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GPS Satellite Clock Bias Prediction Based on Metabolic Grey Model

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Abstract: To address the issue of accuracy degradation caused by error accumulation over time in satellite clock bias forecasting using the grey model, a metabolic grey model is proposed. This model continuously updates satellite clock bias data sequences by removing outdated data, maintaining the system in a state of constant renewal to enhance forecasting accuracy. Forecasting trials were carried out utilizing satellite clock bias data with a 30-second sampling frequency, provided by the GNSS Analysis Center at Wuhan University. Forecasting approaches such as the linear polynomial model, quadratic polynomial model, grey model, and metabolic grey model were employed to perform 6-hour-ahead predictions, with actual clock bias data used as the reference benchmark for validation. Experimental results demonstrate that the metabolic grey model achieves significantly improved forecasting accuracy and stability. Achieving an average 6-hour prediction accuracy and stability of 0.17 ns and 0.32 ns, respectively, the proposed model demonstrates significant improvements over the linear polynomial, quadratic polynomial, and conventional grey models. Specifically, the average prediction accuracy is enhanced by 50.00%, 83.67%, and 39.29%, while prediction stability is improved by 41.82%, 83.51%, and 28.89% compared to these models.

1. Introduction

The Global Navigation Satellite System (GNSS) is a satellite-based radio navigation system that delivers three-dimensional positioning, velocity, and timing data to users anywhere on or near the Earth's surface, operating reliably under all weather conditions. Satellite clocks serve as one of the fundamental guarantees for delivering high-precision Positioning, Navigation and Timing (PNT) services [1]. Accurate prediction of satellite clock bias (SCB) is essential to maintain dependable positioning, navigation, and timing (PNT) services provided by GNSS. Due to the sensitivity and inherent complexity of satellite atomic clocks, coupled with their susceptibility to external environmental influences, it is challenging to precisely understand their variation patterns, making the development of high-accuracy clock bias prediction models difficult. At the same time, the high-precision clock bias information supplied by the International GNSS Service (IGS) frequently falls short of satisfying real-time application demands. Therefore, developing prediction models to generate high-precision clock bias products is critically important [2-3].

Currently, to enhance the accuracy of the SCB prediction, researchers have developed numerous prediction models, such as the Linear Polynomial Model (LPM), Quadratic Polynomial Model (QPM), and Grey Model (GM (1,1)), and so on [4-10]. These approaches can be used to forecast the clock offset characteristics of navigation satellites under different operational conditions, yet each comes with specific applicability and inherent constraints. For instance, the LPM requires minimal fitting data but exhibits decreasing forecast accuracy with extended fitting periods; the QPM is relatively simple to construct and can effectively enhance forecast precision by increasing modeling data volume, demonstrating significant effectiveness in short-term clock bias prediction. However, due to error accumulation, its prediction accuracy gradually declines with extended prediction durations; the GM (1,1) offers advantages such as minimal data requirements and rapid modeling speed, delivering satisfactory forecasting results for both short-term and long-term clock bias. However, over time, random disturbances continuously enter the system, diminishing the influence of earlier data on later data. This simultaneously reduces the model's predictive accuracy, leading to progressively larger prediction errors.

This paper considers that the GM (1,1) possess the advantages of requiring fewer samples for modeling and being suitable for long-term clock bias forecasting. However, as time progresses, all forecast results utilize the same segment of outdated information. Consequently, the model exhibits weak descriptive capability for locally varying regions within the predicted data, fails to accurately fit the actual curve, and consequently experiences increasing errors. To address this issue and further enhance the accuracy and stability of SCB forecasting while mitigating the impact of random disturbances on the system's evolution over time, the metabolic grey model (MGM (1,1)) was developed. This model continuously updates the SCB data sequence by removing outdated data, keeping the entire system in a state of constant renewal and development. This approach not only accounts for localized effects of random disturbances but also achieves high alignment with the overall variation curve.

2. Metabolic grey model-based approach for predicting clock bias

2.1 Establishing the grey model

Consider a complete, continuous and high-quality original SCB data sequence:

$$X^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\} \tag{1}$$

Performing a single accumulation yields the newly generated data sequence:

$$X^{(1)}=\left\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\right\} \tag{2}$$

where $x^{(1)}(m)=\sum_{i=1}^{m}x^{(0)}(i) \quad m=1,2,\cdots,n$.

The following equation is referred to as the original form of the GM (1,1) model.

$$x^{(0)}(k)+ax^{(1)}(k)=b \quad k=1,2,\cdots,n \tag{3}$$

The following equation is referred to as the basic form of the GM (1,1) model.

$$x^{(0)}(k)+az^{(1)}(k)=b \quad k=1,2,\cdots,n \tag{4}$$

where $z^{(1)}(k)=\frac{1}{2}\left[x^{(1)}(k)+x^{(1)}(k-1)\right] \quad k=2,3,\cdots,n$.

The following first-order linear differential equation is called the whitened equation of equation

(4).

$$\frac{dx^{(1)}}{dt}+az^{(1)}\left(k\right)=b \tag{5}$$

The parameters a and b are estimated using the least squares method from equation (4):

$$\hat{a}=\left(B^{T}B\right)^{-1}B^{T}Y \tag{6}$$

where $u=\left[a,b\right]^{T}$ is the parameter vector, and

$$Y=\begin{bmatrix} x^{(0)}\left(1\right) \\ x^{(0)}\left(2\right) \\ \vdots \\ x^{(0)}\left(n\right) \end{bmatrix}, B=\begin{bmatrix} -z^{(1)}\left(2\right) & 1 \\ -z^{(1)}\left(3\right) & 1 \\ \vdots & \\ -z^{(1)}\left(n\right) & 1 \end{bmatrix} \tag{7}$$

Equation (4) can be expressed as $Y=Bu$.

Taking the initial values $\hat{x}^{(1)}\left(t\right)|_{t=1}=x^{(0)}\left(1\right)$, the solution to the whitening equation is obtained as:

$$\hat{x}^{(1)}\left(t\right)=\left(x^{(0)}\left(1\right)-\frac{\hat{a}}{\hat{b}}\right)e^{-\hat{a}\left(t-1\right)}+\frac{\hat{a}}{\hat{b}} \tag{8}$$

Equation (8) can be reduced by successive subtraction to:

$$\hat{x}^{(0)}\left(k+1\right)=\hat{x}^{(1)}\left(k+1\right)-\hat{x}^{(1)}\left(k\right)=\left(1-e^{\hat{a}}\right)\left(x^{(0)}\left(1\right)-\frac{\hat{b}}{\hat{a}}\right)e^{-\hat{a}k} \quad k=1,2,\cdots,n-1 \tag{9}$$

The coefficient a represents the development factor of the GM (1,1), reflecting the trend of $\hat{x}^{(1)}$ and $\hat{x}^{(0)}$. The coefficient b is termed the grey interaction quantity. It is exogenous or derived from the context of practical problems, reflecting the relationship of data changes, whose precise connotation is grey.

2.2 Establishing the metabolic grey model

In the real world, any grey system is subject to random disturbance factors over time. Therefore, it is essential to continuously account for disturbance factors entering the system successively over time. While constantly incorporating new information, outdated information must be promptly discarded, ensuring the entire system remains in a state of continuous renewal and development.

The metabolic model, also known as the dynamic isometric new information model, involves supplementing a single value from the traditional GM (1,1) model into the known sequence while simultaneously removing the oldest data point. A new GM (1,1) model is then established to predict the next value, with the result re-inserted into the original sequence. The oldest data point is removed again, and this iterative cycle continues for sequential predictions until the target forecasting requirements are met. This modeling approach overcomes the limitations of the traditional GM (1,1) model [11-12]. Figure 1 depicts the detailed prediction process of the metabolic GM(1,1) model:

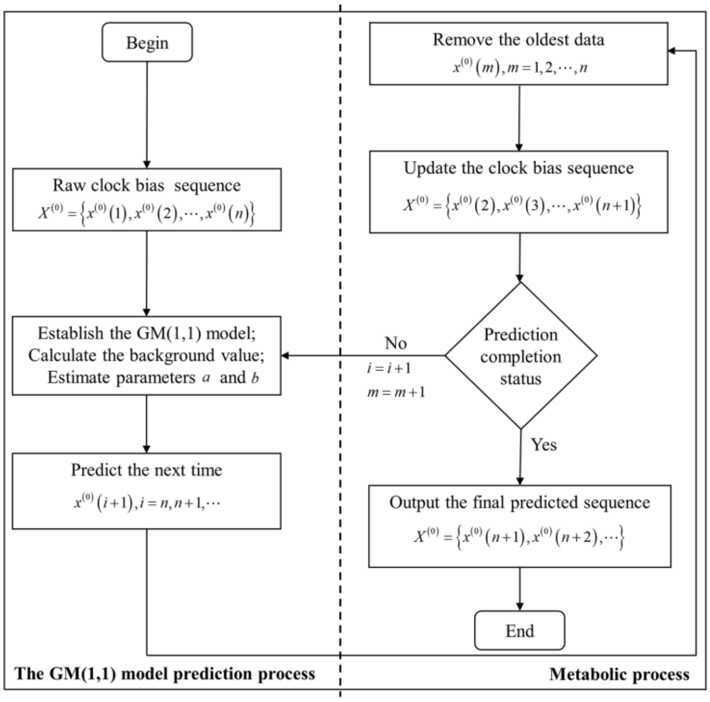

Figure 1: Flow diagram illustrating satellite clock bias forecasting using a metabolic grey model.

3 Experiments and analysis

3.1 Experimental data source

To assess the performance and practicality of the proposed method, high-accuracy SCB observations from GPS week 2377, day 1, supplied by the GNSS Analysis Center at Wuhan University, were utilized as experimental data. These measurements were recorded at a sampling rate of 30 seconds. During this period, 32 satellites were in orbit. Prediction experiments were conducted using randomly chosen clock bias data from six satellites: PRN 06, PRN 16, PRN 23, PRN 24, PRN 25, and PRN 30. Detailed information about them is provided in Table 1.

Table 1: Key satellite-related information selected for analysis.

Satellite ID	Clock type	Launch date	Clock bias trend
PRN 06	II-F -Rb	May 17, 2014	Negative values monotonically decreasing
PRN 16	II-R-Rb	January 29, 2003	Positive values monotonically increasing
PRN 23	III-A-Rb	June 30, 2020	Positive values monotonically increasing
PRN 24	II-F-Rb	October 4, 2012	Negative values monotonically increasing
PRN 25	II-F-Rb	May 28, 2010	Positive values monotonically decreasing
PRN 30	II-F-Rb	February 21, 2014	Negative values monotonically increasing

Figure 2 shows the clock bias variations of six satellites over a continuous 6 h period. The time series of clock bias for PRN 06 and PRN 25 exhibit a steady downward trend, whereas PRN 16, PRN 23, PRN 24, and PRN 30 display a persistent upward movement, reflecting adequate representativeness.

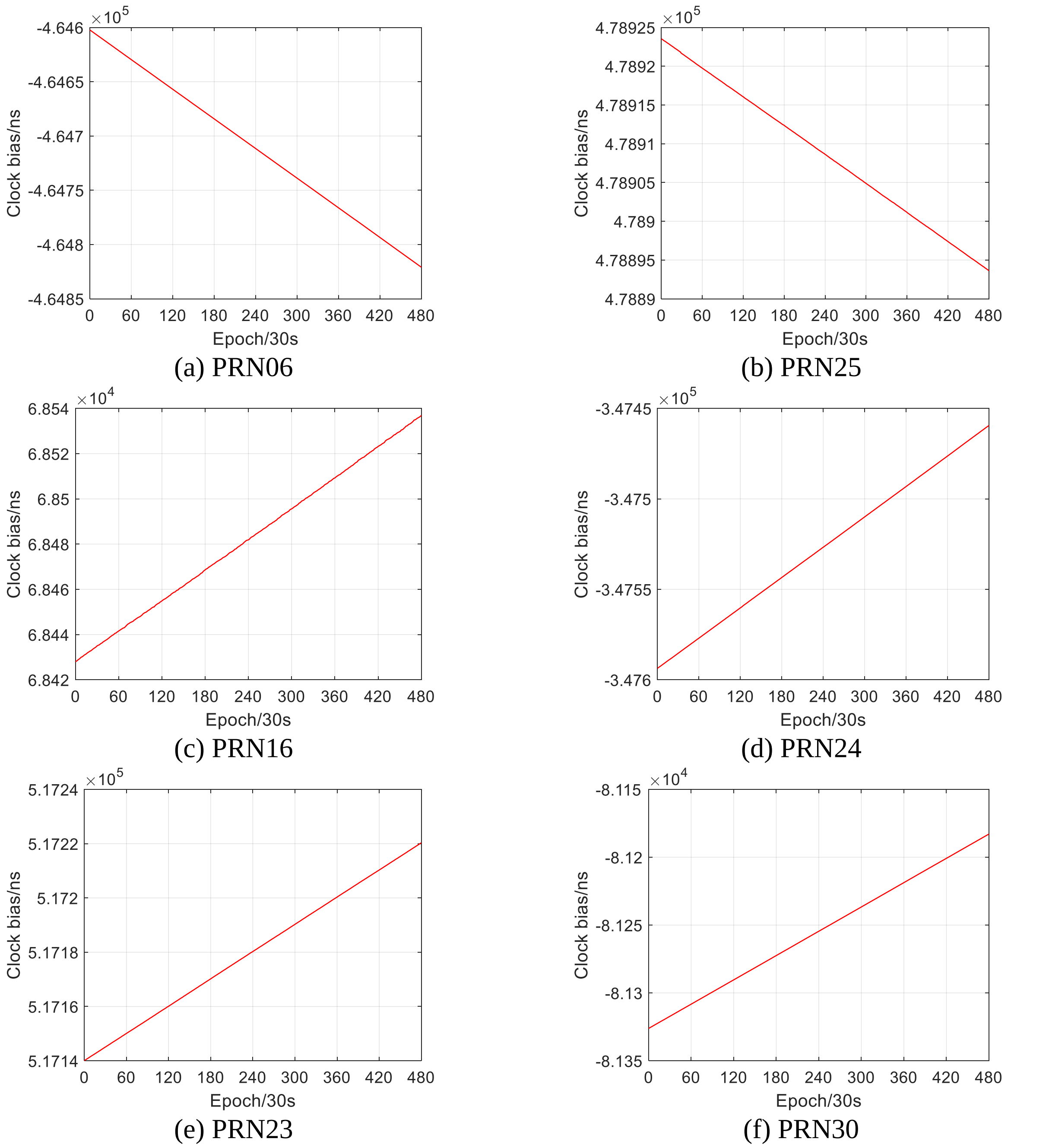

Figure 2: Chart of clock bias variation for the PRN 06, PRN 16, PRN 23, PRN 24, PRN 25 and PRN 30 satellites.

3.2 Prediction results and analysis

To fully evaluate the forecasting performance and feasibility of the proposed method, clock bias data from the first 6 hours of GPS week 2377 were used to establish linear polynomial model (LPM), quadratic polynomial model (QPM), grey models (GM (1,1)) and metabolic grey model (MGM (1,1)), respectively, for forecasting clock bias over the subsequent 6 h. The forecast inaccuracies of each model were calculated by taking the difference between the predicted clock bias from each model and the actual high-precision clock bias data released by the GNSS Analysis Center at Wuhan University for the following 6-hour period. To evaluate and compare the predictive performance of the models, two statistical measures were employed: the root mean square error (RMS) and the extreme value range (Range).

The formulas for calculating RMS and Range are as follows:

$$RMS = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \hat{x}_i\right)^2} \tag{10}$$

$$Range = \max\left(x_i - \hat{x}_i\right) - \min\left(x_i - \hat{x}_i\right) \tag{11}$$

Figure 3 and Table 2 present the fluctuations in forecasting errors and the corresponding statistical outcomes for each model.

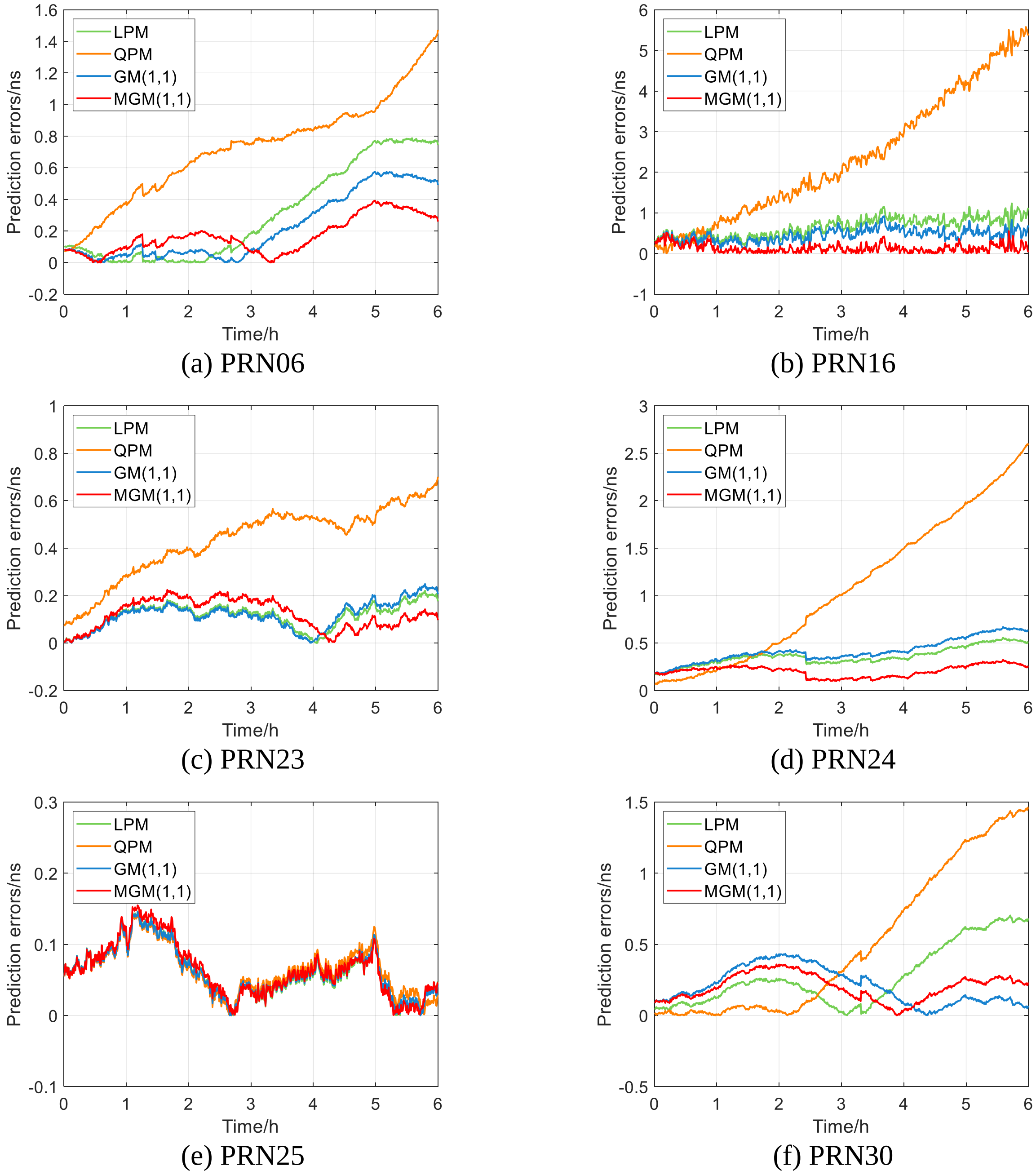

Figure 3: Forecast error variation chart of 6 h satellite clock bias.

Table 2: Statistical analysis results of satellite clock bias prediction error (Unit: ns)

Model	Assessment Metric	PRN 06	PRN 16	PRN 23	PRN 24	PRN 25	PRN 30
LPM	RMS	0.43	0.69	0.13	0.37	0.07	0.35
	Range	0.79	1.05	0.22	0.39	0.15	0.70
QPM	RMS	0.79	2.82	0.46	1.32	0.07	0.72
	Range	1.40	5.57	0.63	2.54	0.14	1.46
GM (1,1)	RMS	0.30	0.48	0.13	0.43	0.07	0.24
	Range	0.57	0.81	0.25	0.50	0.15	0.43
MGM (1,1)	RMS	0.20	0.17	0.14	0.21	0.08	0.22
	Range	0.39	0.55	0.22	0.23	0.15	0.36

Table 3: Mean prediction accuracy, stability and corresponding improvement rates for each model across a 6-hour period.

Model	RMS	Range
MGM (1,1)	0.17	0.32
LPM	0.34	0.55
Improvement (%)	50.00	41.82
QPM	1.03	1.96
Improvement (%)	83.50	83.67
GM (1,1)	0.28	0.45
Improvement (%)	39.29	28.89

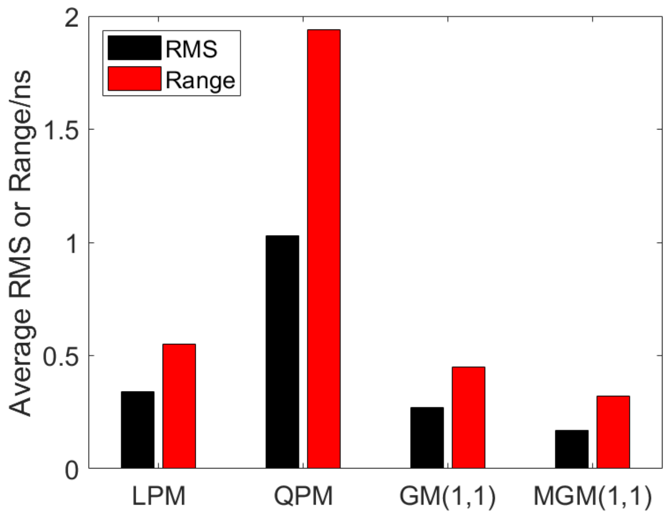

Figure 4: 6 h average prediction accuracy and stability.

Combining Figures 3-4 and Tables 2-3 reveals:

For 6 h short-term forecasts, the linear polynomial model achieved a mean prediction accuracy of 0.34ns and an average prediction stability of 0.55ns; the quadratic polynomial model achieved a mean prediction accuracy of 1.03 ns and an average prediction stability of 1.94 ns; the grey model achieved a mean prediction accuracy of 0.27 ns and an average prediction stability of 0.45 ns; the metabolic grey model achieved a mean prediction accuracy of 0.17 ns and an average prediction stability of 0.32 ns. Compared to the linear polynomial model, quadratic polynomial model and grey model, the metabolic grey model achieved improvements of 50.00%, 83.67% and 39.29% in average forecast accuracy, as well as 41.82%, 83.51% and 28.89% in average forecast stability.

As shown in the forecast error statistics in Table 2, clock bias forecast accuracy varies among different satellites. Further analysis of the satellite launch time information in Table 1 reveals that satellites launched later (e.g., PRN 23, launched in 2020) demonstrate higher forecast accuracy and stability across multiple models. Their forecast accuracy and stability are generally lower than those of earlier satellites (e.g., PRN 16, launched in 2003), a phenomenon potentially related to the technological evolution of satellite clocks. Newer-generation satellites (e.g., PRN 23 equipped with the third-generation rubidium clock) typically exhibit superior frequency stability and lower noise characteristics, resulting in more stable clock offset sequences that facilitate more reliable clock offset predictions. Furthermore, satellites sharing the same II-F type rubidium clock (e.g., PRN 06, PRN 24, PRN 25, PRN 30) launched around the same period (2010–2014) exhibit varying forecast accuracy and stability. This phenomenon may stem from inherent differences in atomic clock manufacturing (e.g., slight variations in initial frequency accuracy, frequency stability and frequency drift rate), coupled with significantly differing operational environments that cause non-uniform rates of performance degradation.

4. Conclusion

To improve the precision and reliability of satellite clock bias forecasting, this study introduces an innovative metabolic grey modeling framework specifically designed to accommodate the unique features of satellite clock bias data while overcoming shortcomings present in current grey models. First, the grey model predicts the clock bias data for the next time step. This predicted data is then added to the original sequence while removing the oldest data point. This process repeats cyclically until prediction completion. This method addresses the limitation of traditional grey models, where prediction errors inevitably increase over time due to random disturbances. The proposed approach mitigates this effect. Finally, a 6 h forecasting experiment was conducted using satellite clock bias data sequences exhibiting two typical trends (monotonically increasing and monotonically decreasing). The results validated the feasibility and stability of this method for satellite clock bias forecasting, demonstrating significantly superior performance compared to commonly used the linear polynomial model, the quadratic polynomial model and grey model. This approach offers a novel perspective for achieving high-precision satellite clock bias forecasting.

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