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Exploration of the Cultivation Path of Critical Thinking in University Mathematics Courses in the Artificial Intelligence Era——Taking Linear Algebra as an Example

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Abstract: Against the backdrop of the deep integration of artificial intelligence (AI) technology into education, the problem of students' mental inertia caused by over-reliance on AI tools has become a core challenge in the teaching reform of university mathematics courses. Focusing on Linear Algebra, this paper designs teaching cases of "AI reverse tasks" targeting core knowledge points such as determinants, matrix inverses, and linear equations. By guiding students to criticize logical flaws, methodological defects, and scenario adaptation issues in AI's problem-solving processes, it explores effective paths for cultivating critical thinking. Practical results show that this teaching model can significantly improve students' ability of active thinking and knowledge application, providing practical references for the teaching reform of mathematics courses in the AI era.

1. Introduction

1.1 Research Background

The rapid development of AI tools such as ChatGPT has brought significant challenges to education and teaching. On the one hand, AI plays a positive role in improving teaching efficiency, enriching teaching resources, and supporting personalized learning. On the other hand, the popularization of AI has plunged mathematics courses into the dilemma of "tool dependence replacing in-depth thinking." When solving mathematical problems, students tend to directly obtain answers through AI, neglecting the derivation process of problem-solving logic and the optimization ideas of methods. This leads to a superficial understanding of knowledge and difficulty in constructing a systematic thinking system [1, 2]. In this context, cultivating students' critical thinking is particularly crucial—although AI can quickly output a large amount of information, it cannot replace humans' in-depth judgment on the authenticity, rationality, and value of information [3, 4].

The Ministry of Education's "Action Plan for Innovation in Artificial Intelligence in Colleges and Universities" clearly states that it is necessary to "cultivate students' ability to use AI tools to solve practical problems, while strengthening critical thinking and innovative awareness" [5]. This requirement points out the direction for the teaching reform of university mathematics courses. This paper takes the Linear Algebra course as a starting point to explore the cultivation path of students' critical thinking in the AI era, aiming to effectively address the problem of students' mental inertia.

1.2 Research Significance

As a core foundational course for science and engineering majors, Linear Algebra, with its knowledge system (such as matrix operations and the structure of solutions to linear equations), serves as the mathematical cornerstone for cutting-edge fields like machine learning and data science. In teaching, rationally utilizing AI as a "teaching assistant" and cultivating students' critical thinking by reshaping homework patterns and assigning "AI reverse tasks" can not only help students deepen their understanding of Linear Algebra knowledge, but also enable them to possess the core ability of "verifying the rationality of AI outputs" when using AI tools to solve practical problems in the future. This realizes a shift in thinking from "passively using AI" to "proactively controlling AI".

2. Design Philosophy and Core Dimensions of "AI Reverse Tasks"

2.1 Design Philosophy

"AI reverse tasks" break the traditional model of "AI-assisted problem-solving" by taking AI output results as "materials for critical analysis" and requiring students to conduct a comprehensive examination of AI's problem-solving steps from three dimensions: "method applicability, logical rigor, and scenario adaptability." Its core logic lies in exposing typical flaws in AI's mathematical reasoning (such as neglecting preconditions, inefficient selection of methods, and incomplete results), thereby forcing students to actively draw on their knowledge reserves in linear algebra. This enables them to complete the thinking cycle of "identifying problems—verifying problems—optimizing solutions" and strengthens their ability to apply knowledge and engage in logical reasoning.

2.2 Core Design Dimensions

Combined with the characteristics of the Linear Algebra course, the design of "AI reverse tasks" should focus on the following three core dimensions:

- Knowledge association dimension: task design should cover core knowledge points of linear algebra (such as determinant calculation techniques, invertible matrix judgment theorems, etc.), ensuring that when students criticize ai outputs, they can deeply draw on course knowledge to achieve the goal of "promoting understanding through criticism."
- Ai vulnerability targeting dimension: aiming at ai's inherent flaws of "emphasizing calculation over logic" and "valuing results over processes," design tasks that easily trigger ai errors. Examples include solving linear equations with special constraints and eigenvalue application problems that need to be combined with practical scenarios. This allows students to grasp knowledge boundaries and application rules while analyzing errors.
- Thinking training progression dimension: design tasks in layers according to the progressive logic of "identifying errors—analyzing causes—optimizing solutions" to meet the needs of students with different thinking levels. Basic-level tasks focus on "identifying obvious AI errors," advanced-level tasks focus on "analyzing the logical roots of errors," and high-level tasks focus on "optimizing AI problem-solving methods," thereby gradually improving students' critical thinking abilities.

3. Case Practice of "AI Reverse Tasks" in Linear Algebra Courses

3.1 Case 1: Calculating the Determinant of a Matrix

AI Reverse Task Design: Given a matrix

$$\boldsymbol{A} = \begin{pmatrix} -1 & -3 & -1 \\ 0 & 1 & 1 \\ -1 & -2 & 0 \end{pmatrix},$$

Find its determinant value $|\boldsymbol{A}|$.The teacher first uses two different AI tools, I and II, to calculate the determinant, with the output results shown in Figures1 and Figure 2 respectively. Then, the teacher assigns the following tasks to the students:

(1) Verify the solution results of the two AI tools and determine which one is correct;
(2) Analyze the problems with the incorrect result;
(3) Sort out the problem-solving methods adopted by the two AI tools;
(4) Explore a simpler method for calculating the determinant.

$$\begin{aligned} |A| &= -1 \times \begin{vmatrix} 1 & 1 \\ -2 & 0 \end{vmatrix} - (-3) \times \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} + (-1) \times \begin{vmatrix} 0 & 1 \\ -1 & -2 \end{vmatrix} \\ &= -1 \times (0+2) + 3 \times (0+1) - 1 \times (0+1) \\ &= -2 + 3 - 1 \\ &= 0 \end{aligned}$$

Figure 1: The calculation result using AI tool I

$$\begin{aligned} |A| &= -1 \cdot (1 \cdot 0 - 1 \cdot (-2)) - (-3) \cdot (0 \cdot 0 - 1 \cdot (-1)) + (-1) \cdot (0 \cdot (-2) - 1 \cdot (-1)) \\ &= -1 \cdot 2 + 3 \cdot (-1) - 1 \cdot 1 \\ &= -2 - 3 - 1 \\ &= -6 \neq 0 \end{aligned}$$

Figure 2: The calculation result using AI tool II

From the teaching feedback, 92% of students can accurately identify the correct result and clearly explain that the error in the incorrect result lies in a sign calculation mistake; 75% of students can use the properties of determinants to transform the matrix into an upper triangular determinant for calculation, verifying the result while optimizing the problem-solving process.

This case effectively helps students establish the awareness that "ai results are not necessarily correct and need to be viewed critically", and at the same time strengthens their mastery of knowledge points such as "the properties of determinants" and "the rule of expanding determinants by rows (columns)", avoiding the mechanical memorization of problem-solving steps.

3.2 Case 2: Determination of Solutions to Nonhomogeneous Linear Systems

AI Reverse Task Design: Given the nonhomogeneous linear system

$$\begin{cases} x_1 - 2x_2 + x_3 = \lambda \\ \qquad x_2 - x_3 = 1 - \lambda \\ \qquad x_2 - x_3 = (\lambda - 1)(\lambda + 2) \end{cases},$$

The problems are as follows:

(1) For which value of the parameter λ is the system of equations unsolvable?

(2) For which value of the parameter λ does the system of equations have infinitely many solutions?

(3) If the system of equations has infinitely many solutions, find its general solution.

The teacher uses an ai tool to solve this problem, with parts of the output steps shown in figure3 and figure 4 . Then, the teacher assigns the following tasks to the students:

(1) Analyze whether the solving method provided by ai is reasonable;

(2) Judge whether this method is efficient; if not, propose a better solving method.

Step 1: System Examination

The given system is:

$$\begin{cases} x_1 - 2x_2 + x_3 = \lambda & (1) \\ x_2 - x_3 = 1 - \lambda & (2) \\ x_2 - x_3 = (\lambda - 1)(\lambda + 2) & (3) \end{cases}$$

Step 2: Consistency Condition

From equations (2) and (3), we have:

$$x_2 - x_3 = 1 - \lambda$$

$$x_2 - x_3 = (\lambda - 1)(\lambda + 2)$$

For the system to be consistent, these must be equal:

$$1 - \lambda = (\lambda - 1)(\lambda + 2)$$

Figure 3: The process of determining the consistency of the system of equations

Step 4: Case Analysis

Case 1: $\lambda = 1$

Equation (2): $x_2 - x_3 = 0 \Rightarrow x_3 = x_2$

Equation (3): $x_2 - x_3 = 0$ ✓ (consistent)

Equation (1): $x_1 - 2x_2 + x_3 = 1$

Substitute $x_3 = x_2$:

$$x_1 - 2x_2 + x_2 = x_1 - x_2 = 1$$

$$x_1 = 1 + x_2$$

Let $x_2 = t$ (free parameter), then:

$$x_3 = t, \quad x_1 = 1 + t$$

General solution:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}$$

Infinitely many solutions.

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Figure 4: The solving process after the parameter λ is assigned a value

From the teaching feedback, 85% of students can independently point out the "inefficiency" of the AI method — when determining the solution status of the system of equations, AI did not prioritize the "elementary row transformation of the augmented matrix" method, but instead derived through direct substitution between equations, resulting in redundant calculation steps and a roundabout logic; 60% of students can completely deduce the calculation process of "solving linear systems by elementary transformation" and clarify the advantages of this method in "determining solutions to parameterized systems of equations".

This case has effectively strengthened students' understanding of "the theorem for determining solutions to linear systems" and "application scenarios of elementary transformation methods", avoided the mental inertia of "mechanically applying AI methods", and improved the flexibility in method selection.

4. Guarantee Measures for the Implementation of Teaching Reform

The smooth advancement of "AI reverse tasks" in the Linear Algebra course requires the construction of a guarantee system from three aspects: teacher competence, evaluation mechanism, and resource development:

- Teacher competence guarantee: teachers need to possess a certain level of ai literacy and instructional design capabilities. On the one hand, they should proactively learn the working principles of mainstream AI tools and master their typical flaws in solving Linear Algebra problems, ensuring that "AI reverse tasks" can accurately trigger AI errors. On the other hand, they should share "guided questioning" skills through teaching and research activities—instead of directly providing answers to students, they guide students to think actively by asking questions such as "What do you think is the logical premise of AI in this step?" and "Can the determinant method be used to simplify the calculation?"
- Evaluation Mechanism Guarantee: Highlight the process-oriented evaluation of critical thinking, and incorporate the completion quality of "AI reverse tasks" into the usual grades. The scoring dimensions include: the accuracy of error identification, the rigor of cause analysis, and the rationality of optimization solutions—rather than focusing only on the final result. At the same time, adopt the model of "student self-evaluation + group mutual evaluation + teacher comment". For example, in group discussions, students are required to evaluate each other on the comprehensiveness of their analysis of AI flaws, and deepen critical thinking through communication and collision of ideas.
- Resource Development Guarantee: Gather the strength of the course team to build a resource library for "AI reverse tasks". Regarding the knowledge points in each chapter of linear algebra, teachers should continuously collect and organize "AI error cases", mark the error types (e.g., calculation errors, logical errors, scenario adaptation errors) and the corresponding knowledge examination points, so as to facilitate direct selection and use in the future. At the same time, teachers should provide students with a screened list of AI tools, clarify the applicable scenarios and limitations of each tool, and guide students to choose and use AI tools rationally and properly.

5. Conclusions and Prospects

5.1 Research Conclusions

Through the teaching reform practice of "AI reverse tasks" in the Linear Algebra course, this paper verifies the effectiveness of this model in combating students' mental inertia and cultivating critical thinking:

In terms of knowledge mastery, by analyzing ai errors, students have deepened their understanding of abstract concepts in linear algebra, such as the determination of solutions to systems of equations and the properties of determinants, and improved the flexibility of knowledge application;

In terms of thinking ability, students have gradually established the awareness of "viewing ai outputs rationally", avoided over-reliance on ai, and acquired initial abilities to "verify ai results";

In terms of application value, this model has laid a foundation for students to use AI tools to solve problems in fields such as machine learning and data science in the future, achieving the coordinated improvement of "mathematical ability" and "AI literacy".

5.2 Future Prospects

Future research can be deepened in three aspects:

▪ Expanding the scope of tasks: extend "ai reverse tasks" from "application of basic knowledge points" to "practical application scenarios of linear algebra". Teachers can expand "ai reverse tasks" from "the application of basic knowledge points" to "practical application scenarios of linear algebra". For instance, by integrating problems such as "matrix singular value decomposition in image compression" and "linear equation modeling in recommendation systems", students can not only critique AI but also understand the connection between mathematics and practical fields.

▪ Constructing a personalized push mechanism: By integrating students' learning data (such as homework error types and knowledge mastery levels), teachers can push tailored "AI reverse tasks" to different students: for students with weak foundational knowledge, tasks focused on "identifying obvious AI errors" are provided; for students who have mastered the basics and seek further challenges, tasks aimed at "optimizing AI problem-solving schemes" are assigned. This approach enables the implementation of "teaching students in accordance with their aptitude".

▪ Exploring multi-model integration: Teachers can integrate "AI reverse tasks" with instructional models such as project-based learning and collaborative group learning. For example, they can assign projects like "collaborative group optimization of AI-based solutions for linear programming problems," which not only fosters students' critical thinking but also enhances their teamwork and innovation capabilities.

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