Cutter Axis Vector Smoothing Algorithm for Five-axis Milling

Li Min, Hongchang Wang*, Ying Chen

Mechanical Engineering, Shenyang Jianzhu University, Shenyang, China

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Abstract: In the five-axis milling process, the existing cutter axis vector planning only adjusts the cutter axis vector and neglects the effect on the cutting bandwidth. Considering this problem, a method of adjusting the cutter axis vector by contrasting the change rate of the cutter axis vector is proposed, which not only ensures the cutting bandwidth but also keeps the change of the cutter axis vector smooth. At the same time, the effective cutting shape and the calculation method of the cutting bandwidth of the flat-end cutter in the feed direction are described. The influence of the change of cutter attitude on the cutting bandwidth is expounded, and the optimal cutter axis vector position is proposed. The MATLAB software is used to simulate and compare the changes of the cutter axis vector before and after optimization. It shows that the algorithm can effectively improve the processing efficiency and processing quality of the curved surface.

1. Introduction

Five-axis CNC machines have two degrees of freedom compared to three-axis CNC machines in five-axis milling. The cutter axis has a better spatial variable range, making it easier to process complex surfaces. However, with the increase of the degree of freedom, the influence of the cutter axis-vector planning on the surface machining quality and machining efficiency is also increasing.

The study of the cutter axis vector planning problem has long attracted the attention of many scholars. L.L. Li [1] proposed a cutter-path planning method for free surface high-surface finish, and elaborated the cutter-path and cutter axis vector planning. Liu [2] proposed a method of smooth transition for the cutter-contact point correction based on kinematics constraint optimization of cutter axis vector and optimized the change of cutter axis vector. Lin [3] proposed a five-axis no-singular cutter attitude optimization algorithm to plan the cutter axis vector. In the process of cutter axis vector planning, the smoothness of the cutter axis vector and the size of the cutting bandwidth are important factors affecting the surface machining quality and machining efficiency. The current cutter axis vector planning is adjusted only for certain aspects, and the two have not taken into consideration. Considering this problem, this paper proposes a method of selectively adjusting the cutter axis vector by comparing the cutter vector change rate, which not only ensures the cutting bandwidth but also keeps the change of the cutter axis vector smooth. Through the smooth control of the cutter axis vector, the cutter axis vector without interference, continuity and smoothness is obtained, thereby improving the machining quality and machining efficiency of the curved surface.
2. Effective Cutting Shape and Calculation of Cutting Bandwidth of Flat-end Cutter

In five-axis milling, if the cutting bandwidth is greater than the cutter-path line spacing, it will increase the processing time and reduce the processing efficiency. If the cutting bandwidth is smaller than the cutter-path line spacing, the scallop height will be too large, the quality of the curved surface will be rough, and undercut will occur when it is more serious. Therefore, a reasonable cutting bandwidth is an important guarantee for improving processing efficiency.

In order to calculate the cutting bandwidth, a local coordinate system is first established at the cutter-contact (CC) point O1. In order to avoid interference, the cutter first rotates around the y1 axis at an inclination angle \( \lambda \) and then rotates around the z1 axis at a rotation angle \( \omega \). \( r \) is the radius of the tip cutter and its bottom contour is projected towards the y1-z1 plane. This projection is the effective cutting shape of the flat-end cutter at the cutter-contact point, also known as the effective cutting ellipse.

![Figure 1 Geometric analysis of effective cutting shapes.](image)

The surface cross-section perpendicular to the cutting direction x1 of the cutter is approximated by a circular arc. Let the radius of curvature of the surface along the y1-direction be \( R_y \). If the maximum scallop height is \( h \), the maximum machining surface error is \( R_y-h \). In this way, the cutting bandwidth can be calculated by finding the intersection point between the effective cutting ellipse and the virtual arc [4].

![Figure 2 The calculation of cutting bandwidth.](image)

The cutting bandwidth of flat-end cutter is calculated as follows:

\[
W = W_l + W_r = 2r \sqrt{\frac{2hR_y - h^2}{rR_y - r^2} - \left(\frac{2hR_y - h^2}{2rR_y - 2r^2}\right)^2}
\]

Since \( |R_y| \gg h \) in actual processing, the amount of \( h^2, h^3 \), and \( h^4 \) in the above equation is negligible. Among them, the cutter radius \( r \) is replaced by the effective cutter radius \( r_e \) [5].
\[
\begin{align*}
    r_e &= r \cos^2 \omega, \quad \lambda \neq 0^\circ \\
    r_e &= r, \quad \lambda = 0^\circ
\end{align*}
\]

This paper simplifies the cutting bandwidth, shown in Equation (1).

\[
W = \frac{8hr_eR_y}{R_y-r_e}
\]  \hspace{1cm} (1)

3. Optimal Cutter Axis Vector Position

In the five-axis milling process, the cutting bandwidth is an important factor influencing the machining efficiency. The flat-end cutter has a larger cutting bandwidth than the ball-end cutter. That is, using the flat-end cutter to process can obtain higher cutting efficiency. Changing the cutter attitude angle \((\lambda, \omega)\) changes the shape of the effective cutting ellipse, which in turn causes a change in the cutting bandwidth [6]. It is assumed that the surface normal vector of the cutter-contact point is used as the initial cutter axis vector, and the cutter attitude angle at this time is \((\lambda_0, \omega_0)\), and the cutting bandwidth is \(W\). If the inclination angle of the cutter \(\lambda\) is changed so that \(\lambda_1 > \lambda_0\), the cutting bandwidth \(W\) decreases. If the inclination angle of the cutter \(\lambda\) does not change, the rotation angle of the cutter \(\omega\) is changed so that \(\omega_1 > \omega_0\) and the cutting bandwidth \(W\) is reduced. Thus, in order to obtain the maximum cutting bandwidth, the optimal cutter axis vector position is the surface normal vector of the cutter-contact point.

Figure 3 Cutting bandwidth in different cutter attitudes.

In order to optimize the cutting bandwidth, \(\lambda\) and \(\omega\) are usually specified [7] so that the radius of curvature at the cutter-contact point can be better matched. For a convex surface or a flat surface, the inclination angle of the cutter \(\lambda\) takes \(0^\circ\) to \(2^\circ\), and the rotation angle of the cutter \(\omega\) is set to \(0^\circ\). If the surface is non-convex, you need to adjust the non-zero inclination angle of the cutter \(\lambda\) to avoid interference. The rotation angle of the cutter \(\omega\) takes \(0^\circ\) to \(5^\circ\). The minimum inclination angle of the cutter \(\lambda\) to avoid interference is calculated.

\[
\lambda_{\min} = \max_{-\pi/2 \leq \phi \leq \pi/2} \lambda_{\phi}
\]

\[
\lambda_{\phi} = \arcsin \left( \frac{r}{R_{\phi}} \right) = \arcsin (rk_{\phi})
\]

Figure 4 The calculation of minimum inclination angle of the cutter.
After finishing, the minimum inclination angle of the cutter $\lambda$ that avoids interference can be expressed as the equation.

$$\lambda_{\min} = \max_{-\pi/2 \leq \phi \leq \pi/2} \lambda_{\phi} = \max_{-\pi/2 \leq \phi \leq \pi/2} \arcsin(rk_{\phi}) = \arcsin(rk_{\max})$$

In the formula, $k_{\max}$ is the maximum surface curvature at CC point.

4. The Calculation of the Initial Cutter Axis Vector

Let $S \equiv S(u,v)$ be the desired machining surface, then the normal vector $n$ of the surface can be expressed as Equation (2).

$$n = \frac{s_u \times s_v}{|s_u \times s_v|}$$

(2)

Let $O_1$ be a CC point on the surface, and $n$ is the normal vector of the surface at the cutter-contact point. Taking the $O_1$ point as the origin, $n$ is the local coordinate system of the $z_1$ axis. The $x_1$ and $y_1$ axes are in the plane of the $O_1$ point, and $a$ and $b$ are the unit vectors of the $x_1$ and $y_1$ axes.

$$T = nc\cos\lambda + (acos\omega + bsin\omega)sin\lambda$$

(3)

5 Cutter Axis Vector Smoothing Algorithm

Through the above analysis of the cutting bandwidth and theoretically optimal cutter axis vector, we can obtain the initial cutter axis vector of any cutter-contact points on the cutter-path. The obtained cutter axis vector can achieve the maximum cutting bandwidth under the premise of avoiding interference. However, when all the cutter axis vectors are viewed on a cutter-path, there is a phenomenon that the initial cutter axis vector change is not smooth. The abrupt change of the cutter axis occurs at a place where the change of curvature is large, so that the position of the cutter shaft or the jig frequently changes during the machining, which affects the machining quality. In this paper, a cutter axis vector smoothing algorithm is proposed, which can obtain a smoother cutter axis vector on the premise of satisfying the cutting bandwidth as much as possible.

In order to measure the smoothness of the cutter axis vector, the change rate of the cutter axis vector of the adjacent cutter-contact points needs to be calculated. In the processing of the curved surface, the smaller the change rate of the cutter axis vector, the smoother the obtained machining surface. The cutter axis vector rate of change can be calculated by Equation (4).

$$TCR_i = \frac{|T_{i+1} - T_i|}{|CC_{i+1} - CC_i|}$$

(4)

In Equation (4), $CC_i$ represents the cutter-contact points and $T_i$ represents the cutter axis vector.
at point CC_i.

According to the calculation of the initial cutter axis vector, the cutter attitude angle (λ,ω) is an important parameter for adjusting the cutter axis vector. When the rate of change of the cutter axis vector is too large, it is necessary to increase the attitude angle of the cutter and readjust the cutter axis vector so that the change rate of the cutter axis vector satisfies the threshold φ of TCR_i. At the same time, with the change of the cutter attitude angle, the shape of the effective cutting ellipse also changes, which in turn causes changes in the cutting bandwidth. This paper derives the cutter axis vector that satisfies the maximum cutting bandwidth within the variable range of cutter vector variation rate.

Let the coordinates of a cutter-contact point CC_1 on the surface be (x_1,y_1,z_1), and the normal vector n_1=(u_1,v_1,w_1). The initial cutter axis vector calculated by Eq. (3) is: T_1=(u_1\cos{\lambda}_1,v_1\cos{\lambda}_1,w_1\cos{\lambda}_1+C\sin{\lambda}_1)=(d_1,d_2,d_3).

CC_2 is the next cutter-contact point of CC_1 with coordinates (x_2,y_2,z_2) and its normal vector n_2=(u_2,v_2,w_2). The initial cutter axis vector is calculated by Eq. (3) is: T_2=(u_2\cos{\lambda}_2,v_2\cos{\lambda}_2,w_2\cos{\lambda}_2+C\sin{\lambda}_2)=(e_1,e_2,e_3).

For convex surfaces or flat surfaces, the inclination angle λ_1 takes 0°~2°. For non-convex surfaces, a non-zero inclination angle of the cutter is calculated to avoid interference λ_1=\arcsin(r_{k_{max}}). When TCR_i = \frac{|T_2-T_1|}{|CC_2-CC_1|} = \sqrt{(e_1-d_1)^2+(e_2-d_2)^2+(e_3-d_3)^2} \leq \varphi$, the cutter attitude angle λ_2 satisfying the TCR_i threshold at the CC_2 point is solved by the least square method.

The specific algorithm is summarized as follows:

Step 1: Select a flat area on the initial cutter-path to determine its initial CC_i point. Determine the surface type at the CC_i point by analyzing the Gaussian curvature and mean curvature characteristics at the CC_i point. For convex surfaces or planes, the cutter axis vector T_1 is calculated by Eq. (3). For non-convex surfaces, the minimum cutter attitude angle at which it avoids interference is first calculated, and then the cutter axis vector T_1 is calculated by Eq. (3).

Step 2: Determine next CC_{i+1} point of CC_i, and calculate the cutter axis vector T_{i+1} through the method of step one.

Step 3: Calculate the cutter axis vector change rate TCR_i of CC_{i+1} and CC_i by Eq. (4). If TCR_i \leq \varphi, then keep T_{i+1}. If TCR_i > \varphi, then by Eq. (4,3) simultaneously solving the cutter attitude angle that satisfies the TCR_i threshold, and calculating the newly generated cutter axis vector T_{i+1}.

Step 4: Use CC_{i+1} as the initial CC point, perform step 2 and step 3, and so on to the end of the cutter-path.
6. Examples

In order to verify the effectiveness of the proposed algorithm, simulations using MATLAB software, TCR ≤ 5%. The simulated surface is generated from the post-processing of the point cloud data. The initial cutter-path is generated using the Iso-scallop strategy [8], and the algorithm is applied to the entire surface.

<table>
<thead>
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<th>TCR</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not optimized</td>
<td>1.12%</td>
<td>0.81%</td>
<td>0.14%</td>
<td>0.05%</td>
<td>0.21%</td>
<td>0.54%</td>
<td>0.70%</td>
<td>0.87%</td>
<td>1.28%</td>
</tr>
<tr>
<td>Optimized</td>
<td>1.12%</td>
<td>0.61%</td>
<td>0.14%</td>
<td>0.05%</td>
<td>0.22%</td>
<td>0.54%</td>
<td>0.70%</td>
<td>0.87%</td>
<td>1.28%</td>
</tr>
</tbody>
</table>

Comparing the cutter axis vector diagram before and after optimization (Fig. 7), when the cutter
moves along the trajectory before optimization, the inclination angle of the cutter occurs a more severe deflection, and the cutter axis vector abruptly changes. The optimized cutter axis vector keeps better smoothness. Comparing the change rate of the cutter axis vector before and after optimization (Table 1), it can be found that the maximum change rate of the arbor axis vector exceeded 12%, far exceeding the threshold of TCR1. During machining, the surface of the workpiece will be marked with scratches. The rate of change of the cutter axis vector after optimization is lower than the threshold of TCR1, and the smoothness of the cutter axis vector is improved, which verifies the effectiveness of the algorithm. By comparing the cutting bandwidth curves before and after optimization (Figure 9), it can be found that after the algorithm optimization, the cutting bandwidth decreases slightly at the abrupt change of the cutter axis vector, but the overall cutting bandwidth is maintained, which verifies the effectiveness of the algorithm.

![Figure 10 Five-axis milling experiment.](image)

In order to further verify the effectiveness of the proposed algorithm, the DMU50 five-axis CNC machining center is used for the machining experiment. The surface quality of the workpiece is smooth and the machining efficiency is high. The smoothness of the tool axis vector in the experimental machining is consistent with the results obtained by the simulation model. It is verified that the algorithm can provide a theoretical reference for actual processing.

### 7. Conclusion

Under the premise of guaranteeing the cutting bandwidth, the generated cutter axis vector can obtain better smoothness and solve the problem that the quality of the surface processing is degraded due to the unsmoothness of the cutter axis vector. Through the optimization of the cutter axis vector smoothing algorithm, the cutter axis vector without interference, continuity and smoothness is obtained, which improves the processing efficiency and processing quality of the curved surface.

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