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# *Reliability of Deep Excavation Heave Stability with Incomplete Data*

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***Keywords:*** Deep Foundation Pit; Bottom Heave; Reliability; Incomplete Data; Interval Reliability

***Abstract:*** Overall sliding failure is a critical failure mode in deep foundation pit engineering, and the reliability of overall sliding is a key component in deep foundation pit risk assessment. Due to constraints in investigation conditions, soil parameter data for foundation pit projects are often incomplete, making it difficult to accurately obtain statistical moment information of soil parameters. Traditional methods for analyzing the reliability of overall sliding in deep foundation pits require assumptions about the moments of soil parameters, which cannot fully reflect actual engineering conditions. To better assess the sliding reliability of real projects, this study employs interval descriptions of soil parameter statistical moments based on actual engineering investigation data. Combining interval reliability theory, an interval evaluation method for the reliability index of overall sliding in deep foundation pits is developed. Using actual engineering data, the reliability analysis and parameter analysis of overall sliding failure modes in deep foundation pits were conducted, exploring the influence patterns of soil parameters in different layers on the reliability index interval of overall sliding. The results indicate: under conditions of incomplete data, the reliability index interval for this foundation pit is [5.02, 8.18]; increasing the mean internal friction angle of the soil significantly improves the reliability index but also enlarges the interval length, with a sharp increase in interval length when the mean reaches 37°; an increase in the coefficient of variation of the internal friction angle reduces both the reliability index and its interval length; raising the mean cohesion increases the reliability index linearly but reduces the interval length, with relatively minor effects; an increase in the coefficient of variation of cohesion decreases the reliability index while expanding the interval length. For multi-layer soil conditions, the variability of parameters in deep and thick soil layers has a more pronounced impact on the overall reliability level and the credibility of the assessment.

## 1. Introduction

With the continuous expansion of urban underground space development, the failure modes faced by deep foundation pit engineering are becoming increasingly complex. Among them, the overall sliding failure of the side wall is one of the most serious forms of damage, often leading to the overturning of supporting structures, subsidence of surrounding strata, and even overall collapse, posing a great threat to engineering safety[4][5].The overall slip failure involves a wider range of soil, and is a complex system instability process under the combined action of retaining structures, soil strength, and external loads[3]. In recent years, a large number of engineering practices and accident analyses have shown that the frequent occurrence of foundation pit sliding accidents in soft soil areas is fundamentally due to insufficient understanding of the variability of soil parameters and the collaborative working mechanism of support structures[4][5]. Therefore, conducting in-depth research on the overall sliding failure mode of deep foundation pits is of great theoretical significance and engineering value for revealing their instability mechanism and ensuring the safety of the foundation pit and surrounding environment.

Domestic and foreign scholars have conducted systematic research on the analysis methods and influencing factors of overall sliding failure in deep foundation pits. The traditional stability verification is mainly based on the limit equilibrium theory, which solves the safety factor by assuming a circular or polygonal sliding surface, such as the Swedish strip method and the circular sliding method recommended in the specifications[6][1]. However, these methods often struggle to accurately reflect the effects of complex soil layers, support structures, and spatial effects on slip surfaces. To this end, some scholars have introduced numerical simulation and model testing methods. Su[1] et al. and Hu[7] et al. respectively used finite element software and model testing to analyze the stress and deformation characteristics of pile anchor support systems and soil nail support structures under complex working conditions, revealing the collaborative working mechanism between support structures and soil; Liu et al.[2] studied the instability range and failure mode of ultra-deep excavation slopes in strong earthquake zones through on-site experiments and three-dimensional numerical analysis. In terms of influencing factors, research has shown that support structure parameters such as anchor length, inclination angle[1], soil nail parameters[7], and excavation geometry and boundary conditions[8][9] significantly affect the position and shape of potential slip surfaces. In addition, the variability of soil parameters and the effect of dynamic loads are also key factors in controlling slip stability. Guo et al.[5] studied the influence of pile driving vibration on the strength of loess and the soil nail soil interface strength through model experiments, revealing the strength attenuation mechanism of the foundation pit support system under dynamic loads; Cheng et al.[6] systematically analyzed the overall stability of the drilled pile and steel support system in combination with deep foundation pit engineering in sandy gravel formations. The above research deepens the understanding of foundation pit sliding failure, but current analysis methods rely heavily on determined soil parameters, and insufficient consideration is given to the statistical uncertainty of parameters caused by limited survey data or construction disturbances, making it difficult to comprehensively assess the risks of actual engineering. Therefore, it is necessary to analyze the overall sliding failure problem of deep foundation pits using reliability.

Unlike above ground buildings, the development of underground spaces requires careful consideration of uncertain factors such as soil type, geological conditions, and groundwater level[10][11]. Therefore, it is necessary to use reliability analysis methods to analyze the failure mode of overall slip. Hou Xiaoliang et al.[12] used the first-order second moment method to analyze the reliability of failure modes in deep foundation pits and conducted sensitivity analysis of soil parameters. Meng Kaiqi et al.[13] used a soil hardening model and combined response surface methodology with Monte Carlo simulation to conduct reliability analysis of deep foundation pits, and obtained the influence of different soil layers and parameters on stability. The above studies assume that the statistical moments of random variables are known in reliability analysis. However, due to cost and practical constraints, there are often relatively few soil parameter data in actual engineering, making it difficult to accurately calculate the statistical moments of random variables.

This article uses interval variables to describe the statistical moments of soil parameters under insufficient data conditions, calculates the reliability of the overall sliding failure mode of deep foundation pits, and analyzes the influence of two important parameters, soil cohesion and internal friction angle, on the reliability of foundation pits, as well as the relationship between the length of the statistical moment interval and the length of the overall sliding failure probability interval.

## 2. Overall sliding function of deep foundation pit

During the excavation process of the foundation pit, the height difference between the soil inside and outside the pit causes lateral pressure on the supporting structure from the outer soil. At the same time, the excavation of the soil inside the pit causes horizontal unloading, resulting in horizontal displacement of the supporting structure into the pit. When the displacement of the supporting structure is too large or the soil strength is insufficient, the soil around the pit will have a downward trend along the potential sliding surface, forming shear stress concentration on the sliding surface. When the overall sliding force of the soil on the sliding surface exceeds the anti sliding force on that surface, the soil and supporting structure will slide along the weak surface as a whole, causing instability and damage to the side wall of the foundation pit, and a large amount of soil will flood into the pit.

Unlike traditional radial stress, the shear stress distributed on the weak surface of the soil is distributed along the arc direction. When the structure fails, the overall sliding motion of the soil is a rotation around the center of the arc, which is the intersection point between the lowest support and the enclosure structure[14]. Treating the sliding soil as a whole, external loads and external resistance act as bending moments on the entire soil, and the loads and resistance are presented in the form of bending moments in the calculation. When the torque that hinders the overall sliding of the soil is less than the torque that drives the overall sliding of the soil, the overall sliding failure of the deep foundation pit will occur. The corresponding functional functions are:

$$G(c,\varphi)=M_{RLK}(c,\varphi)-M_{SLK}(c,\varphi) \tag{1}$$

In the formula: $M_{RLK}(c,\varphi)$ and $M_{SLK}(c,\varphi)$ They are respectively anti slip torque and slip torque, are functions of soil cohesion $c=[c_1, c_2, \ldots, c_n]$ and internal friction angle $\varphi=[\varphi_1, \varphi_2, \ldots, \varphi_n]$.$c_i$ and $\varphi_i$ are the cohesion and internal friction angle of the i-th layer of soil. Based on the circular sliding method, $M_{RLK}(c,\varphi)$ and $M_{SLK}(c,\varphi)$ are calculated using the following formulas[15]:

$$M_{SLK}(c,\varphi)=\sum_{j=1}^{n} G_j R \sin\theta_j \tag{2}$$

$$M_{RLK}(c,\varphi)=c_0 H+\sum_{i=1}^{n} \epsilon_i S_i+\sum_{j=1}^{n} (\tan\varphi_j) G_j \sin\theta_j \tag{3}$$

In the formula, $Z$ is the value of the overall sliding stability function; $H$ is the excavation depth of the foundation pit (m); $H$ is the horizontal projection length of the sliding surface (m); $R$ is the sliding arc radius (m); $N$ is the number of soil strips divided; $c_0$ is the cohesive force of the soil at the bottom of the pit (kPa); $c_i$ is the cohesive force of the i-th layer of soil (kPa); $S_i$ is the arc length (m) corresponding to the i-th layer of soil on the sliding surface; The internal friction angle of the soil at the location of the jth soil strip is denoted as $\varphi_j$; $G_j$ is the self weight of the jth soil strip (kN/m); $\theta_j$ is the central angle corresponding to the jth soil strip.

The decisive factors in the functional function $G$ ($c$, $\varphi$) are the values of $c$ and $\varphi$[12][16][17]. In actual engineering surveys, the soil parameters of each soil layer can be obtained; How to correctly weight the soil according to the actual failure mode determines whether the influence of each layer of soil on the overall slip failure mode can be accurately described. The traditional weighting method based on soil layer thickness is most widely used in engineering practice. This method assumes that the contribution of each soil layer to the overall slip failure is determined by its thickness, and therefore cannot accurately reflect the contribution of the soil in the arc direction. For this situation, weighting should be based on the proportion of the soil layer's arc length along the arc direction. The proportion of the central angle of an arc to the total central angle is the proportion of the arc length to the arc. Therefore, for the weighting of radians, the weighting formula for $c$ and $\varphi$ is [18]:

$$\varphi_k=\frac{\sum_{i=m}^{n}(\alpha_{1i}+\alpha_{12})\varphi_i}{\sum_{i=m}^{n}(\alpha_{1i}+\alpha_{12})} \tag{4}$$

$$c_k=\frac{\sum_{i=m}^{n}(\alpha_{i1}+\alpha_{i2})c_i}{\sum_{i=m}^{n}(\alpha_{i1}+\alpha_{i2})} \tag{5}$$

Among them, $\alpha_{i1}$ is the curvature occupied by the active zone soil layer; $\alpha_{i2}$ is the curvature of the passive zone soil layer.

## 3. Analysis method for overall slip reliability under incomplete data conditions

### 3.1 Failure probability based on first-order second-order moment method

Due to the relatively small nonlinearity of the overall slip failure function in Gaussian space, this paper adopts the First Order Reliability Method (FORM) for calculation[19]. This method approximates the first-order Taylor expansion of the limit state surface in standard Gaussian space at the verification point u *, and then converts the calculation of the reliability index $\beta$ into finding the shortest distance from the spatial origin to the limit state surface. The formula is as follows:

$$\beta=\|\mathbf{u}^*\| \tag{6}$$

Based on reliable indicators, the failure probability can be obtained as follows:

$$P_f=\Phi(-\beta) \tag{7}$$

The key to reliability evaluation based on FORM is to find the verification point $u^*$ in the standard Gaussian space. Based on statistical information of random variables, functional functions in physical space can be mapped to the standard Gaussian space, and Newton's iterative method can be used to find $u^*$.

In the failure mode of overall sliding in deep foundation pits, the random variables in the functional function are the cohesion c and internal friction angle $\varphi$ of the soil. The soil has extremely complex variability in the depth direction, horizontal direction, and geological composition in different regions, making it difficult to determine the distribution of physical properties of the soil. Moreover, due to economic and time cost constraints, the exploration data for deep foundation pits is often insufficient or has blindness in direction and quantity. This series of restrictions makes it impossible to determine the statistical distribution and moments of $c$ and $\varphi$. By using mathematical methods, statistical moment intervals within a certain confidence interval can be obtained through actual engineering exploration data. Therefore, this article combines interval reliability theory to calculate the interval of reliability index $\beta$ under the condition that the statistical moments of cohesion c and internal friction angle $\varphi$ are interval conditions

### 3.2 Reliability interval calculation method

When the statistical information of a random variable is interval, the mapping relationship of the functional function to the standard Gaussian space is non unique. To find the functional function mapping values in the standard normal space corresponding to the upper and lower bounds of reliable indicators, this paper adopts the First Three Moments as Interval method (TMI) based on a simplified third-order moment polynomial pseudo normal transformation model[21] combined with FORM for reliability analysis. The TMI method is based on the sensitivity analysis of failure probability to the statistical moments of random variables, and derives the combination of upper and lower bounds of the statistical moments of random variables corresponding to the upper and lower bounds of the reliability index. The upper and lower bounds of reliability index are expressed as follows:

$$\beta^{upper} = \| u_{upper}^{*} \|, \beta^{lower} = \| u_{lower}^{*} \| \tag{8}$$

$\boldsymbol{u}^*_{\mathrm{upper}}$ and $\boldsymbol{u}^*_{lower}$ Points calculated based on $\{c_{\mathrm{upper}}, \varphi_{\mathrm{upper}}\}$ and $\{c_{\mathrm{lower}}, \varphi_{\mathrm{lower}}\}$.$\{c_{\mathrm{upper}}, \varphi_{\mathrm{upper}}\}$ and $\{c_{\mathrm{lower}}, \varphi_{\mathrm{lower}}\}$can be determined by the sign of the partial derivatives of the functional function $G(c,\varphi)$ with respect to $c$ and $\varphi$ at the mean of $c$ and $\varphi$. The specific description is as follows:

(1) Calculate the partial derivatives of the functional function $G(c, \varphi)$ with respect to $c$ and $\varphi$ at the mean of $c$ and $\varphi$:

$$\mathbf{x}=\frac{\partial G(c,\varphi)}{\partial c}\Big|_{c=\bar{c}},\mathbf{y}=\frac{\partial G(c,\varphi)}{\partial \varphi}\Big|_{\varphi=\bar{\varphi}} \tag{9}$$

(2) Based on the values of these two vectors $x$ and $y$, determine the minimum and maximum values of the moments of the random variable corresponding to the upper and lower bounds of the reliability calculation. The specific corresponding methods are shown in Table 1, where $\mu$, $\sigma$, and $\delta$ represent the first, second, and third moments of the random variable, $X$ is the random variable, $U$ is the random variable transformed into a standard normal space, and $p$ and $q$ are the intermediate parameters calculated through the third moment [21].

(3) Calculate two FORM based on $\{c_{\mathrm{upper}}, \varphi_{\mathrm{upper}}\}$ and $\{c_{\mathrm{lower}}, \varphi_{\mathrm{lower}}\}$, respectively, to obtain the corresponding reliability index and failure probability.

## 4. Analysis of actual engineering cases

To verify the accuracy of the method proposed in this article, the Beijing Urban Sub center Station Comprehensive Transportation Hub Project was selected for calculation and analysis.

### 4.1 Failure probability based on first-order second-order moment method

The Beijing Urban Sub center Station Comprehensive Transportation Hub Project (as shown in Figure 1) is an important national strategic project located in Tongzhou District, Beijing, at the intersection of the "Belt and Road" spatial structure of the urban sub center. The deep excavation project of this project is one of the largest underground comprehensive transportation hubs in Asia, with an average depth of 32 meters.

This project is one of the largest underground comprehensive transportation hubs in Asia, with an average depth of 32 meters in the foundation pit, which is a typical super large and ultra deep foundation pit. Its overall sliding stability problem is particularly prominent, and it has high engineering representativeness and research value. The project is located in Tongzhou District, Beijing, with complex geological conditions and obvious soil layering. The mechanical parameters such as cohesion and internal friction angle of each layer of soil have significant differences, providing an ideal engineering background for studying the influence of parameter variability on reliability under multi-layer soil conditions. In addition, as a national strategic project, this project has extremely high safety requirements, and traditional deterministic analysis methods are difficult to comprehensively evaluate its potential risks. It is urgent to introduce reliability evaluation methods that can consider data incompleteness. Engineering investigation provides detailed soil parameter data, laying a solid foundation for establishing statistical moment intervals and conducting parameter analysis, enabling the interval reliability evaluation method proposed in this study to be validated and applied in practical engineering.

Figure 1: Beijing Sub-Center Railway Station Comprehensive Transportation Hub

Table 1: The Correspondence between Statistical Moments of Random Variables and Bounds for Reliability Intervals

| Uncertain moment | $\frac{\partial G(X)}{\partial x}\Big\vert_{X=\mu}$ | U | | Calculate the moment corresponding to the lower bound of reliability | Calculate the upper bound of reliability Corresponding moment |
|---|---|---|---|---|---|
| | | $\delta < 0$ | $\delta > 0$ | | |
| μ | ≤0 | $(-\infty, +\infty)$ | | $\underline{\mu}$ | $\overline{\mu}$ |
| | >0 | | | $\overline{\mu}$ | $\underline{\mu}$ |
| σ | ≤0 | $(-\infty, 0) \cup (q, +\infty)$ | $[p, 0)$ | $\overline{\sigma}$ | $\underline{\sigma}$ |
| | | $[0, q)$ | $(-\infty, p) \cup (0, +\infty)$ | $\underline{\sigma}$ | $\overline{\sigma}$ |
| | ≤0 | $(-\infty, 0) \cup (q, +\infty)$ | $[p, 0)$ | $\underline{\sigma}$ | $\overline{\sigma}$ |
| | | $[0, q)$ | $(-\infty, p) \cup (0, +\infty)$ | $\overline{\sigma}$ | $\underline{\sigma}$ |
| δ | ≤0 | $(-\infty, 1)$ | $[1, +\infty)$ | $\underline{\delta}$ | $\overline{\delta}$ |
| | | $[-1,0)$ | $(0,1]$ | $\overline{\delta}$ | $\underline{\delta}$ |
| | ≤0 | $(-\infty, 1)$ | $[1, +\infty)$ | $\overline{\delta}$ | $\underline{\delta}$ |
| | | $[-1,0)$ | $(0,1]$ | $\underline{\delta}$ | $\overline{\delta}$ |

## 4.2 Actual Data Processing

To evaluate the probability of overall sliding failure in deep foundation pits, the moment of the random variable is first calculated based on actual engineering data, and then the interval radius length of the given moment is used to calculate the interval of the moment.

According to the geological survey report of the second section of the Beijing Urban Sub center Station Comprehensive Transportation Hub Project, the soil is divided into 6 layers, and the weighting coefficients of each group of soil are calculated separately. Then, the cohesion $c$ and internal friction angle $\varphi$ of each layer of soil are obtained through the geological survey data, as shown in Table 2. Based on the above assumptions, we calculate the interval of the moment using the exact moment of the random variable.

Due to inconsistent construction conditions in actual excavation projects, this study considers the loose normal situation of ground overload $q_k$=10kPa and the dense stacking situation of $q_k$=30kPa. The other parameters of the foundation pit are shown in Table 3.

According to the reference statistical data, it can be seen that in civil engineering, the coefficient of variation of soil cohesion is approximately 0.228[22]; The coefficient of variation of internal friction angle of soil is approximately 0.1 according to statistical data[20]. The actual internal friction angle and cohesion of the soil are both numbers greater than zero. Therefore, in the calculation, both cohesion and internal friction angle are treated as truncated normal distributions (truncation range is 0 to +∞), and the interval length of plus or minus one standard deviation is taken as the mean. The interval length of the variance is located at 5% of the value of the statistical moment itself. This interval length is input, and the moment of the random variable is changed by multiplying it with a coefficient $k$ to change the moment. The corresponding reliability and reliability interval are calculated.

Table 2: Statistical information on cohesion and internal friction angle of different soil layers

| Basic variables | distribution type | Mean interval | Coefficient of variation interval |
|---|---|---|---|
| 1 Cohesion $c_1$(kPa) | TN | [37.525, 41.475] | [0.217,0.239] |
| 2 Cohesion $c_2$(kPa) | TN | [0,1] | [0.217,0.239] |
| 3 Cohesion $c_3$(kPa) | TN | [44.175,48.825] | [0.217,0.239] |
| 4 Cohesion $c_4$(kPa) | TN | [0,1] | [0.217,0.239] |
| 5 Cohesion $c_5$(kPa) | TN | [43.225, 47.775] | [0.217,0.239] |
| 6 Cohesion $c_6$(kPa) | TN | [23.75,26.25] | [0.217,0.239] |
| 1 Friction Angle $\varphi_1$ | TN | [0.218,0.24] | [0.095,0.105] |
| 2 Friction Angle $\varphi_2$ | TN | [0.371,0.411] | [0.095,0.105] |
| 3 Friction Angle $\varphi_3$ | TN | [0.597,0.659] | [0.095,0.105] |
| 4 Friction Angle $\varphi_4$ | TN | [0.212,0.234] | [0.095,0.105] |
| 5 Friction Angle $\varphi_5$ | TN | [0.597,0.659] | [0.095,0.105] |
| 6 Friction Angle $\varphi_6$ | TN | [0.511,0.565] | [0.095,0.105] |

Table 3: Statistical information of non random variables in functional functions

| Constant | Value |
|---|---|
| Ground Overload Standard $q_k$(kPa) | 10, 30 |
| Severe Soil Mass γ(kN/m3) | 20 |
| Distance From Bottom Support Ground $h_0$'(m) | 48 |
| Bottom Support of Retaining Wall Below D'(m) | 10 |
| Allowable Bending Moment Value of Enclosure Structure $M_{sk}$(kN) | 100 |

### 4.3 Failure probability interval

By processing the actual data in section 3.2 and using the interval reliability calculation method in section 2.2, the reliability interval for this failure mode is determined to be [5.02,8.18]. In practical engineering, conservative evaluations are often made, and in most cases, the lower limit of reliability is considered. The lower limit of the reliability index is 4.97 and the failure probability is about $3.4\times10^{-7}$ under the condition of the highest foundation pit overload ($q_k$=30kPa). The lower limit of the reliability index is 5.02 and the failure probability is about $2.58\times10^{-7}$ under the condition of small foundation pit overload ($qk$=10kPa). It can be seen that the variation of foundation pit overload has little impact on reliability. To simplify the calculation, the following analysis will be conducted using the less favorable $q_k$=30kPa. By changing the mean and coefficient of variation of soil cohesion and internal friction angle, a parameter analysis was conducted on the influence of soil parameters on the reliability of overall slip. The influence of soil mean on reliability is shown in Figure 2, and the influence of soil coefficient of variation on reliability is shown in Figure 3.

As shown in Figures 2-3, in the overall slip failure mode, the mean changes in internal friction angle and cohesion have a significant positive impact on the reliability index β, but there are significant differences in their strength and contribution. With the increase of the mean internal friction angle, the upper and lower bounds of the reliability index β show a continuous and stable upward trend, reflecting that the improvement of the internal friction angle as the core parameter of soil shear strength can directly enhance the ability of the foundation pit to resist overall sliding failure. Specifically, as the average internal friction angle gradually increases from a lower level, the growth rate of the reliability index is considerable. For example, for every 5 ° increase in the average internal friction angle, the reliability index β can increase by about 0.6 on average. This change rate remains relatively stable throughout the entire range of mean variation, indicating that the effect of internal friction angle on improving reliability is sustained and reliable. In contrast, the change in the mean cohesive force has a relatively gentle effect on the reliability index *β,* although it also shows a positive promoting effect, the magnitude of the improvement is significantly smaller than that of the internal friction angle.

(a) The influence of the midpoint of the mean interval of internal friction angle on the lower bound value of reliability index

(b) The influence of the midpoint of the mean interval of internal friction angle on the upper bound value of reliability index

(c) The influence of the midpoint of the mean cohesion interval on the lower bound value of the reliability index

(d) The influence of the midpoint of the mean cohesion interval on the upper bound value of the reliability index

Figure 2 The influence of the midpoint of the mean interval of soil parameters on reliability index

(a) The influence of the midpoint of the coefficient of variation interval of internal friction angle on the lower bound value of reliability index

(b) The influence of the midpoint of the coefficient of variation interval of internal friction angle on the upper bound value of reliability index

(c) The influence of the midpoint of the coefficient of variation of cohesion on the lower bound value of the reliability index

(d) The influence of the midpoint of the coefficient of variation of cohesion on the upper bound value of reliability index

Figure 3 The influence of midpoint of soil parameter variation coefficient interval on reliability index

In the overall slip failure mode, there is a significant negative correlation between the coefficient of variation of random variables and the reliability index *β*, and the degree of influence generated by the variability of internal friction angle and cohesion shows significant differences. From Figure 3, it can be seen that as the coefficient of variation of the internal friction angle continues to increase, the upper and lower bounds of the reliability index *β* show a rapid downward trend, especially in the lower bound region where the decrease is more severe. This indicates that the discreteness of the internal friction angle is the main factor threatening the overall sliding stability reliability of the foundation pit. For example, as the coefficient of variation of the internal friction angle gradually increases from a lower level to a higher level, the lower bound of the reliability index may decrease by up to 50%. This significant attenuation means that in the case of limited data leading to increased uncertainty in parameter statistics, the variability of the internal friction angle will significantly compress the lower bound space of reliability, causing a sharp increase in the risk of foundation pit failure. In sharp contrast, the change in the coefficient of variation of cohesion has a relatively mild impact on the reliability index *β*, although it also shows a downward trend. However, both the speed and final magnitude of the decline are significantly smaller than the internal friction angle, indicating that the variability of cohesion has a relatively minor controlling effect on the volatility of reliability.

Overall, the reliability index will increase due to the increase in the mean values of internal friction angle and cohesion, and decrease due to the increase in the coefficient of variation of internal friction angle and cohesion. The coefficient of soil layers at different depths has different degrees of influence, especially the deepest layer of soil has the greatest impact.

In practical engineering, considering conservative and risk factors, the minimum value of the reliability interval is generally taken. However, the analysis results of the reliability interval are still meaningful, and the length of the reliability interval represents the credibility of using this evaluation method. The smaller the length of the reliability interval, the higher the credibility of the evaluation and the more engineering reference significance it has. The variation of the interval length of the overall slip reliability index with respect to the friction angle and cohesion statistical moment inside the soil is shown in Figures 4 and 5. The changes in the statistical moment of both types of random variables will have a certain impact on the interval length, which in turn will affect the accuracy of the analysis results.

As shown in Figure 4, the influence of the statistical moment of internal friction angle on the length of the reliability index interval exhibits significant nonlinear characteristics. As the midpoint of the mean interval of the internal friction angle increases, the length of the reliability index interval shows a non-linear growth trend. When the mean reaches about 37°, the interval length begins to increase sharply, and the maximum reliability index interval length reaches 8, indicating a significant decrease in the accuracy of the reliability analysis results at this time. The increase of the midpoint of the coefficient of variation of internal friction angle reduces the length of the reliability index interval, and the degree of influence also shows a nonlinear relationship. It is worth noting that the deeper and thicker the soil layer, the more severe the influence of the mean internal friction angle and coefficient of variation on the length of the reliability interval, and the parameter variability of deep soil has a more prominent control effect on the credibility of the evaluation results.

From Figure 5, it can be seen that the influence of the statistical moment of cohesion on the length of the reliability index interval is different from that of the internal friction angle. The increase of the midpoint of the mean cohesion interval will increase the length of the reliability index interval, but the impact is relatively small and shows a good linear relationship, which is consistent with the linear manifestation of cohesion in the functional function. An increase in the midpoint of the coefficient of variation of cohesion will result in a decrease in the length of the reliability index interval, and its impact is mainly linear. Similar to the internal friction angle, the changes in the mean cohesion and coefficient of variation of soil layers with greater depth and thickness have a more significant impact on the length of the reliability interval. In multi-layer soil conditions, the parameter statistical characteristics of deep soil play a dominant role in the overall reliability evaluation.

(a) The influence of the midpoint of the mean interval of internal friction angle on the length of the reliability index interval

(b) The influence of the midpoint of the coefficient of variation of internal friction angle on the length of the reliability index interval

Figure 4: The influence of the midpoint between the mean internal friction angle and the coefficient of variation on the length of the reliability interval

(a) The influence of the midpoint of the mean cohesion interval on the length of the reliability index interval

(b) The influence of the midpoint of the coefficient of variation of cohesion on the length of the reliability index interval

Figure 5: The influence of the midpoint between the mean cohesion and coefficient of variation interval on the length of the reliability interval

## 5. Conclusion

This article takes the actual project of the comprehensive transportation hub of Beijing Urban Sub center Station as the analysis object, and conducts reliability analysis and parameter analysis of the overall sliding failure mode of deep foundation pit. Research has found that considering statistical moments as interval variables for reliability assessment can better analyze overall slip reliability when effective data is limited.

The increase of the midpoint of the average friction angle interval within the soil will lead to an increase in the reliability index of the overall sliding of the foundation pit, and the length of the reliability index interval will also increase. The increase of the midpoint of the average cohesion interval of the soil will cause a linear increase in the reliability index of the overall sliding of the foundation pit, and the length of the reliability index interval will decrease. The increase of the midpoint of the coefficient of variation of the internal friction angle in the soil will cause a rapid decrease in the reliability index of the overall sliding of the foundation pit, and the length of the reliability index interval will decrease. The increase of the midpoint of the coefficient of variation of soil cohesion will lead to a decrease in the reliability index of the overall sliding of the foundation pit, and the length of the reliability interval will increase. For multi-layer soil conditions, the greater the depth and thickness of the soil layer, the greater the impact on the overall reliability level.

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