Inventory Optimization Models of Equipment Spare Parts Based on Computer Simulation

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Keywords: availability, confidence values of the readiness rate, support probability, support expense, inventory optimization, computer simulation

Abstract: Through making an analysis of the randomicity of the spare parts demand, probability theory and mathematical statistics is applied to establish the support expense minimum model based on equipment availability, confidence values of the equipment readiness rate and support probability of spare parts. The arithmetic of spare parts joint support storage based on expense is presented. Then inventory optimization method of spare parts joint support based on computer simulation is determined. Applicability of the method is given by way of a numerical example. The inventory optimization method based on computer simulation provides a theoretical basis for solving the problems of inventory and expense in other relative areas.

1. Introduction

Many scholars have done a lot of work in the field of inventory and cost modeling, and have achieved fruitful results. Liu Xiaoqun established the two-hierarchy stochastic inventory model with multi-varieties based on the stochastic inventory model of single goods. In this inventory system, the total costs was composed of goods transport costs, storage costs and loss costs due to lack of goods. The transport costs consisted of the costs of goods transported from the central warehouse to the sub-depots, and that of the goods transshipped among the sub-depots [1]. Wang Liang gave a heuristic algorithm for analyzing the relationship between the cost of transportation and two-level inventory, aiming at how to minimize the total cost of supply chain transportation and two-level inventory under the condition of satisfying the given customer service level [2]. Lv Fei established a mixed integer nonlinear programming model for the location allocation problem of logistics systems and gives a heuristic algorithm for its solution, based on the monocyclic stochastic inventory control strategy adopted by retail outlets [3].

Through the analysis of the related literature in this field, it could be found that establishing inventory optimization models of goods based on various influence factors through making a analysis of the goods demand presenting random change is very rare.

Taking spare parts maintenance as an example, this paper mainly studies how to determine the optimal stock of spare parts maintenance in the base unit warehouse on the basis of the full consideration of such factors as the cost of spare parts maintenance, the availability of equipment, the reliability of equipment and the maintenance procedure.
2. Model Establishment

2.1 Cost Models

Suppose when storing the maintenance spare parts of the basic unit warehouse, the average purchase unit price of the maintenance spare parts is \( a \), the average inventory cost of the maintenance spare parts is \( b \), the average backlog loss cost of the maintenance spare parts is \( c \), the average shortage loss cost of the maintenance spare parts is \( d \). The storage quantity of maintenance spare parts in warehouse is \( X \), and the demand distribution function of this kind of maintenance spare parts is \( f(r) = \frac{\beta^a}{\Gamma(a)} r^{a-1} e^{-\beta r} dr \).

1. The cost of spare parts purchase and spare parts inventory is shown in Equation (1).

\[
C_1(X) = (a + b)X
\]  

(1)

2. The average cost of spare parts backlog loss in storage is shown in Equation (2).

\[
C_2(X) = c \int_0^X (X - r) \frac{\beta^a}{\Gamma(a)} r^{a-1} e^{-\beta r} dr
\]  

(2)

3. The average cost of spare parts shortage loss is shown in Equation (3).

\[
C_3(X) = d \int_X^{+\infty} (r - X) \frac{\beta^a}{\Gamma(a)} r^{a-1} e^{-\beta r} dr
\]  

(3)

4. The total cost is shown in Equation (4).

\[
C(X) = C_1(X) + C_2(X) + C_3(X)
\]

\[
= (a + b)X + c \int_0^X (X - r) \frac{\beta^a}{\Gamma(a)} r^{a-1} e^{-\beta r} dr + d \int_X^{+\infty} (r - X) \frac{\beta^a}{\Gamma(a)} r^{a-1} e^{-\beta r} dr
\]  

(4)

2.2 Availability Models

Availability means the extent to which the equipment is in a working or serviceable state at any time [4]. Suppose the average fault interval time of the equipment is \( T_1 \), the average repair time after the equipment breakdown is \( T_2 \), the maintenance spare parts support degree (support probability) is \( P \) and the average purchase time due to the shortage of maintenance spare parts is \( T_3 \).

When the probability density function of the average fault interval is \( f_1(t_1) \), the average fault interval time of the equipment is shown in Equation (5).

\[
T_1 = \int_0^{+\infty} t_1 f_1(t_1) dt_1
\]  

(5)

When the probability density function of the average repair time after equipment failure is \( f_2(t_2) \), the average repair time after the equipment breakdown is shown in Equation (6).

\[
T_2 = \int_0^{+\infty} t_2 f_2(t_2) dt_2
\]  

(6)

The average serviceability of the equipment can be expressed as Equation (7).
2.3 Confidence Values of the Readiness Rate

Suppose the standard for the equipment integrity of a basic unit is $\eta$ and the number of units equipped with a certain type of equipment is $N$. Then the threshold for the number of units equipped with a certain type of equipment to meet the requirement of the basic unit is shown in Equation (8).

$$N' = \lceil \eta N \rceil$$

In Equation (8), the expression $\lceil \eta N \rceil$ is taken up and adjusted.

The equipment integrity rate is the percentage of equipment in the equipment group that is in good condition at a given point in time, but it is changing over time and the concept of confidence is used to measure the likelihood that the equipment will reach the required standard at any time.

The confidence function of a type of equipment provided by a basic unit to reach the standard of integrity is shown in Equation (9).

$$\alpha = \sum_{x=N'}^{N} C_N \left[ \frac{\int_0^{+\infty} t_1 f_1(t_1) dt_1}{\int_0^{+\infty} t_1 f_1(t_1) dt_1 + \int_0^{+\infty} t_2 f_2(t_2) dt_2 + (1-P)T_3} \right] ^x \times \left[ \frac{\int_0^{+\infty} t_1 f_1(t_1) dt_1}{\int_0^{+\infty} t_1 f_1(t_1) dt_1 + \int_0^{+\infty} t_2 f_2(t_2) dt_2 + (1-P)T_3} \right] ^{N-x}$$

3. Objective Function

Suppose the minimum limit for the reliability of a type of equipment provided by a base unit to reach the standard of integrity is $\alpha_0$, the minimum limit for the average serviceability of a single piece of equipment is $A_0$, the minimum limit for the support degree for the maintenance of spare parts is $P_3$. In addition, the number of maintenance spare parts in stock of a base unit must be an integer greater than or equal to 0.

Considering the military and economic benefits of spare parts, the corresponding inventory scheme is the optimal one when the maintenance spare parts support cost reaches the minimum under the above constraints.

Therefore, its target function can be expressed as Equation (10).

$$\min \ C(X) = C_1(X) + C_2(X) + C_3(X)$$
4. Analysis of an Example

The number of some type of equipment in a unit is 120, the average spare parts quantity requirement of one piece of equipment within two years obeys the gamma distribution. The shape parameter of the gamma distribution is \( \alpha = 3 \) and the scale parameter of the gamma distribution is \( \beta = 2 \). The unit price of the spare parts is 1000, the average cost of the spare parts inventory is 150, the average cost of the spare parts backlog loss is 200, and the average cost of the spare parts shortage loss is 1200. The probability density function of the average fault interval is defined as

\[
10 \frac{1}{\sqrt{10\pi}} e^{-\frac{(t_1-2500)^2}{10}} dt_1,
\]

the probability density function of the average repair time after equipment failure is defined as

\[
20 \frac{1}{\sqrt{20\pi}} e^{-\frac{(t_2-200)^2}{20}} dt_2,
\]

the minimum support level of maintenance spare parts is 92 percent, the average time taken for urgent procurement due to the maintenance spare parts shortage is 190 hours, the equipment integrity rating is above 90 percent, the equipment reliability threshold is 97 percent and the average serviceability threshold is 95 percent. How much of this kind of spare parts can be stored in the base unit warehouse to make the total guaranteed cost minimum within two years?

According to the meaning, we know that the average equipment demand for spare parts distribution is

\[
f(r) = \frac{2}{\Gamma(3)} r^2 e^{-2r} dr
\]

Then the demand for spare parts for 120 pieces of equipment must also be subject to gamma distribution and the distribution function is
According to Equation (8), the threshold for the number of units equipped with a certain type of equipment to meet the requirement of the basic unit is

\[ N' = \left[ 90\% \times 120 \right] = 108 \]

Based on Equation (10), the specific values of the parameters are added and the following objective function is

\[
\text{min } 1150X + 200 \int_{0}^{X} (X - r) \frac{2^3}{\Gamma(360)} r^{360-1} e^{-2r} dr + 1200 \int_{X}^{\infty} (r - X) \frac{2^3}{\Gamma(360)} r^{360-1} e^{-2r} dr
\]

\[
s.t. \quad \int_{0}^{X} \frac{2^3}{\Gamma(360)} r^{360-1} e^{-2r} dr \geq 92\%
\]

\[
\frac{2500}{2500 + 200 + 190(1 - P)} \geq 95\%
\]

\[
\sum_{x=108}^{120} C_{120}^{x} \left[ \frac{2500}{2500 + 200 + 190(1 - P)} \right] \left[ \frac{2500}{2500 + 200 + 190(1 - P)} \right]^{x-1} \geq 97\%
\]

\[ X \geq 0 \text{ and } X \in N \]

Matlab software programming based on computer simulation is used to solve total support cost and the total support cost of this kind of spare parts is minimal within two years. So the optimal quantity of spare parts is 197.

5. Conclusion

Through making an analysis of the randomicity of the spare parts demand, probability theory and mathematical statistics is applied to establish the support expense minimum model based on equipment availability, confidence values of the equipment readiness rate and support probability of spare parts. The arithmetic of spare parts support storage based on expense is presented. Then inventory optimization method of spare parts support based on computer simulation is determined. Applicability of the method is given by way of a numerical example. The inventory optimization method based on computer simulation provides a theoretical basis for solving the problems of inventory and expense in other relative areas.

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