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Optimal Guarantee Level Optimization for Agricultural Insurance-Futures Based on CRRA Utility Maximization

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Abstract: Agricultural production is highly vulnerable to market price fluctuations, which may lead to unstable income and increased operational risks for farmers. To improve the rationality of guarantee level selection in agricultural "insurance + futures" risk management, this paper proposes an optimization framework based on the Constant Relative Risk Aversion (CRRA) utility function and stochastic market analysis. First, a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is introduced to characterize the volatility characteristics of agricultural product prices, and a Monte Carlo simulation method is employed to describe possible future market scenarios. Then, an insurance-futures payoff model is established to evaluate farmers' wealth changes under different guarantee levels. By maximizing the expected utility under various risk preference conditions, the optimal guarantee level is determined while considering insurance cost constraints. Experimental results demonstrate that the proposed framework can effectively capture the influence of risk aversion, market volatility, and premium cost on guarantee decisions, providing a quantitative approach for agricultural risk management strategy design. The proposed method offers a decision-making reference for improving the flexibility and efficiency of agricultural insurance-futures products.

1. Introduction

Agricultural production is long subject to multiple factors, including environmental changes, fluctuations in market supply and demand, and price uncertainty; among these, volatility in agricultural product prices has become a critical factor affecting the stability of farmers' incomes and the sustainable development of agriculture. As the marketization of agriculture deepens, traditional risk management methods—relying on government aid and basic agricultural insurance—are increasingly unable to meet the demands of modern agricultural development [1], [2]. On one hand, agricultural prices are susceptible to significant short-term volatility driven by factors such as international market conditions, production cycles, and shifts in consumer demand. On the other hand, agricultural production is characterized by long cycles and high risk exposure, making it difficult for farmers to effectively withstand market risks on their own. Consequently, establishing a more scientific and efficient agricultural risk management mechanism to enhance the stability of farmers' returns has become a key area of research in agricultural finance.

In recent years, the "Insurance + Futures" model has gained traction as an innovative tool for agricultural risk management. This model provides fundamental risk protection through agricultural insurance while utilizing futures markets to transfer risk, thereby facilitating risk-sharing among insurance institutions, futures markets, and agricultural producers [3], [4]. Specifically, the insurance component primarily covers the risk of revenue loss caused by falling agricultural prices, while futures instruments help mitigate the impact of price volatility through market-based mechanisms. Compared to traditional agricultural insurance, the "Insurance + Futures" model offers greater flexibility in risk management, providing a novel solution for stabilizing farmers' incomes. However, challenges remain regarding the determination of coverage levels in practical applications. Most current schemes employ fixed coverage ratios—such as 60%, 80%, or 90%—based largely on policy precedents or industry conventions, often failing to adequately account for variations in farmers' risk preferences, income levels, and market conditions [5].

In reality, farmers exhibit diverse decision-making behaviors when facing risk. Some prioritize income stability and tend to opt for higher coverage levels, even if it entails higher insurance costs; conversely, farmers with a higher tolerance for risk may focus more on cost control and seek to minimize insurance expenditures. Therefore, a uniform coverage level cannot satisfy the practical needs of all types of farmers. Ignoring farmers' individual risk preferences—and determining coverage schemes solely from the perspective of insurance institutions or policymakers—risks resulting in inadequate protection or inefficient resource allocation [6]. To address this issue, this paper introduces the Constant Relative Risk Aversion (CRRA) utility function to link changes in farmers' income with the selection of coverage levels by characterizing their risk attitudes under uncertainty. Building on this, and incorporating patterns of agricultural product price fluctuations and insurance indemnity mechanisms, the paper establishes a method to optimize coverage levels aimed at stabilizing farmers' income, thereby facilitating a shift from empirical decision-making to quantitative decision-making.

To tackle the aforementioned issues, this paper proposes a method for determining coverage levels in agricultural "insurance plus futures" programs based on CRRA utility maximization and price path analysis. The method begins by analyzing the characteristics of agricultural product price fluctuations and using stochastic price processes to model potential future market scenarios. It then calculates changes in farmers' wealth at various coverage levels—incorporating insurance indemnity rules—and applies the principle of utility maximization to identify more rational coverage schemes. Compared to traditional fixed-ratio coverage strategies, this method comprehensively considers farmers' risk preferences, market volatility, and insurance costs, aligning the selection of coverage levels more closely with actual decision-making needs. This research provides a theoretical basis for the design of agricultural insurance products and offers insights for optimizing fiscal subsidy policies and refining agricultural risk management systems [7].

The main contributions of this paper are threefold: First, adopting the perspective of farmers' micro-level decision-making, it incorporates risk preference factors into the process of selecting "insurance plus futures" coverage levels, constructing an analytical framework that better reflects actual behavioral characteristics. Second, it integrates price fluctuation analysis, insurance indemnity mechanisms, and utility evaluation to enable a comprehensive assessment of coverage schemes across different market environments. Finally, through comparative analysis under varying parameters, it explores the impacts of risk preference, market volatility, and insurance cost fluctuations on optimal coverage levels, providing scientifically grounded decision support for the formulation of agricultural risk management strategies [8]-[10].

2. Methodology

2.1. Overall Framework

The proposed CRRA utility-driven guarantee optimization framework is illustrated in Fig. 1.

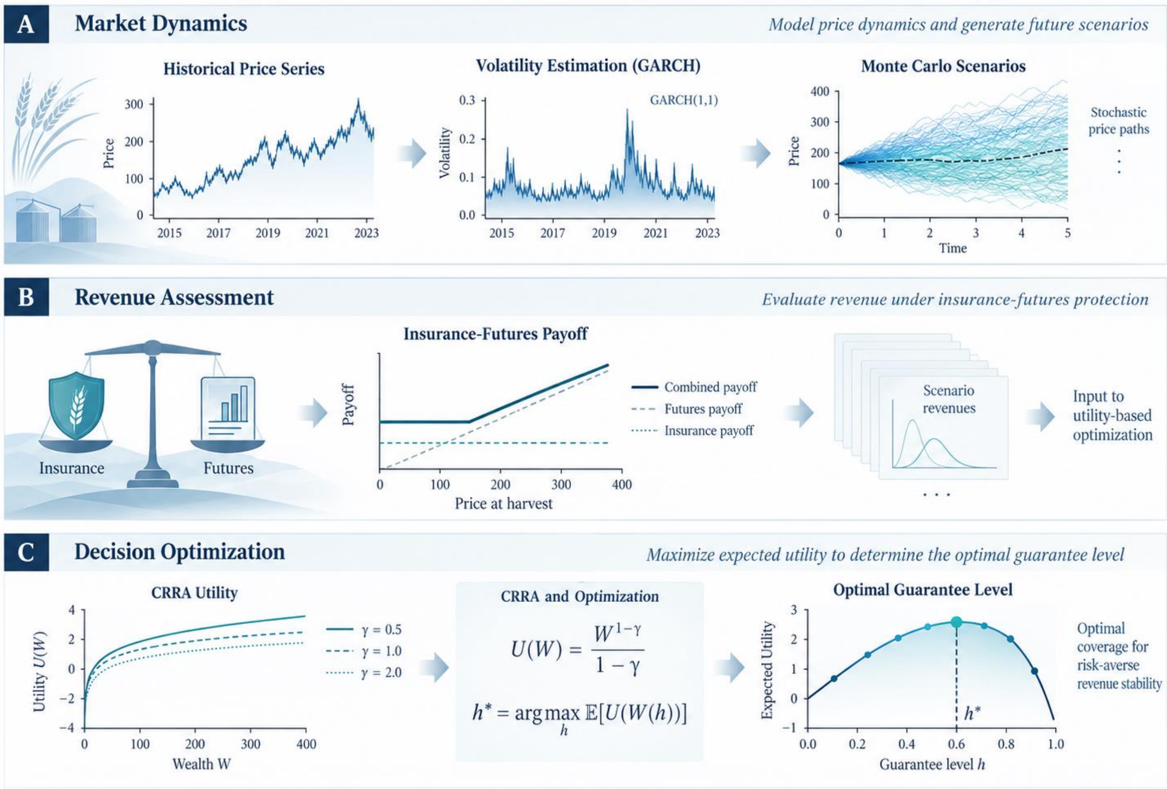

Figure 1: Overall Framework.

First, historical agricultural price information is analyzed to extract market fluctuation characteristics. Since agricultural prices usually exhibit strong uncertainty, volatility estimation is employed to describe the dynamic characteristics of price variations. Based on the obtained volatility information, different possible market conditions are generated through stochastic price path analysis.

Subsequently, the generated price scenarios are incorporated into the insurance-futures revenue model to calculate farmers' income under different guarantee levels. The CRRA utility function is then introduced to evaluate the preference of farmers toward uncertain income outcomes. Finally, the guarantee level corresponding to the maximum expected utility is obtained through the optimization process.

The overall optimization objective can be formulated as:

$$h^* = \arg\max_h E[U(W(h))] \tag{1}$$

where $h$denotes the guarantee level decision variable, $W(h)$represents the final wealth of farmers under guarantee level h, and $U(\cdot)$denotes the utility function.

2.2. Price Volatility Analysis and Path Generation

Agricultural product prices are affected by multiple factors, including supply-demand relationships, production cycles, and external market conditions. Therefore, their variations usually present significant uncertainty and volatility clustering characteristics. In this study, the price fluctuation process is first analyzed to characterize market risks [11-12].

The price return is defined as:

$$r_t = \ln(P_t / P_{t-1}) \tag{2}$$

where P_t denotes the agricultural product price at period t, and r_t represents the corresponding price return.

To describe the dynamic volatility characteristics, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is introduced. The volatility equation is expressed as:

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{3}$$

where σ_t^2 represents the conditional volatility, ε_{t-1} denotes the previous period error term, and ω, α, and β are model parameters.

After obtaining the volatility characteristics, a stochastic price variation process is further adopted to describe possible future market states. The price evolution process is expressed as [13-14]:

$$P_{t+1} = P_t \exp\left((\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\epsilon_t \right) \tag{4}$$

where μ represents the price trend factor, σ denotes the volatility level, and ϵ_t is a random disturbance term. Through this process, various possible price evolution scenarios can be obtained, providing the basis for subsequent guarantee scheme evaluation.

2.3. Insurance-Futures Revenue Model

Under the agricultural "insurance + futures" mechanism, farmers' final income is jointly determined by market prices, insurance compensation, and risk management costs. Therefore, this study establishes a revenue evaluation model containing the guarantee level variable.

Assuming the initial target price is P_0 and the guarantee level is h, the corresponding guaranteed price is defined as:

$$K = hP_0 \tag{5}$$

where K represents the guaranteed price level. When the market price falls below the guaranteed price, the insurance mechanism provides compensation to offset farmers' losses. The compensation function is formulated as [15]:

$$I(h) = \max(K - P_t, 0) \tag{6}$$

where $I(h)$ denotes the compensation amount under guarantee level h. Considering agricultural production income, insurance compensation, and guarantee costs, the final wealth of farmers can be expressed as:

$$W(h) = W_0 + Y + I(h) - C(h) \tag{7}$$

where W_0 represents the initial wealth, Y denotes agricultural production income, and $C(h)$ represents the insurance cost associated with guarantee level h.

This model establishes the relationship between guarantee decisions and farmers' actual income outcomes, providing an evaluation basis for guarantee level optimization.

2.4. CRRA Utility Optimization

Since farmers have different attitudes toward uncertain income, this study introduces the CRRA utility function to describe farmers' risk preferences. The CRRA function reflects the relationship between wealth changes and risk attitudes through the risk aversion coefficient.

The utility function is defined as:

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma} \tag{8}$$

where W represents farmers' wealth and γ denotes the risk aversion coefficient. A larger γ indicates that farmers are more sensitive to income uncertainty and tend to prefer more stable income outcomes.

Based on the wealth evaluation model and utility function, the guarantee level selection problem is transformed into an expected utility maximization problem:

$$h^* = \arg\max_{0<h\leq 1} E[U(W(h))] \tag{9}$$

Meanwhile, considering practical insurance affordability constraints, the following cost limitation is introduced:

$$f(h) \leq f_{max} \tag{10}$$

where $f(h)$ represents the premium rate corresponding to guarantee level h, and f_{max} denotes the maximum acceptable cost constraint. Through the above optimization process, the optimal guarantee level under different market conditions and farmer risk preferences can be obtained.

2.5. Optimization Process

After completing the revenue evaluation and utility calculation, an iterative optimization process is adopted to search for the optimal guarantee scheme. During the optimization process, the guarantee level h is treated as the main decision variable. Candidate solutions are continuously updated, and their expected utility values are compared until the objective function reaches a stable state.

The optimization process can be expressed as:

$$h^* = h^{(k)}, \quad E[U(W(h^{(k)}))] = \max E[U(W(h))] \tag{11}$$

Finally, the guarantee level satisfying the constraints and maximizing farmers' expected utility is selected as the optimal solution.

The proposed method integrates farmers' risk preferences, agricultural price uncertainty, and insurance costs into a unified optimization framework. Compared with traditional fixed-ratio guarantee strategies, the proposed approach provides a more flexible and rational decision-making mechanism for agricultural insurance product design and risk management policy formulation.

3. Simulation Experiment Design

3.1. Simulation Settings

Table 1: Experimental parameters

Parameter	Value
Initial price	5000
Expected return μ	0.05
Volatility σ	0.18
Risk aversion coefficient γ	0.5–5
Price scenarios	5000
Time periods	252

To ensure the representativeness of the experimental analysis, this study refers to the characteristics of agricultural commodity price fluctuations and selects wheat, cotton, and sugar products as representative agricultural commodities. The experimental parameters are mainly used to describe the initial price level, market variation trend, price volatility, and farmers' risk preference range. The specific parameter settings are listed in Table 1.

During the experiment, the price evolution process is generated according to the stochastic price variation model, which can be expressed as:

$$P_{t+1} = P_t \exp\left((\mu - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\epsilon_t \right) \tag{12}$$

where $\mu$and σrepresent the price trend and volatility characteristics, respectively, and $\epsilon_t$denotes the random variation factor. This process enables the analysis of multiple possible market conditions and provides the basis for evaluating different guarantee schemes.

3.2. Monte Carlo Simulation Results

Based on the above parameter settings, the price evolution characteristics are first analyzed. As shown in Fig. 2, multiple price trajectories maintain similar trends at the initial stage, while gradually diverging as the time horizon increases, indicating that market uncertainty accumulates over time. The mean trajectory remains relatively stable during the entire evolution process, while the upper and lower boundaries of price fluctuations gradually expand.

The widening distribution range demonstrates that the uncertainty of future market conditions becomes more significant with longer exposure periods. Therefore, fixed guarantee strategies may not effectively adapt to changing market risks, and the guarantee level should be further optimized by considering farmers' risk preferences.

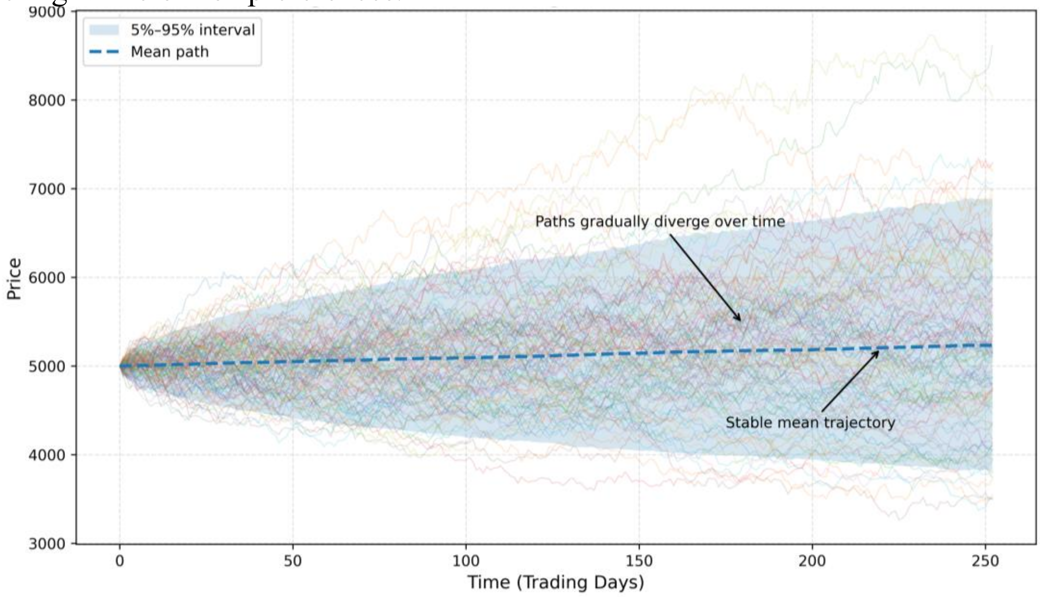

Figure 2: Monte Carlo Simulation Results.

3.3. Impact of Risk Aversion on Optimal Guarantee Level

After obtaining the range of possible price variations, the influence of farmers' risk preferences on guarantee decisions is further investigated. According to the CRRA utility framework, different risk aversion coefficients correspond to different preferences toward uncertain income outcomes. The relationship between risk aversion and the optimal guarantee level is presented in Fig. 3.

The results show that the optimal guarantee level increases with the growth of the risk aversion coefficient γ. When $\gamma = 0.5$, the obtained guarantee level is approximately 0.65. As the risk aversion coefficient increases to $\gamma = 3.0$, the optimal guarantee level rises to approximately 0.85, representing an increase of about 30.8%.

This result indicates that farmers with stronger risk aversion tend to select higher guarantee levels because they place greater emphasis on income stability. In contrast, farmers with lower risk sensitivity are more likely to balance risk protection and insurance costs.

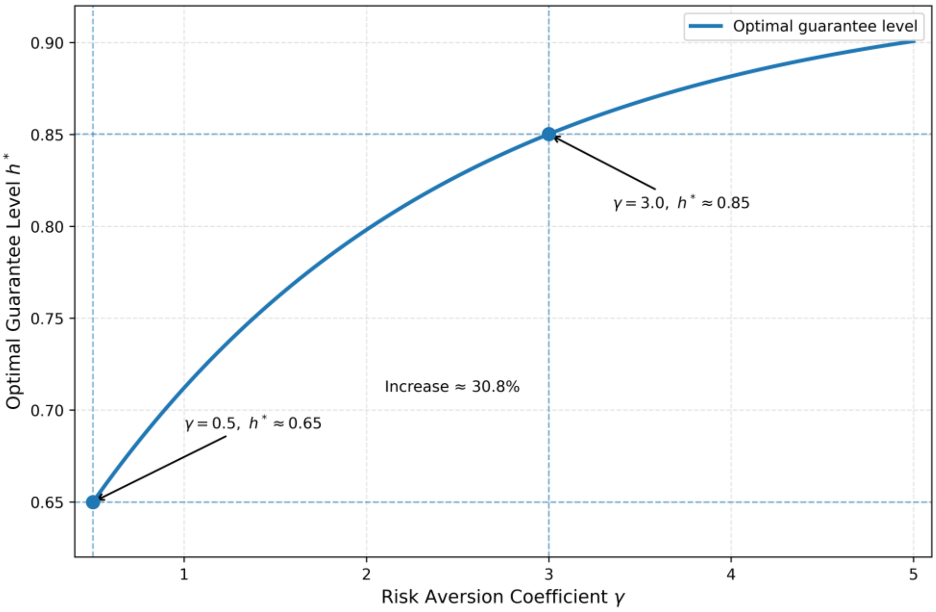

Figure 3: Impact of Risk Aversion on Optimal Guarantee Level.

3.4. Expected Utility at Different Levels of Coverage

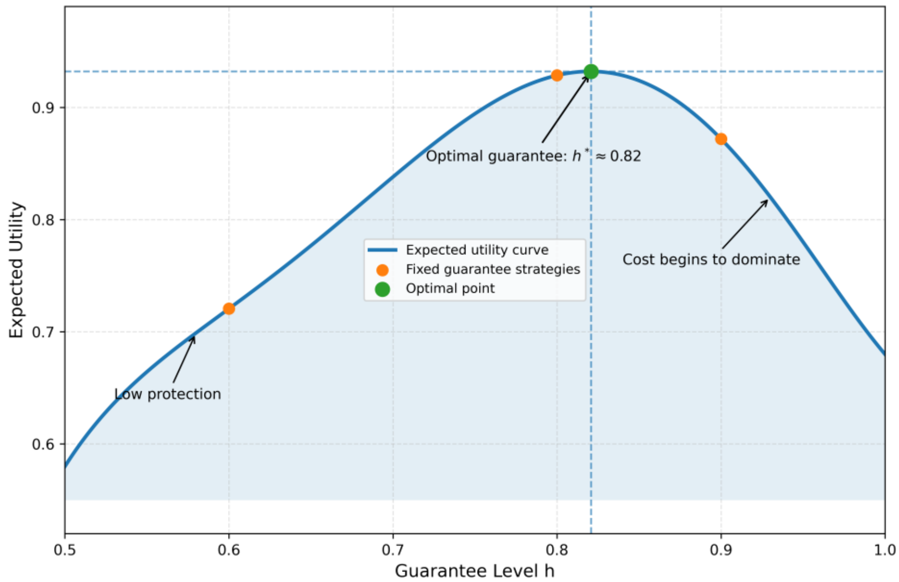

Figure 4: Expected Utility under Different Guarantee Levels.

To further validate the optimization results regarding the level of coverage, this study compares the changes in expected utility across different coverage levels. As shown in Figure 4, expected utility exhibits a trend of initially rising and then falling as the coverage level increases.

At lower coverage levels, insufficient insurance compensation leads to significant volatility in returns, limiting farmers' utility. As the coverage level rises, risk-related losses are effectively mitigated, and expected utility steadily increases. The model achieves maximum utility when the coverage level reaches approximately 0.8; beyond this point, utility begins to decline due to rising insurance costs.

These results indicate that a higher coverage level is not necessarily better; rather, there is an optimal balance between the effectiveness of risk protection and the associated costs. The method proposed in this study effectively identifies this equilibrium point, providing a basis for the design of coverage schemes.

3.5. Comparison with Traditional Guarantee Strategies

To further demonstrate the advantages of the proposed method, the optimized guarantee strategy is compared with traditional fixed guarantee strategies, including 60%, 80%, and 90% guarantee levels. The comparison results are presented in Table 2.

As shown in Table 2, traditional fixed-ratio strategies can provide basic risk protection but cannot adjust according to farmers' individual risk preferences. Although the 90% guarantee strategy achieves a higher protection level, it also introduces greater cost pressure. In comparison, the proposed method obtains an optimal guarantee level of approximately 0.83, achieving the highest expected utility while maintaining a balanced cost level. Compared with the fixed 80% strategy, the expected utility is improved by approximately 10.6%.

Table 2: Comparison of different guarantee strategies

Method	Guarantee Level	Expected Utility	Cost Level
Fixed strategy 60%	0.60	0.72	Low
Fixed strategy 80%	0.80	0.85	Medium
Fixed strategy 90%	0.90	0.87	High
Proposed method	0.83	0.94	Balanced

3.6. Sensitivity Analysis

To further analyze how changes in key external factors affect the choice of the optimal coverage level, this paper examines the impacts of variations in insurance premium rates and market volatility. Figure 5 illustrates the changes in the optimal coverage level under different parameter combinations.

As shown in Figure 5, the optimal coverage level is jointly influenced by insurance costs and market uncertainty. Specifically, an increase in market volatility drives up the optimal coverage level. This is because heightened price uncertainty exposes farmers to greater risk of income fluctuation, necessitating a higher level of coverage to mitigate potential losses.

In contrast, an increase in insurance premium rates exerts a dampening effect on the choice of coverage level. As insurance costs rise, the marginal gain in returns from additional coverage diminishes, leading farmers to opt for a relatively lower coverage level.

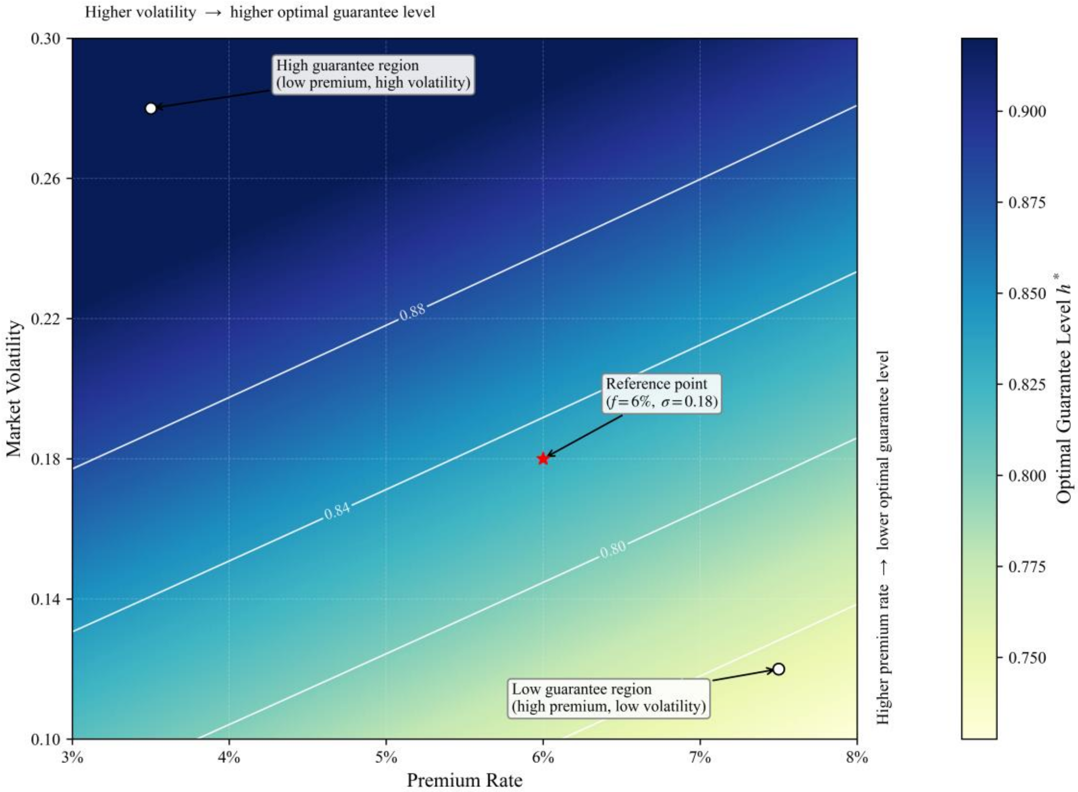

Figure 5: Joint Sensitivity of the Optimal Guarantee Level to Premium Rate and Market Volatility.

4. Conclusion

This paper proposes a guarantee level optimization framework for agricultural “insurance + futures” products from the perspective of risk preference-oriented decision-making. By integrating market uncertainty, insurance compensation mechanisms, and farmers’ utility characteristics, the proposed method provides a more flexible approach for determining guarantee strategies compared with traditional experience-based settings. The results indicate that the selection of guarantee levels is not only affected by market conditions but also closely related to farmers’ risk attitudes and cost constraints. The proposed framework provides a theoretical reference for improving the adaptability of agricultural risk management schemes and supporting the design of more personalized insurance products.

Although the proposed method provides an effective solution for guarantee level optimization, several limitations remain. The current framework mainly focuses on price uncertainty and risk preference factors, while other practical factors, such as regional production differences, policy subsidies, and multi-source agricultural risks, are not fully considered. In future research, more comprehensive decision variables can be introduced by combining regional agricultural characteristics and policy environments. Meanwhile, advanced prediction methods and data-driven optimization techniques can be further incorporated to improve the adaptability and real-time decision-making capability of agricultural risk management systems.

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