Investment Decision Model Based on Multi-objective Programming

Yukun Lu[1], Yuqian Hu[2], Chen Ai[3]

[1]College of Mathematics Science, Huaqiao University, Quanzhou, Fujian, 362021

[2]Ant Apartment in Nanli Street, Huaibei, Anhui, 235000

[3]College of Science, Nanchang University Nanchang, Jiangxi, 330002

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Abstract: The design of a portfolio investment strategy for multiple venture capital and a risk-free asset in the market needs to consider two objectives: the overall return is as large as possible and the overall risk is as small as possible. However, the two objectives are not mutually reinforcing. In a certain sense, they are opposite. In this paper, the multi-objective decision-making method is used to establish the model, and the investment benefit is the goal. The optimization model is established for the investment problem. Different investment methods have different risks and benefits. According to the principle of the optimization model, the model proposes two criteria, so that the economic benefits are as large as possible and the risks are as small as possible under certain conditions of investment. At the same time, a linear programming model for portfolio investment scheme design is given. The main idea is to integrate two design goals through linear weighting: assuming that the transaction fee function is approximately linearized on the basis of considerable investment scale, and the risk function is solved by decision variables.

1. Introduction

With the development of the economy, people's living standards are getting higher and higher, so there are surplus funds. In order to make the remaining funds get more profits, people will invest the remaining funds. There are n kinds of assets in the market for investors to choose. A company with a considerable amount of money in M can be used as a period of investment. The company's financial analysts evaluated the n assets and needed to design a portfolio plan for the company, using a given amount of money M, selectively purchasing several assets or depositing bank interest, making the net income as large as possible. And the overall risk is as small as possible.

Due to the uncertainty of the expected return of the asset, its risk characteristics, so that the investor's net income is as large as possible, and the risk loss is as small as possible, the first solution is to carry out the investment portfolio, diversify the risk, and expect to obtain High returns, the goal of building a model is to solve the optimal portfolio. At the same time, the optimal investment is also determined by the individual factors, that is, the investor's preference for risk and return, and how to solve the relationship between the two is also an important problem to be solved by the model.

2. Establishment of Multi-objective Decision Model

2.1 Investment risk income calculation

This paper uses a certain enterprise data, calculation and analysis to obtain a investment plan that is as satisfactory as possible, and promotes it to the general situation and carries out verification. The following are two situations that should be considered in practice: in the case of certain risks, the maximum benefit is obtained. In the case of a certain income, the risk is minimal. Different investors have different emphasis on interests and risks, and will be regarded as normal within a certain scope. Therefore, it is only necessary to give a model that is as good as possible, that is, the risk is as small as possible and the income is as large as possible.

The company can invest in five kinds of projects, in which the bank is risk-free, and the return r0=5% is fixed. It will not change during the investment period. Other investment projects have certain risks, but their income may be greater than the bank. Interest rate, we intend to establish a model that is applicable to the average investor, and propose multiple investment plans for various types of people according to their risk tolerance (general investors are divided into: risky and conservative) The more risky the person is, the more he is able to withstand the risk loss.

The net income of the investor is the purchase of various assets and the bank's revenue minus the transaction costs in this process. When investing in asset Si, for the difference in the amount of investment xi, the transaction fee paid is also not paid when the investment is not synchronized. When the investment amount is greater than ui, the transaction fee is xi, remember:

$$\varphi_{\mathrm{i}} = \begin{cases} 0, & \mathrm{x_i} = 0; \\ \mathrm{u_i} & 0 < \mathrm{r}_i < x_i; \\ x_i & x_i > u_i \end{cases}$$

For the largest ui, u2=198<200 yuan, and M is known to be a considerable amount of money, and the transaction rate pi is also small. Even when xi<ui, the transaction fee is calculated by ui. It is almost the same as using xi to directly calculate the transaction fee. Therefore, in order to simplify the ui constraint, the transaction fee is calculated by replacing ui with xi.

Set purchase S_i is the amount x_i transaction fee paid $c_i(x_i)$ for $c_0(x_0)$=0.

$$c_i(x_i) = \begin{cases} 0 x_i = 0 \\ p_i u_i 0 < x_i < u_i (i = 1 \sim n) \\ p_i x_i x_i \geq u_i \end{cases}$$

Because the investment amount m is quite large, it can always be assumed for each S_i investment $x_i \geq u_i$, at this time, the above formula can be reduced to:

$$c_i(x_i) = p_i x_i (i = 1 \sim n)$$

Net income from investment in Si:

$$R_i(x_i) = r_i x_i - c_i(x_i) = (r_i - p_i) x_i$$

Correct S_i is risk of investment:

$$Q_i(x_i) = q_i x_i$$

When buying S_i the amount is x_i (i=0~n), portfolio x=(x_0, x_1,……, x_n) are total net income:

$$R(x) = \sum_{i=0}^{n} R_i(x_i)$$

2.2 Multi-objective mathematical programming model

In order to make the total net income R(x) as large as possible and the overall risk Q(x) as small as possible, the mathematical model of the problem can be planned as a multi-objective programming model, namely:

$$\begin{cases} \max R(x) \\ \min Q(x) \\ s.t F(x) = M \\ x \geq 0 \end{cases}$$

Assume the average risk level of the investment $\bar{\mathrm{q}}$, if the overall risk Q(x) is required to be limited to risk k, ie Q(x)<=k, the above formula can be converted into:

$$\begin{cases} \max R(x) \\ s.t Q(x) \le k \\ F(x) = M \\ x \ge 0 \end{cases}$$

2.3 Model solving

According to the multi-objective programming theory, the necessary condition for the non-inferior solution of the model (Kuhn-Tucker condition) is that λ_1, λ_2, μ>0 makes:

$$\begin{cases} \lambda_1 \nabla R(x) + \lambda_2(-\nabla Q(x)) + \mu(F(x) - M) = 0 \\ \mu(F(x) - M) = 0, x \ge 0 \end{cases}$$

Due to the constraint q(x) in the model $\le$k, ie:

$$\max Q_i(x_i) \le k$$

So this constraint can be converted to:

$$Q_i(x_i) \le k(i = 1 \sim n)$$

Eventually it can be transformed into the following linear plan:

$$\begin{cases} \max \sum_{i=0}^{n} (r_i - p_i) x_i \\ s.t \sum_{i=0}^{n} (1 + p_i) x_i = M \\ q_i x_i \le k(i = 1 \sim n) \\ x \ge 0 \end{cases}$$

For specific calculation, let m=1, then (1+ p_i) x_i can be regarded as an investment S_i proportion.

3. Linear Programming Model

After investing in various assets, not only should the income be as large as possible, but the overall risk should be as small as possible, so the objective function should be two functions of income and risk, due to the average rate of return and risk of various assets in the general time. The loss rate is known, so a mathematical model can be built:

$$\text{Goal 1: max } f_i = \sum_{i=1}^{n+1} (r_i X_i - Y_i)$$

$$\text{Goal 2: min} f_i = \max_{1 \le i \le n} (q_i X_i)$$

$$\text{s.t.: } \sum_{i=1}^{n+1} (X_i + Y_i) = 1$$

This is a multi-objective nonlinear mathematical programming model, and f_i is not a continuous function of xi, the optimization is difficult, and the linear weighting below transforms it into a linear programming model:

$$\text{Total objective function: min } f = \lambda f_2 + (1 - \lambda) \quad (-f_1)$$

λ reflecting the subjective factors of investors in venture capital, λ is the smaller the investment, the more risky the investment, when λ=0 means that regardless of the risk, regardless of the risk, such a person is likely to get the most benefit; λ=1 means that regardless of the risk regardless of the income, such people will deposit all the funds into the bank.

Optimized solution because the objective function optimizes $f\max_{1\le i\le n}(q_i X_i)$ achieve X_{n+2} to get the final linear programming model:

$$\mathrm{Min}\, f = (1-\lambda)\sum_{i=1}^{n+1}(p_i - r_i)\, X_i + \lambda X_{n+2}$$

$$\text{s. t}\quad \begin{cases} \sum_{i=1}^{n+1}(1+p_i)\, X_i = 1, & \\ q_i X_i - X_{n+2} \le 0, & i = 1,2,3\ldots n, \\ X_i \ge 0, & i = 1,2,3\ldots n+2 \end{cases}$$

4. Model Results

The risk-benefit diagram for n=4 is as follows:

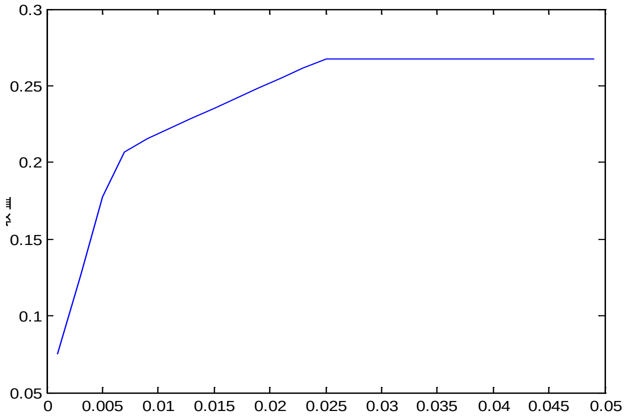

Figure 1. Risk-benefit results

It can be seen from Fig. 1 that the profit y increases with the increase of the risk upper limit k, and the growth rate is the fastest around 0~0.007, and then the growth rate becomes slow. The benefits are discrete with respect to risk. With the increase of the investment risk ceiling k, the income y gradually increases. Investors can choose the k and y that meet the requirements according to their own preferences, and invest in effective portfolio investment. Considering that y should be as large as possible, k should be as small as possible. At the same time, the risk-return curve is analyzed. When the income increases with the increase of risk, this is because as the risk increases, the income gradually increases, people's aversion to risk slows down, and investors gradually move toward risk. When the rising curve is gradually gradual, this is because when the risk is large enough, the risk-reward assets are already invested, and the returns are not changed much.

The function of the net income and the risk-related weighting factor is plotted to obtain the following figure:

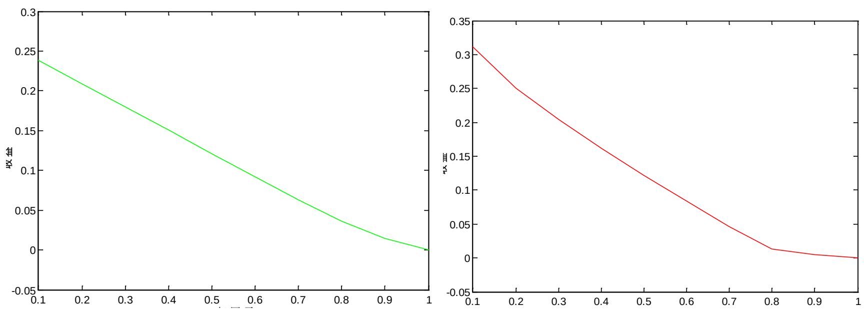

Figure 2. Function of net income and risk related weight factors

From the results in Figure 2, the net income and risk are both monotonic decreasing functions of k (weight factor), which means that the stronger the degree of caution, the smaller the risk, but the smaller the benefit, the clear practical significance. As can be seen from Figure 3 below, the more detailed calculation results are obtained. In this paper, the optimal investment portfolio and the effective investment curve are obtained by using 300 equal points in k=0~1. Any point on this curve indicates the maximum possible return for this level of risk and the minimum risk required for that benefit. In fact, it is found that their effective investment curves are discrete because they correspond to the maximum benefit in the case of slower risk growth and can be considered as the optimal solution in the general sense.

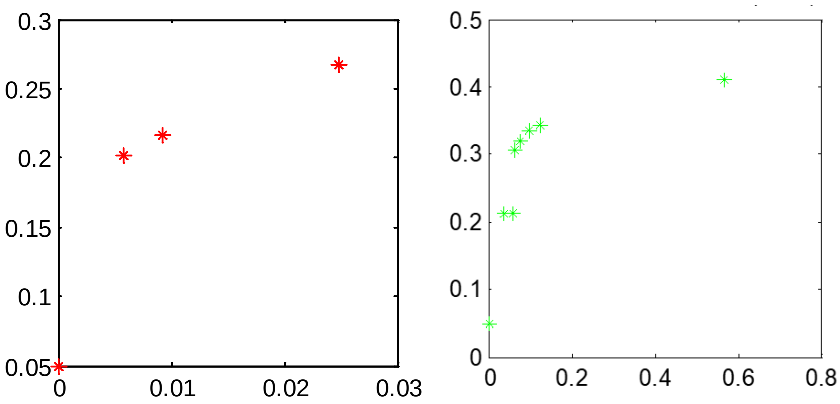

Figure 3. Optimal portfolio

5. Conclusion

By transforming the risk function into inequality constraints, this paper establishes a linear programming model, directly uses the program to calculate, and obtains an optimal decision-making scheme, and gives an effective investment curve. According to the subjective preference of investors, the investment direction is selected. Using the linear programming model, the multi-objective plan is transformed into a single-objective plan, and the upper risk limit is selected to determine the return. According to the income risk map, investors can choose the investment direction according to their own preferences. When the linear weighted model is used to solve the problem, the calculation process is stable and fast. The different weight factors are compared to obtain the optimal decision-making scheme. The effective investment curve is given. According to the subjective preference of investors, the investment direction is selected.

References

[1] Fan Zhengsen, Xie Zhaohong. Mathematical Modeling Technology [M]. Beijing: China Water Resources and Hydropower Press, 2003.

[2] Wang Minsheng, Wang Geng. Modern Mathematical Modeling Method [M]. Beijing: Science Publishing, 2006.

[3] Feng Jie, Huang Liwei. Principles and Cases of Mathematical Modeling [M]. Beijing: Science Press, 2007.

[4] Li Zhilin, Ou Yigui. Mathematical Modeling and Typical Case Analysis [M]. Beijing: Chemical Industry Press, 2006.

[5] Wang Junqiang, Sun Shudong, Yu Xiaoyi, Yang Zhan. Research on New Operational Logic of Product Portfolio Optimization Based on Constraint Theory [J]. Computer Integrated Manufacturing System. 2007(05).

[6] Yin Jianhua.Two-stage portfolio optimization unified model [J]. Foreign investment in China. 2013(16).

[7] Wang Yuping, Xu Chen. Convergence of a new method for solving combinatorial optimization [J]. Journal of Xidian University. 1997(04).

[8] Yue Minyi. Introduction to portfolio optimization [J]. Journal of Operations Research. 1988 (01).

[9] Yu Jing, Zhao Min, Chen Yan. Analysis of decision makers' attitudes under the framework of conflict analysis graph model [J]. Soft Science. 2015(09).

[10] Duan Wenying, Yue Qi. Research on Simulated Annealing Solution Combination Optimization Problem [J]. Forest Engineering. 2004 (04).