Performance Analysis of Positioning and Tracking of Hypersonic Vehicle Based on Improved CS Model

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Abstract: Aiming at the problem of positioning and tracking of hypersonic vehicles in adjacent space, the positioning and tracking performance of space-based infrared low-orbit satellite binary star system for hypersonic vehicles is explored. According to the kinematic equation of the established hypersonic vehicle, based on STK and Matlab simulation software, the real trajectory of the target, the tracking observation scene of the binary star for the target aircraft is built in the STK simulation software. The Cubature Kalman filter algorithm based on the CS model is studied. According to the shortcomings of the CS model, the key parameters are self-determined and adapted to the adjustment. The simulation results show that the parameter adaptive algorithm is improved compared to the original algorithm.

1. Introduction

The adjacent space generally refers to the area between the ground and the height of 20-100km [1]. This area is located between the space-based satellite platform and the traditional aviation aircraft. The adjacent space has extremely important military value. It is land, sea, air and sky. An important part of the six-in-one battle of electricity and electricity. Near-space hypersonic vehicles generally refer to winged or wingless aircraft with a flight speed exceeding Mach 5 and the main voyage in the adjacent space [2]. This type of aircraft has a fast flight speed, such as the US HTV-2 hypersonic vehicle, which is designed to fly at speeds of Mach 20, in response to the US military's "one-hour global strike" strategy. Due to the extremely fast flight speed of this type of aircraft, the radar early warning detection reaction time is short and difficult to find. At the same time, the existing air defense missile system cannot effectively intercept such targets. Because the hypersonic vehicle has extremely high kinetic energy, it is harmful to the target. High efficiency. At present, due to its strong military industrial system and national defense scientific research strength, the United States has developed a large number of advanced and highly advanced near-space hypersonic vehicles, such as the HTV-2 hypersonic vehicle using boost-glide ballistic flight. - The X-51A hypersonic vehicle in cruise flight mode is currently developing other more advanced hypersonic vehicles. These hypersonic vehicles are mainly used to quickly attack enemy high-value targets, posing a major threat to China's national defense security.

In traditional air defense and anti-missile operations, enemy aircraft and missiles are often the focus of attention. Accurate positioning and tracking of targets is the key to air defense and anti-missile operations. Radar as a type of common equipment plays a major role in air defense and anti-missile warfare, but radar has great difficulty in early warning detection of hypersonic vehicles. Firstly, the flying height of hypersonic vehicles is low, and the early warning radar detects the target by the earth. The influence of curvature is large, and it is also affected by geographical location and weather. The hypersonic vehicle itself is a small target during high-speed flight, and a plasma sheath is formed around the fuselage, which forms a scattering effect on the radar wave, making the target
Location tracking is more difficult. Based on the above factors, it is considered that the space-based low-orbit binary star system is a better choice for achieving hypersonic aircraft positioning and tracking.

In the traditional maneuvering target tracking field, the tracking of maneuvering targets requires the use of target tracking algorithms, which mainly achieve tracking of maneuvering targets by combining maneuvering models and corresponding filtering algorithms. Zhou Hongren's CS model [3] has a good description of maneuvering targets, but the CS model also has some problems that need to be further improved to improve the tracking performance for maneuvering targets. In literature [4], the maneuver constant is modeled, and the target maneuver constant is modeled as a zero-mean Gaussian white noise. Combined with the model structure of the CS model, the maneuver frequency is taken as one of the target states, based on the Unscented Kalman Filter. The filter performs target tracking and achieves good results. In the literature [5], for the problem that the CS model has poor tracking performance for non-maneuvering or inorganic moving targets, the bell-shaped function is used as the fuzzy membership function for the acceleration in the model. The adaptive algorithm of extremum correction improves the tracking accuracy of the algorithm; in the literature [6], the common CS-CKF filtering algorithm and the CS-CKS filtering algorithm improved by the backward iterative algorithm are compared. The simulation results prove that the simulation results are improved. The algorithm is superior to existing algorithms. In the literature [7], by introducing the blanking memory filtering theory and the square root theory, the Cubature Kalman filter is improved, and the real-time validity and accuracy of the target tracking are improved by combining the CS model.

2. Target Tracking Observation Scene Construction

2.1 Target Aircraft Ballistic Model Construction

In order to realize the positioning and tracking of the adjacent space hypersonic vehicle based on the low-orbit binary system, it is necessary to design a tracking observation scene of the binary star for the target. The Satellite Tool Kit (STK) is a commonly used software in aerospace simulation. It is very powerful. The STK simulation software and MATLAB will be used to analyze the target positioning and tracking accuracy.

First of all, in order to perform the target tracking, it is necessary to analyze the motion state of the hypersonic vehicle to obtain the real motion state of the target, and based on this, the performance of the tracking algorithm is analyzed. Near-space hypersonic vehicles can maneuver by controlling the angle of attack and the angle of inclination. At present, hypersonic vehicles can be maneuvered in several ways [8]:

1. Maintain the same flight altitude and maximize lateral maneuver
2. Wait for dynamic pressure flight, the inclination angle is constant
3. The angle of attack maintains the maximum lift-to-drag ratio, and the tilt angle is constant.
4. The angle of attack is constant and the angle of inclination is constant.

In this paper, the aircraft angle of attack design in [9] is used to iteratively obtain the real ballistic data of the target based on the fourth-order Runge-Kutta method. The fourth-order Runge-Kutta algorithm is computationally complicated. The amount is large, but the accuracy of the iterative calculation is high, which is often used in simulation calculations. In the Earth's fixed-coordinate coordinate system, the equation of motion of a hypersonic vehicle can be expressed as:
\[
\begin{align*}
\frac{dr}{dt} &= V \sin \gamma, \\
\frac{d\theta}{dt} &= \frac{V \cos \gamma \cos \psi}{r \cos \phi}, \\
\frac{d\phi}{dt} &= \frac{V \cos \gamma \sin \psi}{r}, \\
\frac{dV}{dt} &= -\frac{D}{m} - \frac{\mu}{r^2} \sin \gamma, \\
\frac{d\gamma}{dt} &= \frac{1}{V} \left( \frac{L}{m} \cos \sigma + \left( \frac{V^2}{r} - \frac{\mu}{r^2} \right) \cos \gamma \right), \\
\frac{d\psi}{dt} &= \frac{1}{V} \left( \frac{L}{m} \sin \sigma - \frac{V^2}{r} \cos \gamma \cos \psi \tan \phi \right)
\end{align*}
\] (1)

Among them, \( r \) is the distance between the center of mass of the aircraft and the center of mass of the earth; \( \theta \) is the longitude of the target; \( \phi \) is the latitude of the target; \( V \) is the flight rate of the aircraft; \( \gamma \) is the local ballistic inclination; \( \psi \) is the east-north angle of the velocity vector. What appears in the equation is a constant called the Earth's Gravitational Constant, which is:
\[
\mu = 3.986005 \times 10^{14} \text{ m}^3 / \text{s}^2
\]

Among them, \( D, L \) are the resistance and lift experienced during the movement of the hypersonic vehicle. The expressions are as follows:
\[
D = C_D(\alpha, Ma) \frac{\rho}{2} V^2 S
\]
\[
L = C_L(\alpha, Ma) \frac{\rho}{2} V^2 S
\] (2)

In equation (2), \( \rho \) is the local atmospheric density in the motion of the aircraft. It can be fitted to the atmospheric density of the target and the local sound velocity according to the 1976 US standard atmospheric model. The obtained atmospheric density function is:
\[
\rho = \rho_0 e^{-k h}
\] (3)

Where \( \rho_0 = 1.225 \text{ kg/m}^3 \) is the sea level standard atmospheric pressure, which is a constant.

The height of the target use \( h \) in units, and respectively \( C_D, C_L \) are the resistance coefficient and the lift coefficient of the aircraft. These two parameters are functions of the angle of attack and the target flying Mach number. The functions of the lift coefficient and the drag coefficient are expressed as [10]:
\[
C_D = C_{D_0} + C_{D_1} \alpha^2 + C_{D_2} \exp(C_{D_3} Ma)
\]
\[
C_L = C_{L_0} + C_{L_1} \alpha + C_{L_2} \exp(C_{L_3} Ma)
\] (4)

Including:
\[
C_{D_0} = 0.024, C_{D_1} = 7.24e^{-4},
C_{D_2} = 0.406, C_{D_3} = -0.323,
C_{L_0} = -0.2317, C_{L_1} = 0.0513,
C_{L_2} = 0.2945, C_{L_3} = -0.1028
\] (5)

2.2 Double Satellites Passive Tracking
Since both infrared warning satellites are equipped with an infrared tracking sensor, the target can be tracked for a period of time. At the same time, because the distance between the target aircraft and the satellite is far, sometimes even thousands of kilometers can be reached. At this time, the target is only one pixel point on the infrared sensor of the satellite [11]. In order to facilitate the description of the target observation angle, it is assumed that the orbit coordinate system of the satellite coincides with
the observation coordinate system of the sensor mounted on the star, and the definition of the observation coordinate system is as follows:

The center of the observation coordinate system is the center \( o \) of mass of the satellite. The axis \( o z_o \) is pointed by the centroid of the satellite to the center of mass of the earth (also known as the local vertical line). The axis \( o y_o \) is the negative normal direction of the orbit of the satellite. The axis \( o x_o \) and the other two axes form the right-handed coordinate system, as shown in Figure 1. As shown, the satellite's angular measurement of the target is shown.

![Fig.1 Target measurement in the satellite observation coordinate system](image)

Define each axis angle measurement information:
(1) Azimuth angle \( e \), the angle between the projection of the satellite's line of sight vector in the plane \( x_0y_0o \) and the coordinate axis \( x_0o \), which is positive by pointing from \( x_0o \) to \( oT' \), and its value range is \([0, \pi]\).
(2) Pitch angle \( a \), the angle between the projection \( oT' \) of the satellite line-of-sight vector in the plane \( x_0y_0o \) and the line of sight \( oT \), which is positive by pointing from \( oT' \) to \( oT \), and its value range is \([-\frac{\pi}{2}, \frac{\pi}{2}]\).

3. CS Model and Its Adaptive Algorithm

3.1 CS Model

In [3], it is considered that the Singer model models the probability distribution of the target maneuver acceleration as a zero-mean, uniformly distributed model that is not in accordance with the target's real motion process. It is also unnecessary to consider all maneuver and speed values of the target motion. The maneuvering acceleration at the next moment of the target can only vary within a limited neighborhood around the current target acceleration. The CS model uses a modified Rayleigh distribution function to describe the maneuvering acceleration of the target, with the following formula:

\[
\begin{align*}
\dot{x}(t) &= a(t) + \bar{a} \\
\dot{a}(t) &= -\alpha a(t) + w(t)
\end{align*}
\]

(6)

In equation (6), \( x(t) \) is the position of the target aircraft, \( a(t) \) is the zero-mean colored noise, \( \bar{a} \) is the average of the maneuvering acceleration, \( \alpha \) is the target maneuvering frequency, represents the strength of the target's maneuverability, \( w(t) \) is the zero mean, the variance is \( \sigma_w^2 = 2\alpha\sigma_a^2 \), among them \( \sigma_a^2 \) is target acceleration variance.

\[
X(k) = F(k)X(k-1) + U(k)\bar{a}(k) + W(k)
\]

(7)
In equation (7) \(X(k)\) is the motion state vector of the target aircraft, including the position, velocity, and acceleration parameters of the target; \(F(k)\) is the state transition matrix; \(U(k)\) is the state input control matrix; \(\bar{a}(k)\) is the mean of the current acceleration; \(W(k)\) is the zero mean, variance \(Q(k)\) white Noise, which includes:

\[
F(k) = \begin{bmatrix}
1 & T & (1+\alpha T + e^{-\alpha T})/\alpha^2 \\
0 & 1 & (1-e^{-\alpha T})/\alpha \\
0 & 0 & e^{-\alpha T}
\end{bmatrix}
\]

\[
U(k) = \begin{bmatrix}
-T + \alpha T^2/2 + (1-e^{-\alpha T})/\alpha \\
T - (1-e^{-\alpha T})/\alpha \\
1-e^{-\alpha T}
\end{bmatrix}
\]

\[
Q(k) = E[W(k)W^T(k)] = 2\alpha \sigma_a^2 \begin{bmatrix}
q_{11} & q_{12} & q_{13} \\
q_{12} & q_{22} & q_{23} \\
q_{13} & q_{23} & q_{33}
\end{bmatrix}
\]

\[
\sigma_a^2 = \begin{cases}
\frac{4-\pi}{\pi} [a_{\text{max}} - \bar{a}(k)]^2, \bar{a}(k) > 0 \\
\frac{4-\pi}{\pi} [-a_{\text{max}} - \bar{a}(k)]^2, \bar{a}(k) < 0
\end{cases}
\]

The individual elements in the matrix \(Q(k)\) are:

\[
q_{11} = (1-e^{-2\alpha T} + 2\alpha T + 2\alpha^2 T^3 / 3 - 2\alpha^2 T^2 - 4\alpha T e^{-\alpha T}) / 2\alpha^5
\]

\[
q_{12} = (e^{-2\alpha T} + 1 - 2e^{-\alpha T} + 2\alpha T e^{-\alpha T} + 2\alpha T + \alpha^2 T^2) / 2\alpha^4
\]

\[
q_{13} = (1-e^{-2\alpha T} - 2\alpha T e^{-\alpha T}) / 2\alpha^3
\]

\[
q_{22} = (4e^{-\alpha T} - 3 - e^{-2\alpha T} + 2\alpha T) / 2\alpha^3
\]

\[
q_{23} = (e^{-2\alpha T} + 1 - 2e^{-\alpha T}) / 2\alpha^2
\]

\[
q_{33} = (1-e^{-2\alpha T}) / 2\alpha
\]

At present, there are several nonlinear filtering algorithms that are widely used: including Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), and Cubature Kalman Filter (CKF). The EKF algorithm is difficult to linearize the processing of strong nonlinear systems due to the need to use linearization processing. There are high-order truncation errors, the stability of tracking filtering is insufficient, and it is easy to filter and divergence. The UKF algorithm uses UT transform to generate system sampling points, but its performance is greatly affected by the parameters in the algorithm. The filter often diverge in the face of high-order systems; the CKF algorithm uses the third-order spherical radial rule to obtain the sampled volume points, and the weighted values of the volume points are positive values. It is more suitable for high-dimensional systems, and the filtering accuracy is higher than EKF and UKF, and its application range is more and more extensive. In the algorithm of this paper, CKF algorithm is used as the basic filter of the algorithm.

### 3.2 Improved CS Model

In the introduction of the CS model, we can know the maximum and minimum values of the maneuvering acceleration in the CS model, and the maneuvering frequency has a greater impact on the tracking of the target. These data are often required based on experience and prior. Knowledge determines that these fixed parameters cannot effectively describe the motion of the maneuvering target, or even completely different from the motion state of the real target, so that the tracking error for the target is large, and even the filter divergence occurs. For these situations, it is necessary to adaptively adjust the relevant parameters of the CS model according to the motion state of the target aircraft [12], improve the accuracy of the CS model for the description of the target aircraft motion.
state, and improve the filtering accuracy. Both the maneuvering acceleration extremum and the maneuvering frequency in the CS model need to be adaptively adjusted.

(1) Acceleration maximum value adaptation

The innovation for setting up tracking filters for hypersonic vehicles is:

\[ d(k) = Z(k) - \hat{Z}(k) \]  

In Equation (14) \( Z(k) \) is the actual observation of the satellite to the target, \( \hat{Z}(k) \) is the observation obtained in the observation update in the Kalman filter process, combined with the CKF calculation step to calculate the innovation covariance matrix \( S(k) \), if the target is maneuvered. The innovation should be increased, and the adaptive adjustment of the acceleration maximum value is based on the change of the innovation:

\[ \mu(k) = \frac{\text{Trace}(d(k)d^T(k))}{\text{Trace}(S(k))} \]

\[ a_{max} = \mu(k)a_{max,0} \]

\( \text{Track}() \) is to get the trace of the matrix, \( a_{max,0} \) is the maximum value of the initial acceleration. The parameter is used to represent the difference between the actual value of the innovation and the theoretical value, so that the maximum value of the maneuver acceleration can be adaptively adjusted in real time [13]. When the target is maneuvered, the innovation increases, which leads to \( a_{max} \) increases, increasing the tracking ability of the CS model for high maneuvering targets such as hypersonic vehicles, and reducing the model error resulting from the CS model using the initial set fixed parameters, which improves the accuracy of position tracking.

(2) Maneuver frequency adaptation

The literature [14] pointed out that the maneuvering frequency directly affects the accuracy of the CS model for the target's maneuverability description. The simulation verifies the influence of the maneuvering frequency on the target tracking accuracy in the CS model, and the influence of the inaccurate maneuvering frequency on the target location tracking. Larger so that the filter diverges. In [15], the acceleration mean adaptive algorithm is used, and there are calculation methods such as \( \bar{a}(k) = \bar{x}(k|k-1) \), but this approximation is only applicable when the target's maneuverability is weak. When the target is maneuvered, the filtering accuracy will decrease. It is known in equation (13) that when the motion state of the target is abrupt, the tracking filter's new interest will increase, and the filtered root mean square error will be greatly deviated from the theoretical value. The real motion state is inconsistent with the motion state of the original maneuver frequency \( \alpha \). The maneuver frequency \( \alpha \) needs to be adaptively adjusted. According to the Kalman filter innovation in equation (13), the maneuver frequency is adaptively adjusted based on the simplified minimum mean square error algorithm. [16], that is, the adaptive adjustment formula of the maneuvering frequency:

\[ \alpha(k+1) = \alpha(k) + 2\lambda d(k) \]

Among equation (16) \( \lambda \) is the convergence coefficient. After the algorithm is adjusted and converged, the value of the maneuver frequency \( \alpha \) can be obtained. The value of the maneuver frequency \( \alpha \) can be adaptively adjusted by filtering the new information to realize the state transition matrix and the state error covariance matrix of the CS model. The adjustment makes the CS model more accurately describe the real motion state of the target, and the filtering precision is higher.

4. Simulation Analysis

4.1 Establishment of Target Trajectory and Double-satellites Positioning and Tracking Scene

In this paper, the initial value of the motion state of the target is set by itself, and the fourth-order Runge-Kutta method is used to perform the ballistic simulation of the target. Although the Runge-Kutta algorithm is slightly complicated but has high precision, the initial latitude and longitude of the target are set 0, the target height is 50km, the initial velocity of the target is 6000m/s, the ballistic inclination and the ballistic declination are -0.1rad and \(-\pi / 6 \) rad respectively, and the
target aircraft moves in space for 2001s. After the fourth-order Runge-Kutta The simulated trajectory after the iteration of the method is shown below:

![Fig.2 Target aircraft space motion track](image)

![Fig.3 Target aircraft height change](image)

The ballistic data generated by the fourth-order Runge-Kutta iteration is written into the ephemeris file and imported into the STK, and the real trajectory of the target in the STK is obtained by simulation, as shown in Fig. 4.
Based on the description of the constellation design in the US STSS satellite system in [17], the satellites in the STSS use polar orbits to meet the global coverage requirements, and the constellation orbital height is about 1600 km. In practice, the relative geometry of the constellation is described, where \( T \) is the total number of satellites in the constellation, \( P \) is the number of orbital orbital planes, and \( F \) is the metric of the relative phase of the satellites of adjacent orbital planes. The value of \( F \) is \([0, P-1]\), any integer between \([0, P-1]\), the constellation configuration is set to 24/3/1 in this simulation, and the target location analysis is performed based on the corresponding data given in this observation scene and STK. Figure 5 is a location tracking scenario built in STK.

4.2 Double Satellites Tracking Filter Simulation Analysis
Since the scene is based on the binary satellites system to achieve the positioning and tracking of the target aircraft, the binary star is required to be visible to the target at the same time. The double star seed127 and seed128 which are visible to the target are selected, and the visible time is 6 minutes at the same time, and the target position speed within 6 minutes is performed. Simulation analysis.

Both stars are equipped with infrared cameras. Since the devices on the star can only obtain four angle measurement values such as the azimuth and elevation angle of the target, the position error of the observation binary star is 15m (3\( \sigma \)), and the speed error is 0.2m/s(3\( \sigma \)), the angle of error of the observation device itself is 0.15° (3\( \sigma \)). At the same time, in order to better compare the above two algorithms, the root mean square error (RMSE) commonly used in error calculation is used for simulation calculation. The calculation formula of RMSE is as follows:
\[
RMSE_{\text{pos}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [(\hat{x} - x)^2 + (\hat{y} - y)^2 + (\hat{z} - z)^2]}
\] (17)

\[
RMSE_{\text{vel}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [(\hat{v}_x - v_x)^2 + (\hat{v}_y - v_y)^2 + (\hat{v}_z - v_z)^2]}
\] (18)

Equation (17) is used to calculate the RMSE of the position, and Equation (18) is used to calculate the RMSE of the velocity. The N in the equation is the number of times the system simulation is calculated, where \((x, y, z)\) is the real position of the target, \((\hat{x}, \hat{y}, \hat{z})\) is the target position obtained by the tracking filter algorithm, \((v_x, v_y, v_z)\) is the real speed of the target, and \((\hat{v}_x, \hat{v}_y, \hat{v}_z)\) is the target speed obtained by the tracking filter algorithm. After Matlab simulation analysis, compare two algorithms for the positioning and tracking performance of hypersonic vehicles.

![Fig.5 CKF-CS filtering to obtain position and velocity RMSE](image)

![Fig.6 Improved CKF-CS filtering to obtain position and velocity RMSE](image)

5. Conclusion

In this paper, aiming at the problem of positioning and tracking of hypersonic vehicles, the ballistic model of hypersonic vehicle is introduced. The fourth-stage Runge-Kutta method is used to simulate the ballistics of hypersonic vehicles, and the target classic jump-gliding trajectory is obtained. The simulated ballistic data is used as the real data of the target tracking. At the same time, it is written into the ephemeris e file required by STK, and the low-orbit satellite constellation is set. The simulation analysis of the target tracking accuracy based on the binary stars in the constellation is combined with the common target tracking field. The CS model, the maneuver model, is aimed at the influence of fixed maneuvering acceleration extremes and maneuvering frequency on tracking
filtering, and improves the adaptive algorithm of the CS model. The simulation results prove that the target is improved by the adaptive algorithm. The tracking accuracy is obvious.

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