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## Introduction

Vibration as a phenomenon has been studied by many researchers from different fields. This phenomenon exists in both mechanical systems and various civil structures. Vibration in civil structures can cause health problems for humans such as dizziness, nausea and anxiety [1]. If the amplitude of vibration is large enough, it may lead to catastrophic consequence. For example, the Tacoma Narrows Bridge (**Figure 1.1**) was destroyed only four months after its opening in 1940 due to vibrations caused by wind [2].



**Figure 1.1** Tacoma Narrow Bridge of Unite States in 1940 [2]

Compared to vibration in civil structures, vibrations within mechanical systems are far more common, and also cause tremendous problems such as shortened machine life span, reduced manufacturing precision, poor product quality, and noise [3]. In particular for modern transportation devices, the existence of vibration causes more severe problems than simply the discomfort of passengers. In some extreme cases, such vibrations will cause the malfunction of the device, which may inevitably lead to fatal

accidents [4].

Based on all the above reasons, vibration control in civil structures and mechanical systems is extremely important. In the last several decades, researchers have been achieving remarkable progress in these fields, especially for mechanical vibration control. The classical vibration control of mechanical systems can be further divided into vibration isolation and vibration absorption by methods of application.

Vibration isolation is often applied to a system which is fixed at one point. By minimizing the vibration at the attaching point to the excitation source, this approach usually yields good performance. However, when this system is subjected to multiple excitation sources, the control strategy of vibration isolation may become very complicated [5,6].

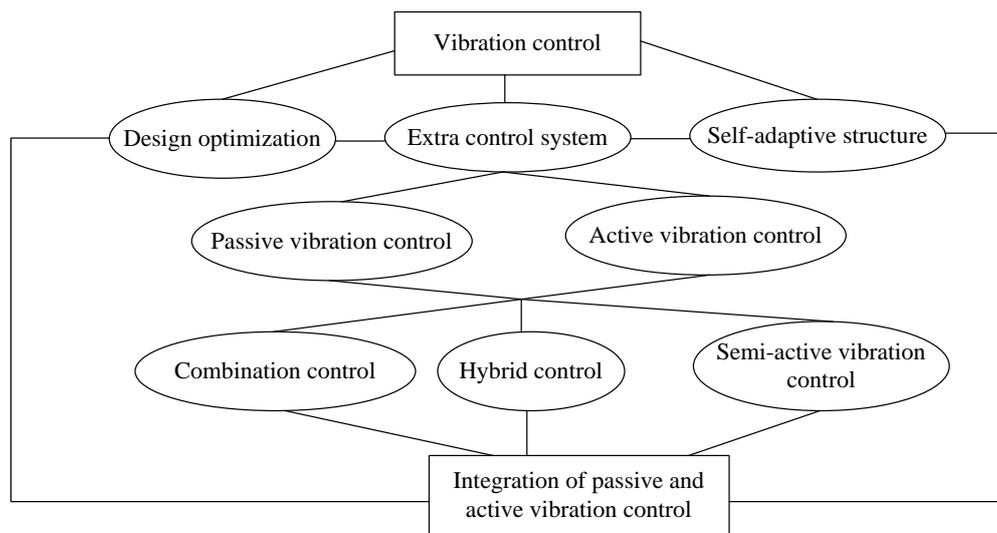
Compared to vibration isolation, vibration absorption achieves the goal of vibration control by using vibration absorbers. In most cases, the vibration absorber is a secondary system which consists of a mass, spring and damper [7,8]. For example, the common vibration absorber, a car shock absorber, dissipates vibration energy to suppress the vibration of the vehicle body, which in turn provides improved ride comfort for the driver and passengers [9,10]. A typical car vibration absorber is shown in **Figure 1.2**.



**Figure 1.2** Classic shock absorber [11]

With the development of simulation and analysis tools, system design can be

optimized by modeling and simulation. The application of modern control theory explores many new ways to achieve effective vibration control [1]. The general development of strategies for vibration control is shown in **Figure 1.3**. From **Figure 1.3**, it can be seen that there are three major ways to achieve vibration control: traditional design optimization which occurs at the system design stage, an additional vibration control system that is added to the structure (extra control system) and design of a self-adaptive structure. As a popular control method, the addition of an extra control system has been attracting much attention. Specifically, passive vibration control and active vibration control are two major approaches to the extra vibration control system. By combining these two vibration control concepts, combination vibration control, hybrid vibration control, often also referred to as semi-active vibration control, has been proposed.



**Figure 1.3** Diagram for strategy of vibration control

Vibration control can be further classified into passive vibration control and active vibration control. Due to its high reliability and low cost, passive vibration control has wide industrial application [12,13]. Usually, passive vibration control is related to the movement of a target system which requires vibration control. By dissipating vibration energy and isolating the transmission of vibration, the intent is for the whole system to

eventually trend towards stability [14] .

On the other hand, active vibration control may be either related to the target system, or independent from the system, depending on the excitation [15,16]. Since there is an actuator that is powered by an outside energy source, an active vibration control system can not only cause the decay of vibrations, but may also cause amplification of vibration, potentially leading to system instability. In order to build a feasible active control system, the determination of controller parameters is critical to the whole design. In most cases, a close-loop feedback control system is chosen and the input excitation of the actuator will be a function of system state [17,18]. The most important feature of active vibration control is high controllability, but due to high cost and low reliability, widespread application of active vibration control is still limited [19]. A comparison between passive and active vibration control is shown in **Table 1.1**.

In the past several decades, researchers have achieved impressive progress in active vibration control [20–23]. Nakano even proposed a self-powered active vibration control system using a piezoelectric actuator [24]. Concomitantly, many researchers have focused on semi-active vibration control by developing variable damping ratio or variable stiffness systems [25–27].

Recently, research on passive vibration control has evolved in many directions [28]. For instance, Behrens et al. presented a design for passive vibration control via electromagnetic shunt damping [13]. By designing an appropriate electrical shunt, a transducer was shown to be capable of significantly reducing mechanical vibrations. Another design adjusts the natural frequency of the system to suppress vibration by adding a large mass. This concept is referred to as a tuned mass damper (TMD) [29–31]. However, both designs require extra components to achieve passive vibration control, which eventually lead to higher cost. In a related theme, many researchers have focused on using flywheels to achieve passive vibration control. Smith [32] proposed a new device named the inerter, and Rivin [33] also developed a spiral flywheel. Both devices transfer linear motion into the rotation of a flywheel, which in turn generates equivalent inertial mass equal to as much as 400 times that of the

gravitational mass of the flywheel. Wang and Wu [34] applied this inerter to vibration control of an optical table, and experimental results were promising. Based on the same theory, Li et al. [35–38] proposed several two-terminal devices to achieve passive vibration control, such as a two-terminal inverse screw transmission system and a two-terminal hydraulic system. One of these designs was shown capable of obtaining a variable two-terminal mass by adjusting an electro-hydraulic proportional valve. Experiments were carried out, and the results agreed with the design requirements. Despite all this progress, none of these approaches can passively generate variable equivalent mass and industrial applications are still rare. Xu investigates a different design for the flywheel, which led to the creation of variable mass [39].

**Table 1.1** Comparisons between active and passive vibration control

Objective Classification	Working principle	Controllability	Robustness	Stability	Reliability	Extra mass	Energy source required	cost
Passive vibration control	Simple	Low	Good	Good	High	Small	None	Low
Active vibration control	Complicated	High	Depends on controller	Depends on controller	Low	Large	High	High

## **Chapter 1. The basic concepts of vibration**

### **1.1. The classification of vibration**

Due to different classification methods, the types of vibration can be variable. In this book, the types of vibration are defined by six classification methods, which are the type of excitation, the physical characteristics of vibration system, vibration period, the characteristics of the vibration excitation, the degree of freedom for vibration system and the characteristics of vibration movement.

#### **1.1.1. The type of excitation**

##### 1.1.1.1. Free vibration

After the initial excitation, the vibration system can vibrate at its natural frequency based on the elastic force without any external excitation. Vibration under this kind of excitation with an initial movement or initial speed is defined as the free vibration. The characteristics of free vibration depends on the physical characteristics of the system, such as the mass, stiffness and damping.

##### 1.1.1.2. Forced vibration

Forced vibration is also named compulsive vibration. Under the continuous action of external periodic forces, the vibration of this system is defined as forced vibration. The external periodic force is also called driving force (or force). After the forced vibration of the object reaches a steady state, the frequency of the vibration is the same as the frequency of the driving force, which is independent with the natural frequency of the object.

### 1.1.1.3. Self-excited vibration

Self-excited vibration is one kind of nonlinear vibration, which is maintained by the excitation generated by the system itself. The self-excited vibration system not only has a vibration element, but also has a non-oscillating energy source, a regulation link, and a feedback link. Therefore, it can also produce a stable periodic vibration when there is no external excitation. The alternating force that sustains self-excited vibration is generated by the movement itself, which is controlled by the feedback and regulation links. Once the movement stops, the alternating force will also disappear, which lead the stop of the self-excited vibration.

Self-excited vibration is another large quantity form of vibration in addition to free vibration and forced vibration. The energy of self-excited vibration system is provided by outside-energy source. However, unlike forced vibration, the energy of self-excited vibration is constant.

## **1.1.2. The physical characteristics of vibration system**

### 1.1.2.1. Linear vibration

The relationship between the constant mass, damping force, elastic force and velocity, displacement is linear in this system, which can be defined as a linear system. The movement of this system can be presented by constant coefficient linear differential equation. The vibration of this linear system under deterministic excitation is named as linear vibration.

### 1.1.2.2. Non-linear vibration

In the contrary, if the relationship between the constant mass, damping force, elastic force and velocity, displacement is non-linear in this system, which will be defined as a non-linear system. The movement of this system can be written as non-linear differential equation. The vibration generated by this system is defined as non-

linear vibration.

### **1.1.3. Vibration period**

#### 1.1.3.1. Periodic vibration

The displacement, velocity and acceleration of a vibration system show periodic changes under an identical time interval, this vibration of this system can be defined as periodic vibration. The simple harmonic vibration is a typical vibration, which can be presented as sinusoidal function or cosine function.

#### 1.1.3.2. Non-periodic vibration

The physical factors of a vibration system show non-periodic changes with time, the vibration of this system can be named as non-periodic vibration. Most vibration are non-periodic vibration, among them, transient vibration is a typical non-periodic vibration.

### **1.1.4. The characteristics of the vibration excitation**

#### 1.1.4.1. Deterministic vibration

If the physical characteristics of this system is deterministic, and the excitation for this system is also certain, which means the response of this system can be determined. The vibration of this system can be defined as deterministic vibration. For a deterministic excitation, the amplitude and changing regulation can be presented as deterministic functions with time, such as periodic function or impulse function.

#### 1.1.4.2. Random vibration

When the excitation for a vibration system is random, the response of this system will be random, which will be named as random vibration. For random excitation, the

amplitude and changing regulation are un-predictable, which cannot be written as deterministic functions with time, such as wind, earth quake, and sea wave. However, there are statistical regularity for random vibration, which can be analyzed by statistical theory.

### **1.1.5. The degree of freedom for vibration system**

#### 1.1.5.1. One degree of freedom vibration

During the vibration, any geometric position of this system can be represented by single coordinate, this system can be defined as system with one degree of freedom. The vibration of this system is the one degree of freedom vibration.

#### 1.1.5.2. Multiple degrees of freedom vibration

In the vibration, the any geometric position of this system can be represented by multiple coordinates, this system will be named as system with multiple degrees of freedom. The vibration on this system can be defined as multiple degrees of freedom vibration.

#### 1.1.5.3. infinite degrees of freedom vibration

The any geometric position of a system can be represented by infinite coordinates during the vibration, this system will be named as system with infinite degrees of freedom. The vibration on this system can be defined as infinite degrees of freedom vibration.

### **1.1.6. The characteristics of vibration movement**

In the industrial case, there are always direction for the vibration. Based on this characteristic, the vibration can also be classified by vibration direction, which are listed as follows:

1) Vertical vibration: The vibration direction is parallel to the axis of the vibration system.

2) Lateral vibration: The vibration direction is perpendicular to the axis of the vibration system.

3) Rotation Vibration: The vibration direction is rotated with the axis of the vibration system.

4) Pendulum vibration: The vibration is occurred near the balance position of pendulum system, which is perpendicular to the axis of the vibration system.

## **1.2. The research of vibration dynamics**

To achieve vibration, other than a vibration system, the external excitation is also needed, which means the vibration of a vibration system is based on the external excitation. Take the external as an input, the response of the vibration system can be defined as output, the relationship between the input and output depends on the characteristics of the vibration system.

As mentioned before, for a complete vibration system, there are three major parts, which are input, output and system characteristics. The vibration status of this system depends on the external excitation and the characteristic of the vibration system. Therefore, for the research of vibration, besides the vibration status and system response, the relationship between excitation and response, and the effect of vibration caused by the system characteristic are also focused by many researchers. The research of vibration dynamics is divided into three parts, which are vibration analysis, system identification and excitation identification respectively.

### **1.2.1. Vibration analysis**

With given external excitation and system characteristics, the response of this system can be determined. This process of solving the response of a vibration system is called vibration analysis. This is the positive research for vibration problem, which

is also the most well-developed and most traditional research method for vibration. As the most common problem in industrial design, there are many effective numerical method and commercial software other than analytical method, which can be applied on engineering design and analysis with excellent results. Vibration analysis is the fundament for structural design, which is also the basic skill for structure or mechanical engineer.

### **1.2.2. System identification**

By given external excitation and system response, the system characteristics of this system can be obtained. This process of the determination of system characteristic is defined as system identification. With certain external excitation, the system characteristic can be determined to satisfy the given response of this system, which can be named as system design. With certain external excitation and system response, the parameters such as frequency, vibration state and damping can be determined with the experimental data, which is defined as parameters identification. System identification is the first type inverse problem in vibration research. The relative hardware and software has been developing rapidly in last ten years, and the technology and theory of identification is relatively complete.

### **1.2.3. Excitation identification**

With the system response and characteristics, the process of finding or identifying the vibration excitation is called excitation identification. Under certain circumstance, the vibration system will be affected by the passive excitation in the environment. The excitation identification for this situation is also defined as vibration environment prediction. In a real vibration case, the vibration situations are usually extremely complicated. The external excitation can be variable, intercoupling and interaction, which means the excitation identification and mathematical model for complicated situation is much more difficult than common situation. To solve this kind of problem,

the research on the first type inverse problem is needed, which will also combine with identification, analysis and design for multiple coordination. This research is the second type of inverse problem on vibration research. At this moment, this research about this problem is still under developing.