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Calculation Method for Shear Capacity of Diagonal Cross-section of Reinforced Concrete Beam with Unreinforced FRP Strips

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Abstract: To investigate the shear capacity of inclined cross-sections of FRP-reinforced recycled concrete beams without stirrups, 31 sets of shear test data were collected to analyze the effects of parameters such as longitudinal reinforcement ratio and shear span ratio on shear capacity. By comparing the calculation results of China Code GB 50608-2020 and American Code ACI 440.1R-15, it was found that both significantly underestimate the actual shear capacity and fail to adequately account for shear span ratio, size effect, and pin bolt effects. Based on the concrete formula in China Code, a shear span ratio correction function was introduced to propose a modified shear capacity calculation formula. The mean ratio of modified calculated values to experimental values was 1.27, with a coefficient of variation of 24.08%, indicating good prediction accuracy. Significant deviations in some specimens may be related to size effect and require further investigation.

1. Introduction

With the acceleration of global urbanization, demand for construction aggregates continues to surge, leading to chronic shortages of natural sand and gravel supplies. Concurrently, strict restrictions on quarrying imposed by ecological conservation policies have resulted in severe resource scarcity in certain regions. Meanwhile, construction-related demolition activities generate increasing volumes of solid waste annually [1]. Construction solid waste constitutes a reusable resource that can be processed through crushing and sorting to produce recycled aggregates, which can be reused in engineering projects. This approach not only alleviates the supply-demand imbalance of natural sand and gravel but also reduces land occupation by solid waste and environmental pollution, making it a crucial measure for promoting green and sustainable infrastructure development. Against this backdrop, the recycling of waste concrete into recycled coarse aggregate (RCA) and its application in structural engineering has emerged as a pivotal approach to sustainable development in civil engineering [2].

Recycled aggregates exhibit multiphase heterogeneity, particularly due to the surface adhered aged mortar, resulting in reduced mechanical properties of recycled concrete compared to natural aggregate concrete. Studies have shown that the strength of recycled concrete is typically 10%–20% lower than that of natural aggregate concrete [3]. At the component level, this deterioration in mechanical properties is further manifested as a reduction in the flexural and shear capacity of recycled concrete beams, with the maximum reduction reaching up to 15% [4].

Beyond mechanical property degradation, recycled concrete exhibits higher internal steel bar corrosion risks compared to conventional concrete due to its elevated porosity and unstable alkaline environment. Corrosion not only shortens structural lifespan but also increases maintenance costs. Therefore, replacing traditional steel bars with fiber-reinforced composite (FRP) bars-known for superior corrosion resistance-has emerged as an effective solution. FRP bars demonstrate advantages such as high tensile strength, lightweight construction, and corrosion resistance [5], effectively mitigating steel bar corrosion issues and demonstrating broad application prospects.

To promote the healthy development of FRP-reinforced concrete structures and guide engineering applications, extensive research has been conducted. Xiao Jianzhuang et al. [6] found through shear tests on recycled concrete beams that their failure process resembles that of conventional concrete beams, but the shear capacity decreases with increasing substitution rates. Based on these findings, they proposed a recommended formula. Choi et al. [7] conducted experiments with shear span ratio, reinforcement ratio, and substitution rate as variables, similarly confirming that shear capacity gradually decreases with increasing substitution rate. Alam found that for FRP-reinforced ordinary concrete beams without stirrups, shear capacity reduced by 70% when the shear span ratio increased from 1.5 to 3.5.Issa et al. [9] conducted shear tests on unreinforced BFRP-reinforced concrete beams and found that the ultimate shear capacity decreases with increasing shear span ratio while increases with higher reinforcement ratio at the same shear span ratio. These studies revealed influencing mechanisms from three independent perspectives: recycled aggregate (reinforced concrete beams), shear span ratio (FRP-reinforced ordinary concrete beams), and FRP reinforcement materials (ordinary concrete beams). However, systematic experimental data under coupled conditions (i.e., FRP-reinforced recycled concrete beams) remain lacking. Consequently, research on shear capacity of inclined cross-sections for such beams remains limited. In summary, the shear performance of FRP-reinforced recycled concrete beams is influenced by coupled factors including shear span ratio and longitudinal reinforcement ratio, necessitating the development of more appropriate calculation formulas. To address this, this study first compares current code provisions for shear capacity calculation of FRP-reinforced concrete beams in China and the United States, and analyzes critical influencing factors not considered in existing standards.

2. Comparison of Shear Bearing Capacity Calculation Formulas for Different Design Codes

For FRP-reinforced concrete structures, several countries have issued relevant design and construction codes. Among them, China's code (GB 50608-2020) [10] and the American code (ACI 440.1R-15) [11] adopt different design methods. Both codes divide the shear capacity of FRP-reinforced concrete beams into two parts, namely

$$V=V_{\mathrm{f}}+V_{\mathrm{c}} \tag{1}$$

In the formula: V_{f} represents the shear bearing capacity partially borne by FRP stirrups, while V_{c} denotes the shear bearing capacity partially borne by concrete.

In China's standard GB 50608-2020, the shear resistance formula for FRP(2)-reinforced concrete beams is shown in Equation (2) ~ (4):

$$V=\frac{A_{f_v} f_{fv} \mathrm{h}_0}{s}+0.86 f_t b_w k \mathrm{h}_0 \tag{2}$$

$$k=\sqrt{2E_f/E_c\rho_f+(E_f/E_c\rho_f)^2}-E_f/E_c\rho_f \tag{3}$$

$$f_{fv}=\min\left(0.4f_{fd},\frac{0.006E_f}{\gamma_f\gamma_e}\right) \tag{4}$$

The formula includes: k as the section shape coefficient; bw as the section width; ρ_f as the longitudinal reinforcement ratio; E_f as the elastic modulus of FRP reinforcement; E_c as the elastic modulus of concrete; h_0 as the effective section height; ft as the concrete axial tensile strength; A_{f_v} as the effective cross-sectional area of FRP stirrups; s as the stirrup spacing; f_{fv} as the shear strength of FRP stirrups; f_{fd} as the tensile strength of FRP stirrups; γ_f as the material subfactor coefficient (safety factor) for FRP stirrups (typically 1.2); and γ_e as the environmental impact factor or durability reduction factor (normally 1.0).

In the U.S. standard ACI 440.1R-15, the shear resistance formula for FRP(5)-reinforced concrete beams is expressed as equations (5) to (8).

$$V=\frac{A_{fv}f_{fv}d}{s}+\frac{2}{5}b_wkd\sqrt{f_c'} \tag{5}$$

$$k=\sqrt{2E_f/E_c\rho_f+(E_f/E_c\rho_f)^2}-E_f/E_c\rho_f \tag{6}$$

$$\rho_f=\frac{A_{fv}}{b_ws} \tag{7}$$

$$f_{fv}=\min\left(0.004E_f,\left(0.3+0.05\frac{r_b}{d_b}\right)\right) \tag{8}$$

Where: k is the section shape coefficient; b_w is the section width; ρ_f is the longitudinal reinforcement ratio; E_f is the elastic modulus of FRP reinforcement; Ec is the elastic modulus of concrete; d is the effective height of the section; f_c' is the compressive strength of concrete; A_{fv} is the effective cross-sectional area of FRP stirrups; s is the stirrup spacing; f_{fv} is the shear strength of FRP stirrups; r_b is the radius of FRP stirrups; d_b is the diameter of FRP stirrups.

By comparing the shear capacity calculation formulas for FRP-reinforced concrete beams in the specifications of China and the United States, it is found that the two standards are largely consistent in the form of the FRP stirrup contribution term V_f, but there are significant differences in the concrete contribution term V_c: the ACI specification adopts the axial compressive strength of concrete, while the China specification adopts the tensile strength of concrete. In addition to the different selection of concrete strength indicators, the two standards also share common shortcomings in the factors considered, as shown in Table 1.

Table 1: Factors corresponding to standardized shear capacity calculation formulas in various countries

source	shear span ratio	Longitudinal reinforcement ratio	elastic modular ratio	regenerated aggregate	dowel action
GB 50608-2020	×	√	√	×	×
ACI440.1R-15	×	√	√	×	×

Neither Chinese nor American codes consider the shear span ratio, recycled aggregate, or bolted effects. For unreinforced FRP-reinforced recycled concrete beams, Vf = 0, meaning the beam's shear capacity is entirely provided by the concrete component Vc. The following section evaluates the code applicability for unreinforced specimens based on this premise.

3. Shear Capacity of Inclined Cross-sections in Reinforced Concrete Beams with Unreinforced FRP Strips

As previously mentioned, current Chinese and American codes do not account for shear span ratio, recycled aggregates, or bolt anchorage effects, with their formulas established based on experimental data from natural aggregate concrete. Consequently, direct application of these formulas to unreinforced FRP-reinforced recycled concrete beams would result in significant deviations. To quantify the impact of these factors, this study collected 31 sets of shear test data as shown in Table 2. This section first analyzes the actual effects of key parameters neglected by existing codes (shear span ratio, bolt anchorage, recycled aggregates) on shear capacity using the collected experimental data. Subsequently, we evaluate the deviation levels of current code formulas and propose revised formulations.

Table 2: Shear test data of rebars-free FRP-reinforced recycled concrete beams

data sources	Longitudinal tendon type	quantity	Concrete strength fc'/MPa	shear span ratio	Longitudinal reinforcement ratio/%	Beam cross-sectional width/mm	Beam section height/mm
Lan Youbin [12]	GFRP	2	33.4	1.5~2.5	2.27	200	400
Liu Huaxin [13]	BFRP	9	41	1.54~2.02	0.77~1.16	200	300~400
Chen Ming [14]	BFRP	4	37.6~42.8	2.1	1.16~1.21	200	300
Peng Changling [15]	BFRP	12	40~50.2	2.1~3.3	1.21~2.42	200	300
Bai Yajia [16]	BFRP	1	40	1.54	1.16	200	300
Zhang Zhijin [8]	BFRP	4	41.3~47.1	1.11~2.02	0.77~1.16	200	300~400

Based on Table 2 data, we first analyze the impact of shear span ratio on shear capacity. To visually demonstrate this relationship, experimental groups with similar parameters (e.g., reinforcement ratio, concrete strength, cross-sectional dimensions) but varying shear span ratios were selected, and scatter plots are shown in Figure 1. The results reveal that the shear capacity of recycled concrete beams exhibits a sharp nonlinear decline with increasing shear span ratios. When the ratio increases from 1.2 to 1.8, the capacity drops dramatically from 178 kN to 90 kN, representing a nearly 50% reduction. For λ <1.8, beams operate within the arch effect-dominated zone with higher capacity; however, when λ> 1.8, arch effects diminish sharply leading to significant capacity reduction. Consequently, if shear capacity calculation formulas disregard shear span ratio effects, results for low shear span ratio specimens may become overly conservative, while designs for high shear span ratio specimens may lack sufficient safety margins.

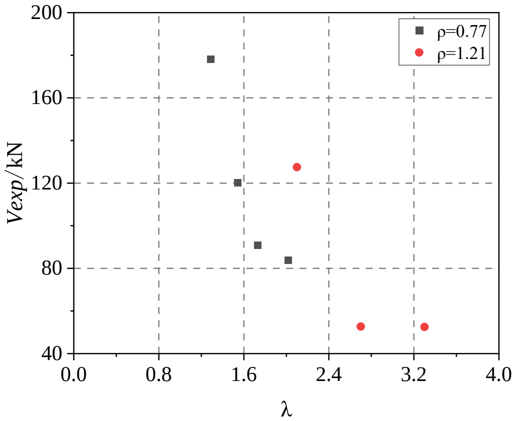

Figure 1: Scatter plot of shear span ratio versus shear capacity

In addition to the shear span ratio, the longitudinal reinforcement ratio is another critical factor affecting the shear capacity of unreinforced beams. Studies indicate that increased longitudinal reinforcement ratios lead to enhanced shear capacity, which can be attributed in part to the pinning effect of longitudinal bars. However, the shear contribution from pinning effects is typically minor and is generally not considered in existing code formulas. Given the challenges in isolating the independent contribution of experimental data, this study temporarily excludes the pinning effect term from the established calculation formula.

Similar to the bolt-reinforcement effect, the influence of recycled aggregates must be considered in formulas but requires distinct treatment approaches. As previously discussed, recycled concrete exhibits interface fragility due to residual old mortar adhesion. In conventional natural aggregate concrete, the extreme hardness of natural gravel causes shear cracks to bypass hard aggregate edges during propagation, forming highly tortuous fracture surfaces. These rough fracture surfaces generate tremendous mechanical interlocking forces between aggregates during shear displacement. Recycled aggregates, however, represent a composite structure combining primary natural aggregates with surface-adhered old mortar. The high porosity and multiple micro-cracks characteristic of old mortar create extremely fragile multi-interface transition zones between new and old mortar layers, as well as between old mortar and primary aggregates. When oblique principal tensile stress acts on recycled concrete beams, cracks no longer bypass aggregates but penetrate them directly due to insufficient strength of adhered old mortar, thereby weakening aggregate interlocking forces. All experimental data collected in this study were obtained from recycled concrete beams, meaning the derived modified formulas implicitly account for load-bearing capacity reduction caused by recycled aggregates without requiring additional correction coefficients.

To evaluate the accuracy of applying the aforementioned two codes directly to unreinforced FRP-reinforced recycled concrete beams, the shear capacity test values Vexp of each specimen were compared with the theoretical values Vpred calculated according to the China code GB 50608-2020 and the American code ACI 440.1R-15. The average ratio of test values to calculated values for the China code was 3.46, with a coefficient of variation of 107.87%. For the American code, the average ratio was 4.53, with a coefficient of variation of 167.11%. As shown in Figures 2 and 3, the shear capacity calculated by the China code GB 50608-2020 and the American code ACI 440.1R-15 was significantly lower than the actual shear capacity of the beams. Although all specimens exhibited calculated values below test values, the coefficients of variation were extremely high (107.87% for the China code and 167.11% for the American code), indicating highly unstable prediction deviations of the code formulas across different specimens. Some specimens showed calculated values too close to test values, resulting in insufficient safety margins, while others were overly conservative, leading to material waste. Therefore, directly applying existing design codes for FRP-reinforced ordinary concrete beams to evaluate unreinforced FRP-reinforced recycled concrete beams would fail to ensure adequate safety margins, necessitating the establishment of dedicated calculation formulas.

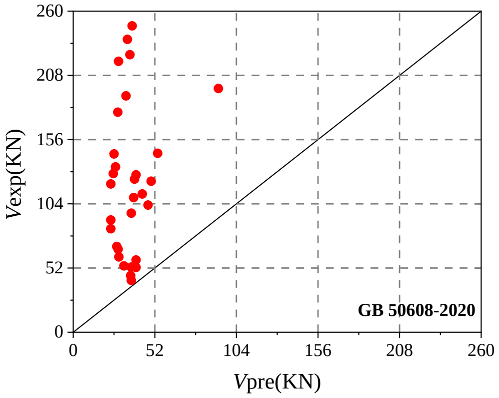

Figure 2: Scatter plot of experimental values and calculated values from the China specification recommended formula

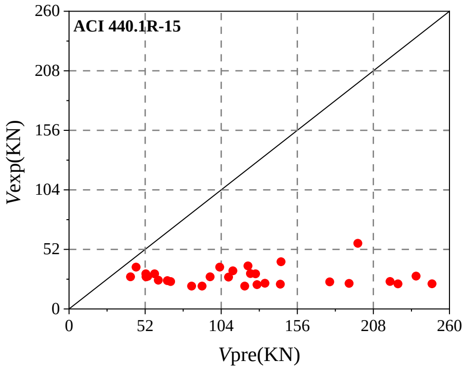

Figure 3: Scatter plot of experimental values versus calculated values using the formula recommended by U.S. specifications

Since the prediction results of China's standard GB 50608-2020 are relatively closer to the experimental values, and it adopts the axial tensile strength of concrete, which better aligns with the brittle failure characteristics of recycled concrete, this paper modifies the formula based on the concrete item in the standard. Based on the above analysis, the following adjustments were made when establishing the modified formula: (1) The shear span ratio significantly affects shear capacity but was not considered in the standard, requiring an independent correction term; (2) The contribution of pinning to shear resistance is minimal and is no longer introduced separately; (3) The influence of recycled aggregates is automatically accounted for in the fitting process due to all specimens using recycled concrete, with their average reduction effect already incorporated.

As shown in Figure 1, the shear capacity decreases with the increase of the shear span ratio, and the two variables approximately satisfy a power function relationship. Based on the concrete formula in China standard GB 50608-2020, a shear span ratio correction function is introduced. Since all (9)specimens (10)were recycled concrete, their reduction effect was implicitly accounted for during the fitting process. Finally, the modified shear capacity formula for unreinforced FRP-reinforced recycled concrete beams is presented as equations (9)~ (10):

$$V=0.86f_t b_w k \mathrm{h}_0 \frac{5.12}{\lambda} \tag{9}$$

$$k=\sqrt{2E_f/E_c\rho_f+(E_f/E_c\rho_f)^2}-E_f/E_c\rho_f \tag{10}$$

Where: k is the section shape coefficient; bw is the section width; ρf is the longitudinal reinforcement ratio; Ef is the elastic modulus of FRP reinforcement; Ec is the elastic modulus of concrete; h0 is the effective height of the section; ft is the axial tensile strength of concrete; λ is the shear span ratio; s is the stirrup spacing.

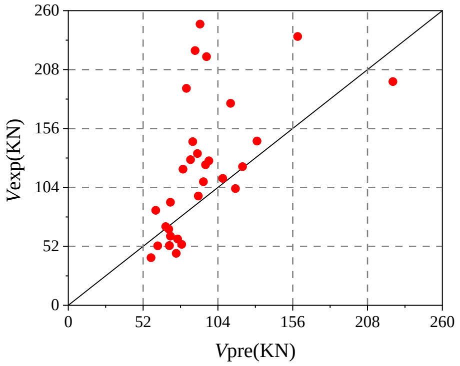

Figure 4: Scatter plot of experimental values versus calculated values using the correction formula

To validate the proposed formula's rationality, Table 2 data were substituted into Equation (10). The scatter plot comparing calculated values with experimental measurements is shown in Figure 4. The mean ratio of experimental to calculated values was 1.27, with a standard deviation of 0.306 and a coefficient of variation of 24.08%. These results demonstrate that the modified formula proposed in this study exhibits excellent predictive accuracy for unreinforced FRP-reinforced recycled concrete beams. Notably, significant deviations between calculated and experimental values were observed in some specimens (Figure 4), which may be attributed to size effect influences. Given the relatively narrow cross-sectional height variation range of collected specimens (300–400 mm), independent quantification of size effect impacts remains unfeasible. Future research should validate the formula across broader size ranges and incorporate size effect correction terms.

4. Conclusion

(1) The shear capacity of unreinforced FRP reinforced concrete beams calculated according to China Standard GB 50608-2020 and American Standard ACI 440.1R-15 is significantly lower than the experimental values, with extremely high coefficient of variation. Direct application may result in insufficient safety margins for some specimens and overly conservative results for others, making it unsuitable for such beams.

(2) The shear span ratio significantly affects the shear capacity. When the shear span ratio increases from 1.2 to 1.8, the capacity decreases by nearly 50%, with an approximate power function relationship between the two. Existing codes do not account for the shear span ratio, necessitating the introduction of an independent correction term.

(3) Based on the concrete item of China standard GB 50608-2020, a shear span ratio correction function was introduced to obtain a modified shear resistance formula. The mean ratio of calculated values to experimental values was 1.27, with a coefficient of variation of 24.08%, indicating good prediction accuracy.

(4) The significant deviations observed in some specimens may be attributed to size effects. However, the cross-sectional heights of specimens collected in this study were relatively narrow

(300−400 mm), making independent quantification unfeasible. It is recommended to incorporate size effect correction terms across a broader size range in future studies.

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