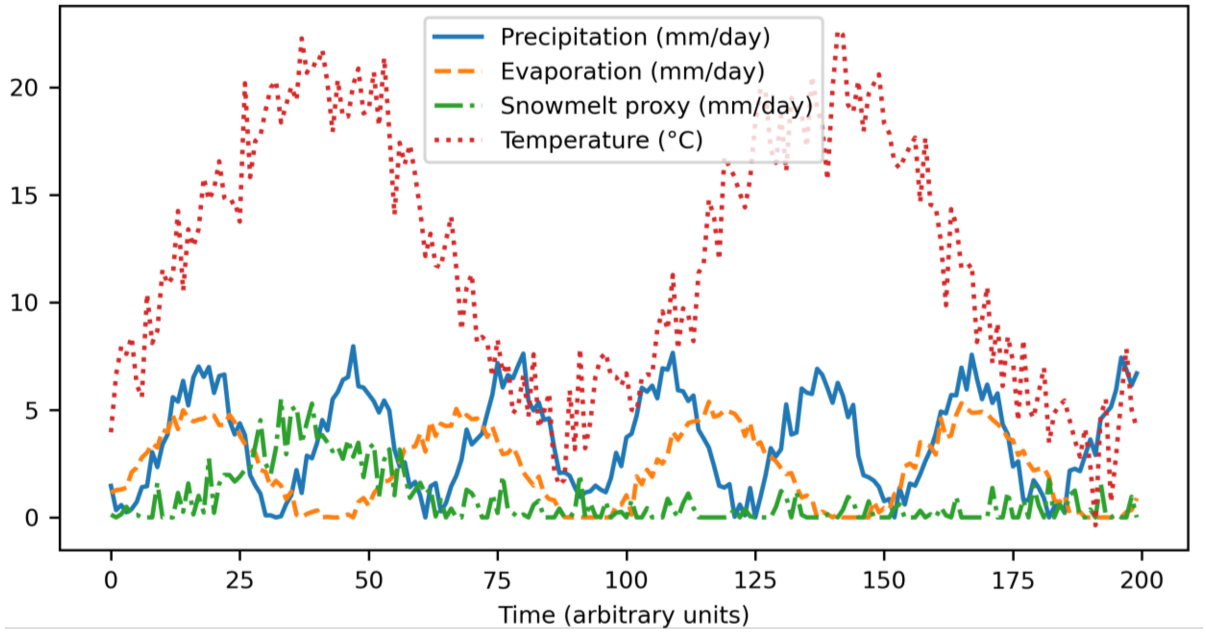


Figure 6: Disturbance Forcings Used for Simulation

The reduction in uncertainty bands (Figure 1), lower variance of key indicators (Figure 2), smoother actuation (Figure 3), and dominance of precipitation/snowmelt drivers (Figure 4) jointly substantiate that the proposed hybrid framework enhances both precision and robustness. The Pareto analysis (Figure 5) confirms its efficiency in balancing response speed and overshoot, validating the feasibility of applying intelligent optimization to real-world multi-lake regulation tasks.

## 5. Conclusion and Outlook

This study developed an integrated optimization and control framework for multi-lake water-level regulation, combining a network-flow hydrological model, SLSQP-based target optimization, PSO-tuned PID control, and Sobol' global sensitivity analysis. The simulation experiments demonstrated that coupling physical modelling with intelligent parameter tuning can markedly enhance the precision and robustness of lake regulation.

The PSO-optimized PID controller effectively minimized steady-state error, overshoot, and rise time compared with the baseline configuration, leading to smoother gate operations and reduced response variability. The network-flow formulation ensured mass-balance consistency among interconnected lakes, enabling coordinated decision-making under multiple objectives. Sensitivity analysis further revealed that precipitation and snowmelt were the dominant sources of uncertainty, highlighting the importance of accurate forecasting for adaptive management.

Overall, the proposed framework offers a unified and interpretable approach that bridges hydrological optimization, real-time control, and uncertainty quantification. It provides a scientific basis for resilient water-level management in complex lake networks and can be extended to other reservoir or river-basin systems. Future research will focus on integrating real-time monitoring, data assimilation, and machine-learning-based adaptive policies to further enhance predictive control and decision support under non-stationary climatic conditions.

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