

# Study on the Quantization Error of a Coherent Inverse Synthetic Aperture Radar Digital Moving Target Simulator

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**Abstract.** When generating the simulation echo by an inverse synthetic aperture radar (ISAR) digital moving target simulator (MTS), the quantization error of time delay caused by the digital-to-analog (DA) devices will destroy the consistency between the simulated echo and the ideal echo, result in image blur. In order to eliminate the quantization error, two compensation approaches are proposed in this paper. The quantization error between the simulated echo and the ideal echo and its influence on the imaging results are analysed. As the quantization error is fixed, the compensation methods in the time domain and the frequency domain are introduced, respectively. The simulation results show that the proposed method is effective and have excellent performance.

## 1. Introduction

Wideband radar has high range resolution which can be obtained by means of pulse compression. Inverse synthetic aperture radar (ISAR) is a typical wideband radar, which has a very good performance in identify targets [1]. Since it can capture two-dimensional images and extract a number of target features from the target echo. So it is widely used in space target imaging, classification, recognition etc. [2-4]. Taking into account the ISAR has a broad application prospects, many scholars do a deep research in this field.

In order to verify the effectiveness of the ISAR algorithm, the echo simulator was designed to solve the problem economically and effectively [5]. Earlier simulator was designed based on delay line, which is simple and effective to simulate the distance Doppler shift, Radar Cross Section (RCS) and other characteristics [6]. With the development of digital circuit technology, the design of the Moving Target Simulator (MTS) simulator is gradually developed [7]. But the quantization error has not been considered in these papers.

This paper studies the quantization error elimination theory in the digital MTS based on the digital signal processor (DSP) with high precision. When generating simulated echo in ISAR MTS, the digital to analog (DA) devices are used to quantify the delay time, where the quantization error will be generated inevitably. So the simulated echo will make ISAR imaging blurred. In order to

eliminate this error, two compensation ways are proposed. At first, the phase error between the simulated echo and the real echo and its influence on the imaging results are analysed theoretically; then, we investigate that each pulse quantization error is fixed, so the phase compensation methods in the time domain and the frequency domain to eliminate the phase error are proposed, the simulation results show that the proposed method is effective and excellent. The contribution of this paper is to investigate the quantization error, and propose two methods of compensation.

## 2. Quantization Error Generation

Chirp pulse signal is commonly used to obtain target image in an ISAR. Assume that the pulse repetition period of the chirp signal is  $T_r$ , the carrier frequency is  $f_c$ , the chirp rate is  $\mu$ , the pulse width is  $T_p$ , the chirp signal can be expressed as:

$$s_t(t_r, t_m) = \text{rect}\left(\frac{t_r}{T_p}\right) \exp(j\pi\mu t_r^2) \exp(j2\pi f_c t) = p(t_r) \exp(j2\pi f_c t) \quad (1)$$

Where  $p(t_r) = \text{rect}(t_r / T_p) \exp(j\pi\mu t_r^2)$ , designates the complex envelope. When  $|t| \leq (T_p / 2)$ , yields 1 and otherwise it is 0.  $t = t_r + t_m$ , where  $t_r$  denotes the fast time,  $t_m = mT_r$  represents the slow time, respectively, R is the distance between ISAR and the target centre. The echo signal received by the ISAR is given as bellow.

$$s_{rb}(t_r, t_m) = p\left(t_r - \frac{2R}{c}\right) \exp(-j2\pi f_c \frac{2R}{c}) = p(t_r - \tau) \exp(-j2\pi f_c \tau) \quad (2)$$

Where  $\tau = 2R/c$  gives the delay time which then quantified by the DA device. Assume the quantization cycle is  $T_{dl}$ , the quantified expression is obtained as:

$$\bar{\tau} = [\tau / T_{dl}] T_{dl} > \tau \quad (3)$$

Where  $[\ ]$  denotes the ceiling operation. According to the Eq.(3), the simulated pulse is transmitted by the MTS at starting time instance  $\bar{\tau}$ . So the time delay error between real time delay and simulated one is written as:

$$\Delta\tau = \bar{\tau} - \tau \quad (4)$$

The time delay error  $\Delta\tau$  is a non-random variable. Since both are deterministic variables with respect to the MTS. the compensation procedure shown below is easy to implement. Substituting Eq. (4) in Eq. (2), we get Eq. (5).

$$\begin{aligned} s_{rb}(t_r, t_m) &= p(t_r - \bar{\tau}) \exp(-j2\pi f_c \tau) = \text{rect}\left(\frac{t_r - \bar{\tau}}{T_p}\right) \exp\left(j\pi\mu(t_r - (\tau + \Delta\tau))^2\right) \exp(-j2\pi f_c \tau) \\ &= \text{rect}\left(\frac{t_r - \bar{\tau}}{T_p}\right) \exp\left(j\pi\mu(t_r - \tau)^2\right) \exp(-j2\pi f_c \tau) \exp\left(j\pi\mu(\Delta\tau^2 - 2\Delta\tau(t_r - \tau))\right) \\ &= s_{rb}(t_r, t_m) \exp\left(j\pi\mu(\Delta\tau^2 - 2\Delta\tau(t_r - \tau))\right) \end{aligned} \quad (5)$$

As seen in Eq.(5), the phase error caused by the quantization operation is written as:

$$\Delta\phi = \pi\mu(\Delta\tau^2 - 2\Delta\tau(t_r - \tau)) \quad (6)$$

This time-variable phase error  $\Delta\phi$  is quite small and its impact on range profile can be neglected. However, the influence on the Doppler shift of the echo signal need to be considered. By removing the high-order item and the time-variable part, Eq. (6) can be approximated as below:

$$\Delta\phi \approx 2\pi\mu\Delta\tau \times \tau \quad (7)$$

As seen from Eq. (7), the frequency rate is generally constant for a specific ISAR. Therefore, the maximum phase error is:

$$\Delta\varphi_{\max} = 2\pi\mu\tau T_{dl} \quad (8)$$

Assumed that the bandwidth is 1GHz, the pulse width is 0.5us, R=100Km,  $T_{dl}$ =0.01ns, we get  $\Delta\varphi_{\max} = 83.7^\circ$ . Such a phase error will blur the image seriously. From the expression (8), we can conclude that the smaller  $T_{dl}$ , the smaller phase error. The results showed that: as for a MTS,  $\bar{\tau}$  and  $\tau$  are fixed, it is feasible to eliminate the quantization error by compensation operation on DSP, which has a high precision.

### 3. Quantization Error Eliminate

Since the existence of quantization error will inevitably lead to ISAR imaging blur, the elimination of quantization error is the key problem that must be considered in the design of ISAR echo simulator. Considered quantization error is fixed, and high accuracy DSP devices can compensate the quantization error, two compensation methods operating in time-domain and frequency-domain are introduced in this part.

#### 3.1. Time-Domain Compensation

In the process of ISAR sampling, the sampling interval of the receiver is larger than the DA interval, that is  $T_{dl} < T_s$ , where  $T_s$  is the sampling period of the receiver. According to Eq. (2) and Eq. (5), the relationship between real pulse and simulated pulse is shown in Figure 1.

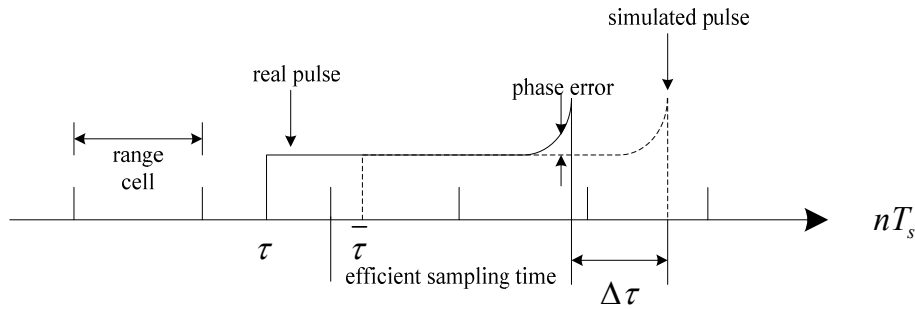


Figure 1. The relationship between real pulse and simulated pulse

From the Figure 1, we can see that the two pulses are parabolic, which indicates that the phase of the current frequency-modulated wave is time dependent. Furthermore we can clearly see that, the two pulses are spaced by the phase error  $\Delta\varphi$ . Due to phase error, the real pulse and the simulated pulse may be located at different range cells and their phases are different at the efficient sampling time instants. This is the source of error. And this error is fixed, therefore, the time domain compensating algorithm is presented as follow. At first, the simulated pulse and real pulse must be placed at the same range cell, and the operation can be implemented by translating the simulated pulse along the time axis as shown in Figure 2.

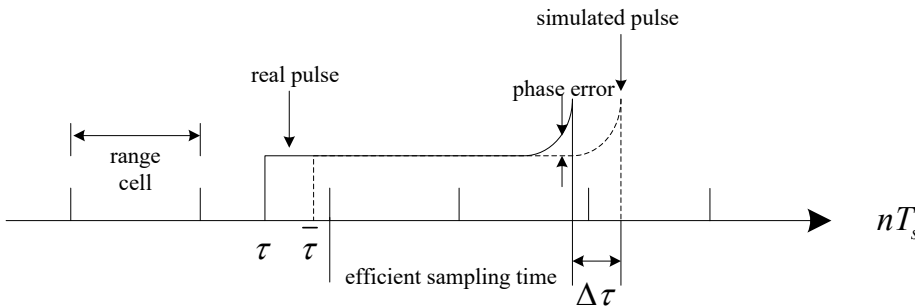


Figure 2. The relationship between real pulse and simulated pulse

In this way, the error between the real pulse and the simulated pulse get smaller and the phase error compensation only need to consider the same range cell. Then, taking advantage of the high precision of the floating-point DSP computing, one can use DSP to calculate the compensation factor  $comp$ . The compensation factor of the solutions can be expressed as:

$$comp = \exp(-j\pi\mu(\Delta\tau^2 - 2\Delta\tau(t_r - \tau))) \approx \exp(-j2\pi\mu\tau \times \Delta\tau) \quad (9)$$

Since the compensation factor is conjugate to the phase error in time domain, it will eliminate the phase error. The compensation results can be derived as:

$$s_{rb}(t_r, t_m) = s_{rb}^-(t_r, t_m) \times \exp(-j\pi\mu(\Delta\tau^2 - 2\Delta\tau(t_r - \tau))) \quad (10)$$

### 3.2. Frequency-Domain Compensation

Time-domain compensation algorithm can eliminate the phase error, we also consider this problem in the frequency domain. We choose the echo signal of the target center point as the reference point, the echo signal is expressed as  $s_{ref}(t_r, t_m)$ . The rest of the scattered points of the echo signal can be expressed as:

$$s_{scatter\_point}(t_r, t_m) = s_{ref}(t_r, t_m) \delta\left(t_r - \frac{\Delta R}{c}\right) \quad (11)$$

Where  $\Delta R$  represents the distance from each scatter point to the reference point. The frequency-domain expression is presented as:

$$s_{scatter\_point}(f_r, t_m) = s_{ref}(f_r, t_m) \exp(-j2\pi f_r \times \frac{\Delta R_n}{c}) \quad (12)$$

Assuming that the target is composed of  $N$  scattering points, the sum of echo signal of all the scattered points is:

$$S_{sum}(t_r, t_m) = \sum_{n=1}^N IFFT\{s_{scatter\_point}(f_r, t_m)\} = \sum_{n=1}^N IFFT\left\{s_{ref}(f_r, t_m) \exp\left(-j2\pi f_r \times \frac{\Delta R_n}{c}\right)\right\} \quad (13)$$

Similarly, the above expression  $\Delta R_n/c$  also produce phase error, but it can be ignored compared to the phase error generated by  $\tau = R/c$ . Therefore, the compensation factor in the frequency domain is same with that in time domain. The results are shown as follow:

$$\begin{aligned} S_{sum}(t_r, t_m) &= \sum_{n=1}^N IFFT\left\{s_{ref}(f_r, t_m) \exp\left(-j2\pi f_r \frac{\Delta R_n}{c}\right)\right\} \\ &= \sum_{n=1}^N IFFT\left\{s_{ref}(f_r, t_m) \exp\left(-j2\pi f_r \frac{\Delta R_n}{c}\right) \times \exp(-j\pi\mu(\Delta\tau^2 + 2\Delta\tau(t_r - \tau)))\right\} \end{aligned} \quad (14)$$

## 4. Simulation Results and Discussion

In this simulation, the aircraft model with 74 scattering points is considered. For ease of calculation, assuming that every scattering point of RCS is  $1m^2$ , and the aircraft only has the speed in radial direction with respect to the ISAR system. The aircraft model is shown in Figure 3. The parameter settings of the simulation are shown in table1.

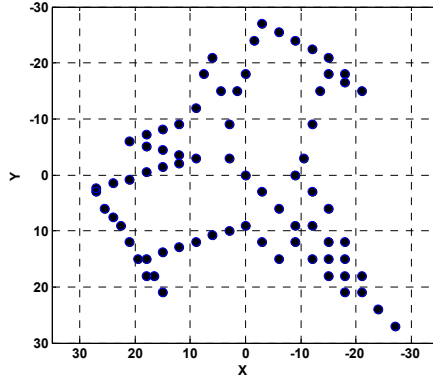


Figure 3. ISAR model of real target

TABLE I Simulation System Parameters

Parameter type	Value	Unit
Carrier frequency	10	GHz
Band width	1	GHz
Pulse width	0.5	us
PRF	6000	Hz
R	100	km
velocity	100	m/s

Assuming that ISAR is at the origin of the coordinates, the distance between the aircraft and the ISAR is 100km. we get the ISAR image of simulated result shown in Fig.4.

The image in Fig.4 is the result when  $T_{dl} = 0.01ns$ . We can see from Fig.4 that the phase error caused by the quantization error has caused the image blur in the Doppler domain, but the range profile of target is clearly displayed. It proved that the quantization error only makes ISAR image in Doppler domain blurred as mentioned in section 2. In addition we can conclude from the Eq. (7) that decreasing the value of the period  $T_{dl}$  can reduce the phase error in Doppler domain and eliminating image blur. So we set  $T_{dl} = 0.0005ns$  and get ISAR image shown in Fig 5. When  $T_{dl}$  is 0.0005ns, which mean  $f_{dl}$  is 2000GHz. Typically, AD sampling frequency cannot raise to this level. So compensation method is used to reduce the phase error and eliminating image blur.

Fig.4 (a) and (b) are the imaging result of ISAR when the time is  $T_{dl} = 0.01ns$  and  $T_{dl} = 0.0005ns$ , this image shows that when the period reduces, the blurred image becomes clear, which is consistent with the Eq.(7). Fig.5 (a) and (b) show the ISAR imaging simulation results obtained by using the compensation algorithm in time domain and frequency domain respectively when  $T_{dl} = 0.01ns$ . Results show that both methods can eliminate the ISAR imaging blur, and they can effectively maintain consistency between simulation echo and ideal echo.

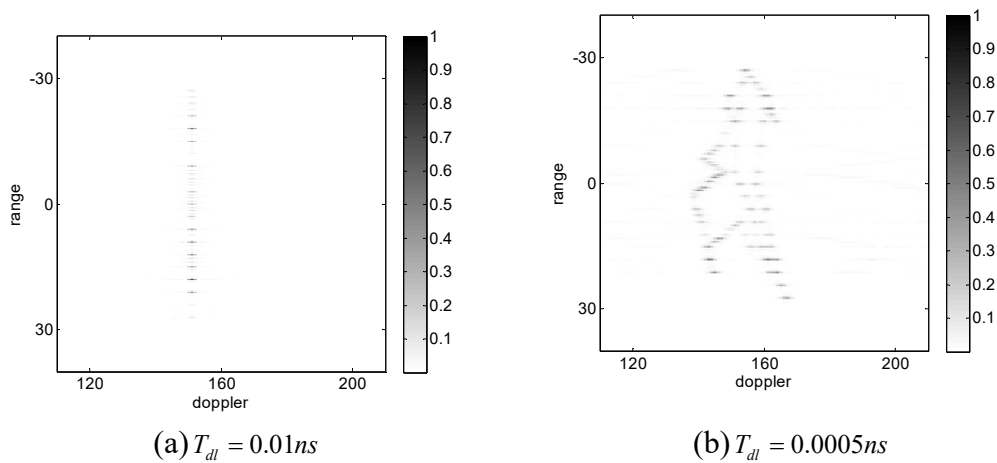


Figure 4. ISAR imaging simulation result

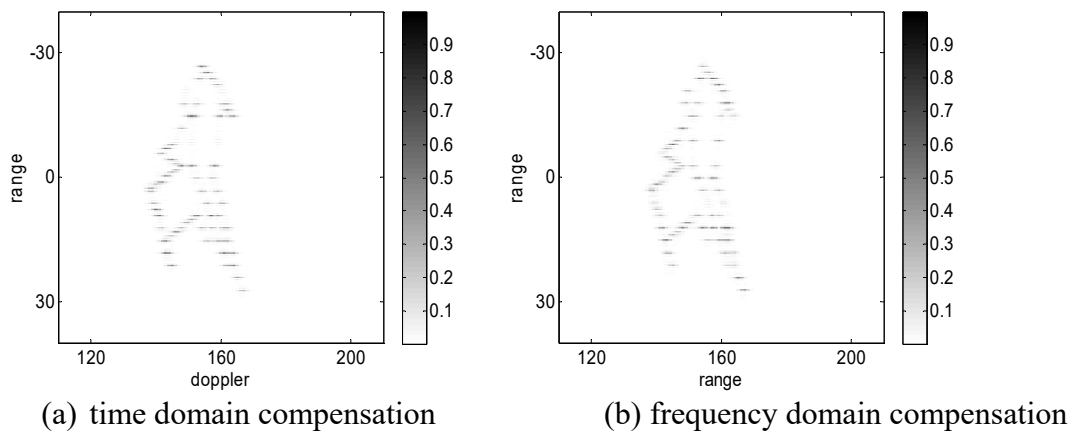


Figure 5. ISAR image of simulation target after compensation

## 5. Conclusions

This paper analyses the quantization error in the real pulse and simulator pulse. The quantization error will lead phase error and ISAR imaging blur. Two kinds of compensation algorithm are proposed and simulated. Both methods can eliminate imaging blur, maintain the consistency between the ideal echo and simulation echo. After employing the proposed method, the problems caused by the quantization error can be resolved. The ISAR MTS can generate an ideal image. The contribution of this paper is to analyze the source of the quantization error and the effect on the ISAR imaging results, and two methods are proposed to eliminate the quantization error.

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