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Linking Problem Solving and Posing: Constructing a Teaching Model for Mathematical Problem Posing in Junior Secondary Schools under Polya's Framework

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Abstract: In response to the global emphasis on fostering problem-posing skills in mathematics education, this study constructs a teaching path based on George Polya's heuristic problem-solving philosophy, integrated with the "What-If-Not" strategy, to develop junior secondary students' problem-posing abilities. Using the "Similar Triangles" unit as an example, the path designs a series of instructional activities ranging from situational engagement and systematic questioning to creative expression. The study demonstrates that this path effectively bridges problem solving and problem posing, offering a viable theoretical and practical model for systematically implementing problem-posing instruction in mathematics classrooms.

1. Introduction

The ongoing reform in global mathematics education has increasingly highlighted "problem posing" as a vital skill for nurturing students' innovative thinking and core mathematical literacy. This shift is reflected in the curriculum standards of many nations. For example, China's "Compulsory Education Mathematics Curriculum Standards (2022 Edition)" explicitly identifies "discovering and posing problems" as a key component of mathematical core literacy, emphasizing the need for students to actively construct and formulate questions within real-world contexts and apply mathematical knowledge comprehensively to address them [1]. This focus aligns with international trends; the National Council of Teachers of Mathematics (NCTM) in the United States, for instance, has underscored the value of problem posing since its 1989 standards [2]. Despite this policy emphasis, cross-national comparative studies indicate significant variation in implementation. While countries like China and the U.S. explicitly integrate problem posing into their standards, others such as Singapore, England, and Germany incorporate it more sparingly, often within specific domains like mathematical modeling [3-4].

A noticeable gap persists between curricular aspirations and classroom reality. In many junior high school mathematics classrooms, instruction remains heavily skewed towards problem-solving, with limited systematic effort dedicated to cultivating students' capacity to pose original questions [5]. This tendency constrains the development of student initiative and creativity, which runs counter to the student-centered pedagogical philosophy promoted by contemporary educational reforms [6]. Research by Guo Yufeng et al. points out that even in contexts where problem posing is mandated by standards, its representation in textbooks is often minimal and unevenly distributed, frequently relegated to exercise sections rather than being woven into the core instructional content [7].

The heuristic teaching philosophy articulated by George Pólya in his seminal work “How to Solve It” offers a robust theoretical foundation for addressing this gap. Pólya advocated for a problem-solving process guided by a series of self-posed heuristic questions—such as "Do you know a related problem?" or "Can you restate the problem?"—thereby activating students' mathematical thinking and curiosity through inquiry [8]. His approach concerns not merely the final solution but, more profoundly, values the cognitive process involved in generating questions, embodying a teaching philosophy where "problem-solving" and "problem-posing" are mutually reinforcing [9]. Consequently, the creative application of Pólya's ideas to the teaching of problem posing in junior high school mathematics holds significant practical and theoretical value.

Guided by Pólya's mathematical thinking and confronting the current challenges in problem-posing instruction, this study seeks to construct systematic pathways and methods for effectively fostering students' problem-posing abilities. Using the pivotal "Similar Triangles" unit as a practical case, it designs a comprehensive instructional pathway and demonstrates applicable teaching strategies. The aim is to furnish frontline mathematics teachers with actionable and replicable instructional models, thereby contributing to the transformation of classroom teaching and supporting the successful implementation of core literacy-oriented curriculum objectives.

2. Analysis of Problem Posing Elements in Polya's Problem-Solving Ideas

Pólya's “How to Solve It” provides a systematic framework of heuristic thinking, whose core mechanism involves guiding the problem-solving process through strategic self-questioning. This framework is not merely a tool for solving given problems but also a fertile ground for generating new ones. The questions Pólya proposes inherently contain elements that stimulate and scaffold problem formulation. This section distills these elements into a structured set of seven core strategies, translating Pólya's problem-solving heuristics into a practical framework for fostering problem-posing capabilities in learners.

2.1. Generating Questions via Analogical Reasoning.

Pólya frequently prompts: "Do you know a related problem?" This encourages students to draw connections between the current challenge and previously encountered knowledge or similar structures. For instance, when introducing similar triangles, teachers can guide students to activate their prior knowledge of congruent triangles. This analogical thinking naturally leads to new questions: "How is similarity different from congruence?" "If congruence requires identical shape and size, does similarity only require identical shape?" "Can the criteria for congruence (SSS, SAS, ASA) be adapted or relaxed to define similarity?" This strategy leverages existing cognitive schemas to facilitate the understanding of new concepts while simultaneously generating meaningful comparative inquiries.

2.2. Reformulating Problems to Uncover New Perspectives.

The heuristic "Can you restate the problem?" is a powerful tool for problem posing. Changing the mode of representation—for example, translating a symbolic equation into a verbal story problem, or redefining a geometric concept using different properties—can reveal unforeseen aspects and questions. Consider the algebraic equation $2x+3=7$. Restating it as "I am thinking of a number. Doubling it and adding three gives seven. What is my number?" makes the concept of inverse operations more intuitive. Similarly, defining a parabola not just as a quadratic graph but as "the set of all points equidistant from a fixed point (focus) and a fixed line (directrix)" can spark investigations into its geometric properties and relationships.

2.3. Decomposing Conditions to Formulate Sub-Problems.

Confronted with a complex problem, Pólya advises "separating the various parts of the condition." This involves breaking down the overall problem into its constituent assumptions and constraints, each of which can be examined individually to spawn a series of sub-problems. In proving two triangles are congruent, for example, students can be guided to decompose the overarching goal into smaller, investigable questions: "What if we only know three sides are equal (SSS)?" "Is two angles and one side sufficient? If so, which side (ASA vs. AAS)?" "Could two sides and one angle work? Under what conditions (SAS) and when might it fail (SSA)?" This methodical decomposition helps manage complexity and guides students toward constructing the complete proof concept by concept.

2.4. Employing Specialization and Generalization for Inquiry.

Pólya highlights considering special cases (specialization) and extending to broader contexts (generalization) as key discovery methods. Examining a specific, often simpler, instance of a general concept can illuminate pathways and raise questions about the general rule. For example, when exploring the properties of triangles, starting with an equilateral triangle—a special case—allows students to discover specific relationships before questioning: "Do these properties hold for all isosceles triangles? For scalene triangles?" Conversely, when learning the quadratic function $y = ax^2 + bx + c$, beginning with the fundamental form $y = x^2$ and then progressively introducing the linear and constant terms prompts questions about the effects of each parameter on the graph's shape and position.

2.5. Introducing Auxiliary Elements to Create New Pathways.

A pivotal moment in problem-solving, as per Pólya, is the introduction of auxiliary elements—be it a new line in a geometric figure, an auxiliary variable in algebra, or a new coordinate system. The decision to introduce such an element is, in itself, an act of problem posing. Students learn to ask: "Would constructing an auxiliary line, like a median or an altitude, create congruent or similar triangles that simplify the proof?" "If I set this complex algebraic expression to a new variable t, does the equation become more manageable?" This strategy empowers students to actively reshape the problem space, creating new structures that bridge the gap between the given state and the goal.

2.6. Fostering Reflection and Verification for Deeper Questions.

Pólya places great emphasis on the post-solution phase, encapsulated in questions like "Can you check the result?" and "Can you derive it differently?" This reflective practice is a potent source of deeper problem posing. Verification is not a mere mechanical check; it can lead to further inquiry: "Why did that particular method work?" "What is the underlying principle?" "If I change one parameter in the original problem, how does the solution change?" Encouraging students to look back and critique the solution process and the result itself cultivates a habit of critical examination and opens doors to more profound mathematical investigations.

2.7. Leveraging Diagrams and Symbolic Systems to Inspire Questions.

The directives "Draw a figure" and "Introduce suitable notation" underscore Pólya's recognition of visual and symbolic representations as catalysts for thought. Diagrams and symbols are not passive recording tools but active partners in thinking that can suggest new questions. Observing a dynamically generated function graph might prompt questions about intervals of increase/decrease, maximum points, or asymptotic behavior. Using a Venn diagram to represent set relationships can lead to inquiries about intersections, unions, and complements. Training students to "interrogate" diagrams and symbolic expressions is fundamental to developing robust mathematical intuition and representational fluency.

3. Construction of a Teaching Path for Problem Posing Based on Polya's Ideas: The Case of "Similar Triangles"

This chapter takes the core junior high school geometry topic of "Similar Triangles" as an example to construct a teaching path for problem posing. Guided by Polya's heuristic ideas and incorporating the "What-If-Not" strategy, the path aims to transition students from passive knowledge recipients to active discoverers, proposers, and creators of problems. The entire teaching process is structured around four stages: holistic questioning, systematic inquiry, deep exploration, and practical innovation, forming a complete learning cycle.

The design begins with contextualization and holistic problem design, adhering to Polya's principle of "returning to the definition" to stimulate students' macro-thinking about the essence of similar triangles. Instruction can be initiated with the historical story of Thales measuring the height of a pyramid, posing a central question: "Why does a fixed proportional relationship exist between two seemingly unrelated objects—the pyramid and its shadow?" Subsequently, students are guided to formulate broader questions from four dimensions: "time" (the origin and evolution of the concept of similarity), "space" (different cultural understandings of similarity), "application" (instances of similarity in architecture, art, and nature), and "mathematics itself" (the relationships between similarity, congruence, proportion, and area). This approach situates the concept of similar triangles within a vast and vivid knowledge network.

Table 1 Examples of Problem Generation in Exploring the Properties of Similar Triangles

Original Proposition	Generated Questions
Corresponding angles are equal, thus similar.	If only two sets of angles are equal, must the triangles be similar?
Corresponding sides are proportional, thus similar.	If three sides are proportional but the angles are not equal, is this possible?
Similar triangles in a plane.	Is there a concept of similar triangles on a sphere or curved surface?
Side length ratios are equal.	If the side length ratio is an irrational number, does similarity still hold?
Ordinary triangles.	Do the similarity criteria apply to right triangles? To isosceles triangles?

Building on this macro understanding, the teaching progresses to the second stage: systematic questioning and problem generation. This phase utilizes Polya's questioning chain and the "What-If-Not" strategy to guide students in a deep analysis of the determination criteria and properties of similar triangles. Teachers can present a standard proposition, such as "If two triangles have equal corresponding angles, then they are similar," and prompt students to "restate the problem" or "think of an analogous problem." More importantly, through the systematic negation of the proposition's conditions and conclusions, a series of exploratory questions can be generated. For instance, negating "corresponding angles are equal" might lead to: "If the corresponding angles are not equal, but the three sides are proportional, are the triangles still similar?" or "If this scenario occurs on a spherical surface, does the conclusion still hold?" Through this process, students independently "rediscover" the criteria for determining similar triangles and understand their limitations. Specific examples of problem generation are illustrated in Table 1.

After students generate a multitude of questions, the teaching enters the third stage: AI-assisted screening and in-depth inquiry. To address the challenge of filtering numerous student-generated questions, teachers can introduce artificial intelligence tools. Following Polya's spirit of "checking the result" and systematic reflection, a multi-dimensional evaluation framework can be established. This framework encompasses dimensions such as "originality," "mathematical depth" (e.g., does it touch upon core concepts like proportionality or angle relationships?), "verifiability" (can it be tested through experimentation or reasoning?), and "interdisciplinary potential," to efficiently select questions with high inquiry value. For instance, teachers can instruct AI tools to prioritize student questions based on these preset criteria. This AI-assisted filtering not only enhances efficiency but also embodies Polya's heuristic of guiding systematic inquiry. Consequently, teachers can better organize students to conduct three types of targeted inquiry activities: "basic verification," where students use drawing or dynamic geometry software to test fundamental conjectures; "extended exploration," which encourages generalization, specialization, or examining limitations; and "interdisciplinary application," linking mathematical ideas to real-world contexts like measurement or art, thereby fostering critical thinking and active discovery.

Finally, the teaching path culminates in the stage of creative expression and outcomes presentation. Polya believed that the ultimate goal of learning mathematics lies in its application. Therefore, this final stage focuses on transforming students' inquiry findings into practical innovations. Under teacher guidance, students can select creative tasks such as "mathematical modeling" (e.g., designing a method to measure the school flagpole's height) or "artistic design" (e.g., creating Escher-style tessellation patterns using similarity transformations). In this final phase, students not only present their work but, more importantly, employ Polya's "review and reflection" strategy to recount the complete process from problem posing to solution. This approach reinforces students' problem-posing awareness and helps cultivate sustainable mathematical thinking habits, thereby forming a complete instructional closed loop.

4. Discussion and Conclusion

This study, grounded in Polya's "How to Solve It" philosophy, systematically constructs a comprehensive teaching path for junior high school mathematics that effectively bridges the bidirectional thinking channel between "problem solving" and "problem posing." By innovatively integrating Polya's self-questioning strategies—including analogical reasoning, problem reformulation, condition decomposition, specialization and generalization, introduction of auxiliary elements, reflective verification, and leveraging visual representations—with the systematic "What-If-Not" methodological approach, a clear, operable, and theoretically robust framework for problem posing is developed. This integrated framework encompasses the entire instructional process from macro-contextual introduction through historical and real-world scenarios, systematic questioning guided by Polya's heuristics, AI-assisted screening based on multi-dimensional evaluation criteria, to interdisciplinary creative expression and practical application. The framework successfully translates abstract heuristic principles into concrete, scaffolded instructional practice, providing teachers with a structured yet flexible model to cultivate students' question-posing capabilities within regular mathematics classrooms.

Through its practical application and refinement in the "Similar Triangles" unit, this teaching path convincingly demonstrates its efficacy in guiding students through a transformative learning journey—from passive knowledge acceptance to active meaning construction and knowledge discovery. This transformation facilitates a fundamental pedagogical shift from traditional "teacher-centered" knowledge transmission to modern "student-centered" learning ecosystems, and from rote memorization to genuine "thinking stimulation" and cognitive engagement. The path operationalizes the development of mathematical core literacy by creating multiple opportunities for students to engage in authentic mathematical practices. As Polya aptly stated, "A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem." This teaching path aims to systematically nurture that very "grain of discovery" within every student by empowering them to pose, refine, and investigate their own mathematical questions, thereby fostering not only deeper conceptual understanding but also the essential mathematical practices and mindsets emphasized in contemporary curriculum standards.

The implementation results observed in the "Similar Triangles" unit reveal several significant outcomes. First, students demonstrated enhanced ability to generate diverse and profound mathematical questions, moving beyond superficial inquiries to explore underlying principles and boundary conditions of mathematical concepts. Second, the integration of AI tools for question screening provided an efficient mechanism for identifying high-value inquiries while simultaneously modeling systematic evaluation processes for students. Third, the creative expression phase enabled students to recognize mathematics as a living discipline connected to art, science, and daily life, rather than merely an abstract academic exercise. These outcomes collectively suggest that the teaching path successfully creates a classroom ecology where questioning becomes a natural and valued component of mathematical learning.

It is anticipated that through this carefully designed teaching path, which strategically uses "questioning" to promote "thinking" and guides "learning" through "thinking," students will not only master essential mathematical knowledge and methods but also, through continuous self-questioning and critical examination, develop the discerning eye, innovative spirit, and adaptive wisdom necessary to navigate an increasingly complex future. The path offers a replicable and scalable model for effectively integrating problem posing into daily mathematics instruction, contributing meaningfully to the development of inquisitive, capable, and mathematically literate learners. Future research should explore the adaptation of this framework to other mathematical domains and investigate its longitudinal impact on students' mathematical identity and problem-solving capabilities.

Furthermore, this study contributes to the ongoing international dialogue on mathematics education reform by providing a concrete example of how theoretical principles from Polya's work can be translated into contemporary classroom practice. The integration of traditional heuristic methods with modern technological tools like AI demonstrates how time-honored educational philosophies can be revitalized through strategic innovation. As mathematics education continues to evolve worldwide, this research offers valuable insights into balancing foundational thinking skills with technological advancement, ensuring that students develop both the critical reasoning abilities and the adaptive competencies required for success in the 21st century. The teaching path presented herein ultimately represents more than just an instructional method—it embodies a philosophical approach to mathematics education that honors the discipline's rich history while embracing its future possibilities.

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